Magnification of Order of Square Matrices

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Abstract: *The study introduces an operation for magnification of order of square matrices and some elementary results from this operator (* $\mathcal{M}$ *). Some topological results are also included. During the study of* $GL_n(F)$ *and* $SL_n(F)$*, operator* $\mathcal{M}$ *shows some interesting results. In this study, failure of* $\mathcal{M}$ *to solve system of equations is observed. Given operator shows isomorphic and topological behavior. As Determinant and Trace are special results of Matrix Theory, using this operator, there are some specific efforts to make comfortable results of Trace of Matrix and Determinant of Matrix.*

Keywords: *Magnification of Order of Matrix, Operator on Matrix, System of Equations, Special Trace, Special Determinant. Magnification of* $GL_n(F)$*, Magnification of* $SL_n(F)$ *.*

1. Introduction

Mathematically this function is itself uniquely defined.For any matrix, trace and its determinant are two most important parts. Here an operator $\mathcal{M}$ defined as in next sectionis an interesting way to compute trace and determinant after some results. An operator $\mathcal{M}$on n $\times$ n square matrix with the properties like linearity, homomorphism and isomorphism is also observed for this operator. Magnifying an object gives clearance to study the properties of that object neatly. Given operator maximizes our square matrixand we can evaluate its determinant and trace. By deep analysis I got a very good result which is helpful to give a constant prediction for determinant

i.e. det(A)=0 for A$\in \mathbb{M}_2(\mathcal{K})$,

Where$\mathcal{K}$=real number system or complex number system.

2. Basic Definitions:

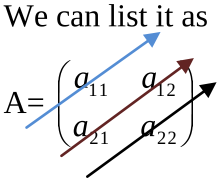
Definition of $\mathcal{M}$:

Let A=$(\boldsymbol{a_{ij}})_{\mathbf{2\times 2}}$ any matrix. Magnification of a square matrix A can be defined by following rule:

If

$$A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

We can list it as

A=


Then list it like after applying an magnifying operator $\mathcal{M}$on given square matrix A, we get

$$\mathcal{M}(A)=\begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{12} & 0 \\ a_{22} & 0 & 0 \end{pmatrix}$$

Again apply this operator on $\mathcal{M}$(A), we get

$$\mathcal{M}^2(A) = \begin{pmatrix} a_{11} & 0 & 0 & 0 & 0 \\ a_{21} & 0 & 0 & 0 & 0 \\ a_{22} & a_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

And so on.

3. Results

Take A,B∈M_2(K); K is some field.

i. If o(A) = n×n then we have o($\mathcal{M}$(A))= (2n-1) × (2n-1).

Proof: By Principal of Mathematical Induction, we can have that given result is true for n=1 and suppose it is true for n=k then we can have result is true for n=k+1.Thus in general for any n∈ ℕ our given result is true.

ii. $\mathcal{M}$(A+B)= $\mathcal{M}$(A)+ $\mathcal{M}$(B)

Proof: Take n=2,we have

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

$$\text{Then } A+B = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{pmatrix}.$$

Applying $\mathcal{M}$ both the sides, we get

$$\mathcal{M}(A+B) = \mathcal{M}(A) + \mathcal{M}(B) .$$

Thus given result is true for n=2.

Note: Using P.M.I, we can prove above result for any real numbers n.

iii. $\mathcal{M}$(A.B) ≠ $\mathcal{M}$(B.A)

Proof: Result can be prove from multiplicative property of matrix.

iv. $\mathcal{M}$(C.A) =C. $\mathcal{M}$(A); C is some constant.

Proof: Result can be prove from scalar multiplicative property of matrix.

v. $\mathcal{M}$(A.B) $\neq$ $\mathcal{M}$(A). $\mathcal{M}$(B)

Proof: Result can be prove from matrix multiplicative property of matrix.

vi. tr($\mathcal{M}$(A)) $\neq$ (tr(A))

Proof: Result can be prove from scalar multiplicative property of matrix.

vii. det(A) $\neq$ 0 but det($\mathcal{M}$(A)) = 0 always.

Proof:

viii. $\mathcal{M}$is an function. $\mathcal{M}$: $M_n(K) \rightarrow M_{(2n-1)}(K)$,

$Where\ K = \mathbb{R}, \mathbb{C}, \mathbb{K}.$

ix. $\mathcal{M}$(A) gives bijective i.e., injective as well as surjective properties.

4. Conclusion

Here Result 1 to Result 9 are itself some observations while magnifying provided matrix in previous way. It gives new way to think about magnification of order of matrix by choosing different pattern to magnify by listing members of experimental matrix. Listed results can be use to upgrade present cryptosystems. This results are not giving any proper conclusion to solve the system of equations. In the next study this concept of magnification will be upgrade by replacing 0 with n (i.e., a real number). This study wants to make given operator in such a way by which hereditary property for trace and determinant of matrix can be carry on after applying magnification operator. Some result of Magnification of GLn(F), Magnification of SLn(F) can be develop in this study.

5. Reference

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