CHAPTER 3

Correlating Fast and Slow Axes of a Quarter Wave Plate to Interference Colours

Abstract

Anisotropic materials exhibit different optical properties in different directions due to their birefringent nature, and they are used to make several optical components in the optical industry. A wave plate is an optical component made up of anisotropic material that changes the state of polarization. Quarter wave plate (QWP) is one of the waveplates that is frequently employed to produce circularly and elliptically polarized light. The orientation of the fast and slow axes of QWP with respect to linearly polarized light decides the resultant polarization.

Usually, quarter wave plates sold at lower prices are not marked with fast and slow axes. Manufacturing companies' advice to perform colour based tests by tilting the quarter wave plate along the fast and slow axes. Even though this technique is frequently recommended, the literature rarely explores the physics underlying the commonly observed colours.

In the present chapter, colours that are observed through the fast and slow axes of the quarter wave plate are correlated to the interference colours, and observations are explained on the basis of the Michel Levy interference colour chart. The aim of this chapter is to explain the physics behind observed colours and make it easily available to undergraduate students and researchers.

3.1 Introduction

Quarter wave plates have been commonly used to generate circularly and elliptically polarized light because of its birefringent property. The key factor that decides the resultant polarization of light is its fast and slow axes orientation with respect to the polarization direction. Usually, the wave plates that are inexpensive and available in undergraduate labs are not marked with fast and slow axes. Users are required to perform an experiment based on colour changes observed while tilting a quarter-wave plate along the fast and slow axes. Axes of QWP are assigned on the basis of observed colours. Although this colour based technique to find fast and slow axes of QWP is commonly advised by manufacturing companies, there is no literature available to explain the physics behind the observed phenomenon.

In the present chapter, the properties of anisotropic materials from which wave plates are made are discussed in detail. The chapter further discusses the birefringence properties of optically anisotropic materials and explains why these materials are used to make waveplates and other optical components. The chapter also focuses on various techniques documented in previous works for determining the fast and slow axes of the wave plates.

The orientation of the fast and slow axes of QWP with respect to the plane polarized light is related to the production of circularly and elliptically polarized light.

In order to distinguish between the fast and slow axes of quarter wave plates, an experimental method suggested by companies is to perform colour based tests by tilting the quarter wave plate along the fast and slow axes. Different interference colours observed in experimental procedure are correlated to the fast and slow axes of a quarter wave plate. The Michel Levy interference colour chart is used to explain the genesis of different interference colours observed. The experiment is performed with minimum resources on quarter wave plates made up of mica and quartz. Finally, we give a thorough explanation of how different colours can be observed while tilting a quarter wave plate and how Michel Levy charts can be used to interpret them.

3.1.1 Optically Anisotropic Crystals and Birefringence

Materials may be classified into three categories based on molecular arrangement: opaque, optically isotropic, and optically anisotropic. Opaque materials do not allow light to pass through them. An optically isotropic material has a uniform refractive index because its molecular arrangement is the same in all principal directions. In comparison, an optically anisotropic material has a direction-dependent refractive index because the molecular arrangement is different in each direction. As a result, a light ray that strikes an optically isotropic material refracts as a single ray, but a beam that strikes an optically anisotropic material splits into two rays. The polarization of the two beams is in mutually perpendicular directions, showing birefringence known as double refraction phenomena, discovered by a Danish physicist, Bartholinus, in 1669.

