

Abstract

An edge labeled graph is a graph G whose edges are labeled with non-zero ideals of a commutative ring R . A Generalized Spline on an edge labeled graph G is a vertex labeling of G by elements of the ring R , such that the difference between any two adjacent vertex labels belongs to the ideal corresponding to the edge joining both the vertices. The set of generalized splines forms a sub ring of the product ring $R^{|V|}$, with respect to the operations of coordinate-wise addition and multiplication and also becomes a module over the ring R . This ring which is also a module is known as the generalized spline ring R_G , defined on the edge labeled graph G , for the commutative ring R . We have considered particular graphs such as complete graphs, complete bipartite graphs and hypercubes, labeling the edges with the non-zero ideals of an integral domain R and have identified the generalized spline ring R_G for these graphs. Also, general algorithms have been developed to find these splines for the above mentioned graphs, for any number of vertices and Python code has been written for finding these splines. We also determine conditions for a subset of $R_{(G,\alpha)}$ to form a basis for the spline module $R_{(G,\alpha)}$, for some classes of graphs such as Dutch Windmill graph and its special cases such as friendship graph, butterfly graph over GCD domain. We find a generating set of flow-up classes for wheel graphs over the ring $\mathbb{Z}/p^k\mathbb{Z}$, where p is prime. Also we classify splines on cycles and wheel graphs over the ring $\mathbb{Z}/m\mathbb{Z}$ when m has few prime factors and find a generating set of flow-up classes on these graphs over $\mathbb{Z}/m\mathbb{Z}$. We also determine conditions for a subset of $R_{(G,\alpha)}$ to form a basis of $R_{(G,\alpha)}$ for some classes of graphs. We have studied basis criteria for generalized splines on some isomorphic graphs over GCD domain and constructed flow-up basis for generalized spline modules on an arbitrary tree.