Student ID: ____________

NAVRACHANA UNIVERSITY
SLSE, BSc PROGRAME
END SEMESTER EXAMINATION
1st Year, Semester - I

Academic Year 2017 – 2018

Subject: DIFFERENTIAL CALCULUS(MAJOR/MINOR)

Course Code: MA123/125 **Marks: 40**

Date: 30/11/2017 **Time: 10:30AM – 11:30PM**

Instructions:

➔ **Calculator is permitted.**

➔ **Write answers in answer book only.**

Q-1 Answer the following questions **(2x4 =08 marks)**

1. The equation of the Tangent line to the curve $y = x^3 - 3x + 2$ at the point $(2,4)$ is

a. $9x - y + 14 = 0$ b. $9x - y - 14 = 0$

c. $9x + y + 14 = 0$ d. $9x + y - 14 = 0$ e.None of the given

2. The limit of the given function $\lim_{x\to 0} x^x$ is

a. -1 b. ∞ c. 0 d. x e.None of the given

3. If $x = 2cost - cos2t$, $y = 2sint - sin2t$; then the value of $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{2}$ is

a. -2 b. 2 c. $3/2$ d. $-3/2$ e.None of the given

4. The n^{th} derivative of $\frac{1}{1-5x+6x^2}$ is

a. $(n!)\left[\left(\frac{3}{1-3x}\right)^{n+1} - \left(\frac{2}{1-2x}\right)^{n+1}\right]$ b. $(n!)\left[\left(\frac{3}{1-3x}\right)^{n+1} + \left(\frac{2}{1-2x}\right)^{n+1}\right]$

c. $(n!)\left[\left(\frac{4}{1-3x}\right)^{n+1} - \left(\frac{3}{1-2x}\right)^{n+1}\right]$ c. $(n!)\left[\left(\frac{4}{1-3x}\right)^{n+1} + \left(\frac{3}{1-2x}\right)^{n+1}\right]$ e.None of the given

Q-2 Answer the following questions (*Any Four*) **(4X4=16 marks)**

1. State Lagrange's Mean Value Theorem and examine the validity of the hypotheses and the conclusion of Roll's theorem for the function

$f(x) = 1 - (x-1)^{\frac{2}{3}}$ on $[0,2]$.

2. Evaluate: $\lim_{x\to 4}\left[\frac{1}{log(x-3)} - \frac{1}{x-4}\right]$.
3. Find the n^{th} derivative of $sin^4 x$.
4. Find the points on the curve $y = x^3 - 3x + 5$ at which the tangent line:
   (a) is parallel to the straight line $y = -2x$;
   (b) is perpendicular to the straight line $y = -\frac{x}{9}$;
5. Expand the function $f(x) = sin^2 x - x^2 e^{-x}$ in positive integral powers of x up to the terms of the fourth order of smallness with respect to x.
6. State Taylor's expansion series for expansion of $f(x)$ and use it to find the expansion of $sinx$.

Q-3 Answer the following questions (*Any Two*) **(6X2= 12 marks)**

1. Find the n^{th} derivative of $\frac{x^2+4x+1}{x^3+2x^2-x-2}$.
2. If u and v are two functions of x having derivatives of the n^{th} order , then prove that
$(u.v)_n = u_n.v + \binom{n}{1}u_{n-1}v_1 + \binom{n}{2}u_{n-2}v_2 + \cdots + \binom{n}{r}u_{n-r}v_r + \cdots + \binom{n}{n}u\, v_n$.
3. If $y = sin(msin^{-1}x)$, prove that
$(1-x^2)y_{n+2} = (2n+1)xy_{n+1} + (n^2-m^2)y_n$.

Q-4 Answer the following question: **(4X1= 04 marks)**

1. The registrar of Kellogg University estimates that the total student enrollment in the Continuing Education division will be given by

$N(t) = \frac{-20{,}000}{\sqrt{1+0.2t}} + 21{,}000$

Where, $N(t)$ denotes the number of students enrolled in the division t yr from now. Using the Taylor series of $N(t)$ at $t = 0$, find an approximation of the average enrollment at Kellogg University between $t = 0$ and $t = 2$.