An Algebraic Study of Generalized Spline Modules on Graphs over Commutative Rings with Identity

A

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7.7 Conclusions

Our work is concluded by developing an algorithm to construct the generalized spline rings for the special graphs such as the complete graphs, complete bipartite graphs and hypercubes.

We found an algorithm for writing the generating set which acts as a basis for the generalized spline modules for cycle graphs and for wheel graphs, taking the base ring as quotient ring of integers.

We gave basis criteria for $R_{(G,\alpha)}$ on edge labeled Dutch windmill graph and special cases of Dutch windmill graph such as Friendship graph and Butterfly graph which have common cut vertices with Cycle graph C_n and triangles respectively, over any GCD domain by using determinantal techniques[8] and flow-up bases.

We have given basis criteria for complete graph K_4 and for wheel graph W_4, which are isomorphic to each other over any GCD domain. These graphs have no common cut vertices with cycle graphs, diamond graphs and trees.We observed that graphs which are isomorphic to each other have same or equivalent basis criteria since zero trails of these graphs are same and Q_G is also same for these graphs. We generalize this result and prove that basis criteria for generalized spline modules on each graph of an arbitrary set of isomorphic graphs is same over any principal ideal domain.Depending upon the type of graph and the base ring, R we can easily use this result to find Q_G as well as basis criteria for generalized spline modules on graphs which are isomorphic to some graphs like cycle graphs,diamond graphs, trees and Dutch windmill graphs.

We extended this result to generalized spline modules on isomorphic trees over any GCD domain and constructed Flow up basis for generalized spline modules on a star graph. An algorithm is developed for indexing the vertices of ordered rooted trees which helps us to generalize the method of constructing flow-up basis for generalized spline modules on any ordered rooted tree and hence on a family of isomorphic trees over a GCD domain.

7.8 Future directions for further research and open questions

The graphs we have used in our research find important applications in network and approximation theory and the present work adds to the existing knowledge and understanding in these and related areas. Also, it opens a vast field for research as we can think

of studying the generalized spline moodules over these and other graphs by changing the base rings to other rings such as the polynomial rings and ring of Laurent polynomials. As these rings are PIDs, we can also try to find suitable bases for the generalized spline modules for these graphs.

We have basis criteria for generalized spline modules on arbitrary graphs over principal ideal domains.Further investigations on arbitrary graphs open a possibility of finding proof for general basis criteria for generalized spline modules on arbitrary graphs over any GCD domain.

Open Questions

- 1. Identify and study generalized spline modules on complete graphs,complete bipartite graphs,hypercube and cycle graph over polynomial rings and ring of Laurent polynomials.
- 2. Find proof of general basis criteria for generalized spline modules on arbitrary graphs over any GCD domain and give an algorithm to determine the entries of a flow-up class with the smallest leading entry on graphs like wheel graph,complete graph etc.