Student ID: ______________

NAVRACHANA UNIVERSITY
SLSE, BSc PROGRAME
END SEMESTER EXAMINATION
3rd Year, Semester - V
Academic Year 2017 – 2018

Subject: LINEAR ALGEBRA **Marks: 40**
Course Code: MA302 **Time: 01:00PM – 03:00PM**
Date: 20/11/2017

Instructions:
- ➔ Calculator is permitted.
- ➔ Write answers in answer book only.

Q-1: Answer the following questions: ....2x2 =04

1. Let $T: V_4 \to V_3$ be a linear map defined by $(e_1) = (1,1,1), T(e_2) = (1,-1,1), T(e_3) = (1,0,0)$, $T(e_4) = (1,0,1). Then$ verify that $r(T) + n(T) = \dim(V_4)$.
2. True of False: "No Linear Transformation from V_4 to V_2 can be one-one." . Why?

Q-2: Answer the following questions: (Any Three)..................3x4 =12

1. Define and find the Range and Kernel of given maps.

   $\mathrm{D}: \mathcal{C}^1(\mathrm{a},\mathrm{b}) \to \mathcal{C}(\mathrm{a},\mathrm{b})$ defined by $\mathrm{D(f)} = \mathrm{f}'$.

2. Prove that the linear map $\mathrm{T}: \mathrm{V}_3 \to \mathrm{V}_3$ defined by

   $$\mathrm{T(e_1)} = \mathrm{e_1} - \mathrm{e_2}, \mathrm{T(e_2)} = 2\mathrm{e_2} + \mathrm{e_3}, \quad \mathrm{T(e_3)} = \mathrm{e_1} + \mathrm{e_2} + \mathrm{e_3}$$

   is neither one-one nor onto .
3. Let T: $\mathrm{U} \to \mathrm{V}$ be a linear map then R(T) is a subspace of V.
4. If T: $\mathrm{U} \to \mathrm{V}$ be a linear map and dim U=dim V=p. Then show that T is onto $\Leftrightarrow$ n(T)=0. 1.
5. Check the linearity of the following maps.

   (a) T:$V_1 \to V_3$ defined by $T(x) = (x, 2x, 3x)$
   (b) $T: \mathcal{C}[0,1] \to V_2$ defined by $T(f) = (f(0), f(1))$.

Q-3: Answer the following questions: (Any Three).... ....3x8=24 Marks

1. Let T: $U \to V$ be a linear map. Then
   (a) If T is one-one and $u_1, u_2, \dots, u_n$ are LI vectors of , then $T(u_1), T(u_2), \dots, T(u_n)$ are LI .
   (b) If $v_1, v_2, \dots, v_n$ are LI vectors of $R(T)$ and $u_1, u_2, \dots, u_n$ are vectors of U such that $(u_1) = v_1, T(u_2) = v_2, \dots, T(u_n) = v_n$, then $u_1, u_2, \dots, u_n$ are LI.
2. Prove if T: $U \to V$ be a nonsingular linear map. Then T^{-1} : $\mathrm{V} \to U$ is a linear, one-one, onto map.
3. State and prove rank nullity theorem.
4. Determine whether there exists a linear map in the following cases, and where it does exist give the general formula.
   (a) $T: V_2 \to V_2$ such that $T(1,2) = (3,0)$ and $T(2,1) = (1,2)$.
   (b) $T: \mathcal{P}_4 \to \mathcal{P}_3$ such that $T(1+x) = 1$, $T(x) = 3$ and $T(x^2) = 4$.

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