Enrollment No.________________

NAVRACHANA UNIVERSITY
a UGC recognized University

School: School of Science
Program/s: BSC Chemistry (CPM)
Year: 2nd **Semester:** 3rd
Examination: End Semester Examination
Examination year: December - 2021

Course Code: MA150 **Course Name:** **Abstract Algebra – I**
Date: 07/12/2021 **Total Marks:** 40
Time: 8:30 am to 10:30 am **Total Pages:** 2

Instructions:

- ➔ *Write each answer on a new page.*
- ➔ *Use of a calculator is permitted.*
- ➔ *Draw all relevant waveforms in answer sheet only.*
- ➔ ** COs=Course Outcome mapping. # BTL=Bloom's Taxonomy Level mapping*

Section – I

Q. No.	Details	Marks	COs*	BTL#
Q.1		[10]		
i)	For a commutative group G, $ab =$ ________ $\forall a, b \in G$.			
ii)	The set of all 2x2 matrices over $\mathbb{Z}$, is a ring with respect to multiplication. True/False			
iii)	The subgroup of a group contains the identity element $e \in G$. True/False		CO1, CO4	1, 2, 3, 5
iv)	Define order of an element $a \in G$			
v)	Define a group and give an example of a group.			
vi)	If $G = (\mathbb{Z}: +)$, $H = 3\mathbb{Z}$, then form all the cosets of H in G. How many distinct cosets will you get?			
Q. 2		[10]		
i)	Define normal subgroup of a group G. What is the group of all cosets of a normal subgroup known as?			
ii)	Show that the mapping $\phi: G \to G'$ defined by $\phi(x) = e^x$, where $G = (\mathbb{R}, +)$ & $G' = (\mathbb{R}, .)$ is an isomorphism.		CO1, CO3, CO4	1, 2, 4, 3, 5
iii)	Define Kernel of a mapping. Prove ϕ is a 1-1 homomorphism iff $Ker\{\phi\} = \{e\}$. **P. T. O.**			

Section - II				
Q. 3	**Attempt any 5 from the following:**	**[5]**		
i)	Consider a group $G = \{1, -1, i, -i\}$ with operation of complex multiplication. What is the inverse of an element $-i$?		CO1, CO2	1, 2, 3, 5
ii)	Which two properties are enough to show that a non-empty set H is a subgroup of a group G?			
iii)	What is the order of the permutation $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 5 & 4 & 6 & 3 \end{bmatrix}$?			
iv)	Define a group homomorphism.			
v)	Write a statement of the Cayley's theorem.			
vi)	Explain in your words, how symmetric groups S_n are useful in the field of Chemistry.			
Q. 4	**Attempt any 5 from the following:**	**[15]**		
i)	Define the group $U(n)$. Write the elements of this group for $n = 15$. Is it a cyclic group?		CO1, CO2, CO3	1, 2, 4, 3, 5
ii)	Define a permutation group. Which operation is defined on a permutation group? Write any one example of a permutation group.			
iii)	What is a symmetric group S_4? What does it represent geometrically? Is it an abelian group? What is order of a symmetric group S_4?			
iv)	Let G be the group of nonzero real numbers under multiplication. Check whether the function from G to G defined by function $\phi(x) = \|x\|$ is homomorphism or not.			
v)	Consider following two permutations: $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{bmatrix}$. Compute $\alpha\beta$ and $\beta\alpha$.			
vi)	Consider a permutations $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 & 1 & 7 & 8 & 6 \end{bmatrix}$ Write it as a product of disjoint cycles and as a product of 2-cycles. Also determine its order.			

************End of Question Paper************