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Accurate numerical modeling for functionally graded (FG) cylinders of finite length subjected to thermo mechanical load

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A simplified and accurate analytical cum numerical model is presented here to investigate the behavior of FG cylinders of finite length subjected to thermo mechanical load. A diaphragm supported FG cylinder under symmetric thermal and mechanical load which is considered as a two dimensional (2D) plane strain problem of thermoelasticity in (r, z) direction. The boundary conditions are satisfied exactly in axial direction (z) by taking an analytical expression in terms of Fourier series expansion. Fundamental (basic) dependent variables are chosen in the radial coordinate of the cylinder. First order simultaneous ordinary differential equations are obtained as mathematical model which are integrated through an effective numerical integration technique by first transforming the BVP into a set of initial value problems (IVPs). For FG cylinders, the material properties have power law dependence in the radial coordinate. Effect of non homogeneity parameters on the stresses and displacements of FG cylinder are studied. The numerical results obtained are also first validated with existing literature for their accuracy. Stresses and displacements in axial and radial directions in cylinders having various l/r_i and r_o/r_i ratios parameter are presented for future reference.

KEYWORDS: Numerical integration; functionally graded materials; boundary value problem; thick cylinder.

The demand for improved structural efficiency in space structures and nuclear reactors has resulted in the development of a new class of materials, called functionally graded materials (FGMs). FGMs have become one of the major research topics in the mechanics of materials community during the past fifteen years. The concept of FGMs was proposed in 1984 by materials' scientists in the Sendai (Japan) area as a means of preparing thermal barrier materials[1]. Continuous changes in the composition, microstructure, porosity, etc. of these materials result in gradients in properties such as mechanical strength and thermal conductivity. Thus, FGMs are heterogeneous materials, characterized by spatially variable microstructures, and thus spatially variable macroscopic properties are introduced to enhance material or structural performance. Particularly, material properties can be designed to vary continuously along structural geometry to prevent delamination and stress concentration in traditional multilayered structures. The basic concept is to mix ceramic and metal such that the material properties continuously vary from one constituent material to the other. The spatially variable material properties make FGMs challenging to analyze. Before these material devices are used in engineering design, it is very important that these are analyzed very accurately. For such a reason, present study focuses the analysis of FG cylinders using the exact approach. The uniqueness of this approach is: it first requires algebraic manipulation of basic elasticity equations like equilibrium, strain displacement and constitute equations. After this manipulation, this becomes the two point boundary value problem which governs the behavior of finite length cylinder which is plane strain