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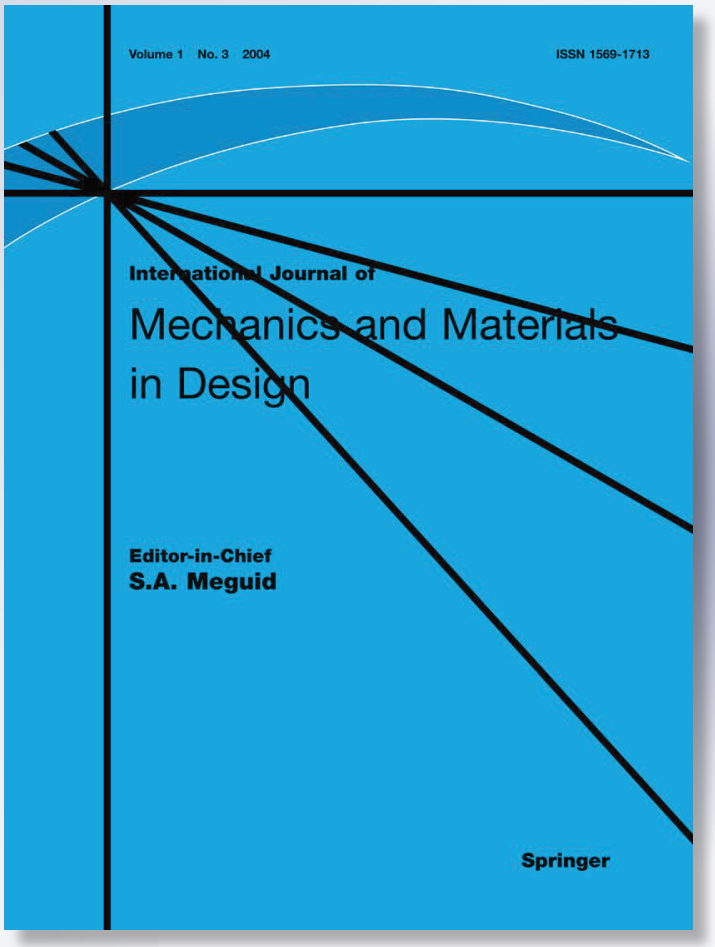

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A mixed semi analytical solution for functionally graded (FG) finite length cylinders of orthotropic materials subjected to thermal load

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Abstract A simplified and accurate analytical cum numerical model is presented here to investigate the behavior of functionally graded (FG) cylinders of finite length subjected to thermal load. A diaphragm supported FG cylinder under symmetric thermal load which is considered as a two dimensional (2D) plane strain problem of thermoelasticity in (r, z) direction. The boundary conditions are satisfied exactly in axial direction (z) by taking an analytical expression in terms of Fourier series expansion. Fundamental (basic) dependent variables are chosen in the radial coordinate of the cylinder. First order simultaneous ordinary differential equations are obtained as mathematical model which are integrated through an effective numerical integration technique by first transforming the boundary value problem into a set of initial value problems. For FG cylinders, the material properties have power law dependence in the radial coordinate. Effect of non homogeneity parameters and orthotropy of the materials on the stresses and displacements of FG cylinder are studied. The numerical results obtained are also first validated with existing literature for their accuracy. Stresses and displacements in axial and radial directions in cylinders having various l/r_{i} and $r_{\mathrm{o}}/r_{\mathrm{i}}$ ratios parameter are presented for future reference.

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Keywords Functionally graded materials · Numerical Integration · Boundary value problems · Thick cylinder

List of symbols

r, θ, z	Cylindrical coordinates
u, v, w	Displacement components
σ_r, σ_θ, σ_z	Normal stress components on planes normal to r, θ, and z axis
τ_{zr}	Shearing stress component in cylindrical coordinates
ε_r, ε_θ, ε_z	Unit elongations (normal strain) components in cylindrical coordinates
γ_{zr}	Shearing strain component in cylindrical coordinates
C_{ij}	Material constants for orthotropic materials
α_i	Coefficient of thermal expansion per degree centigrade for orthotropic materials
T	Temperature rise at any point in a cylinder
v	Poisson's ratio
r_{i}	Inner radius of the cylinder
r_{o}	Outer radius of the cylinder
l	Length of the cylinder
T_{m}	Initial reference temperature
$\bar{u}, \bar{w}$	Nondimensionalized displacement components
$\overline{\sigma_r}, \overline{\sigma_\theta}, \overline{\sigma_z}$	Nondimensionalized normal stress components
$\overline{\tau_{rz}}$	Nondimensionalized shearing stress component in cylindrical coordinates