Wave plates (half wave plates and quarter wave plates) are composed of optically anisotropic materials as they can use the property of birefringence where it is possible to have two waves having certain path difference and a phase difference of $\pi/2$ superpose while exiting the crystal. Every anisotropic crystal has a unique axis along which light rays do not experience double refraction, known as the optic axis of the crystal. The optic axis is a hypothetical line that goes through one of the blunt corners in the crystal as shown in Figure 3.1. At the blunt corner, the surface planes intersect to form three obtuse angles and are inclined to make equal angles with each edge. All lines parallel to the optic axis also act as the optic axis of the crystal.(Subrahmanyam et al., 2010)

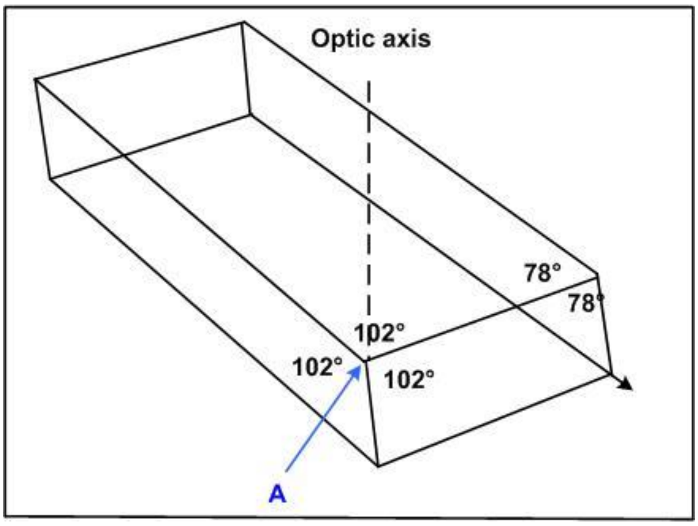

Figure 3.1: The calcite crystal's optical axis and blunt corner are displayed. The blunt corner (Corner A) of the anisotropic crystal is the corner where the surface planes intersect to make three obtuse angles with their edges, and the dotted line going through it is the crystal's optic axis.

Anisotropic materials are classified into two types: uniaxial and biaxial crystals, depending on how many optical axes the crystal has. Uniaxial crystals are those that belong to the trigonal, tetragonal, and hexagonal crystal systems and have one optic axis along which double refraction does not occur. Some examples of uniaxial crystals are tourmaline, calcite, and quartz. The orthorhombic, monoclinic, and triclinic crystal systems, on the other hand, are biaxial crystals, which contain two optical axes along which double refraction does not occur. A few examples of biaxial crystals are mica, muscovite, and talc. Mica is an easily available birefringent crystal that can be used to build quarter wave plates for an undergraduate lab. It also has additional qualities that include flatness, (Poel et al., 2014) transparency, and heat resistance.

Double refraction occurs when an incident ray is refracted into two linearly polarized beams with oscillating electric fields on mutually perpendicular planes. The ordinary ray (o-ray), one of the refracted rays, follows Snell's law and can travel in all directions with the same speed. Extraordinary ray (e-ray), on the other hand, does not follow Snell's law and travels in various directions with different speeds. The polarization of an e-ray is parallel to the optic axis (principal section), whereas that of an o-ray is perpendicular to the optic axis (principal section).

Propagation of e-ray and o-ray within a negative uniaxial crystal is shown in Figure 3.2, and how the orientation of the optic axis affects the incident ray within the anisotropic crystal is discussed. The speed stays the same in all directions for a spherical wave front, which corresponds to an o-ray, while it changes with direction for an elliptical wave front, which corresponds to an e-ray.

When optic axis of the anisotropic crystal is at an arbitrary angle with the incident ray as shown in Figure 3.2a, both refracted rays, o-ray and e-ray, travel in different directions with different speeds within the crystal, but when the incident ray is perpendicular to the optic axis of the crystal, both refracted rays (o-ray and e-ray) move in the same direction but travel with different speeds, as shown in Figure 3.2b. When the incoming ray is parallel to the crystal's optic axis, the behaviour of an anisotropic crystal is completely different; both refracted rays act as if the crystal is isotropic and travel in the same direction with the same speed as shown in Figure 3.2c. It must be observed that only along the optic axis the spherical wave front and elliptical wave front touch each other, suggesting that the e and o rays move in that direction with the same speed.

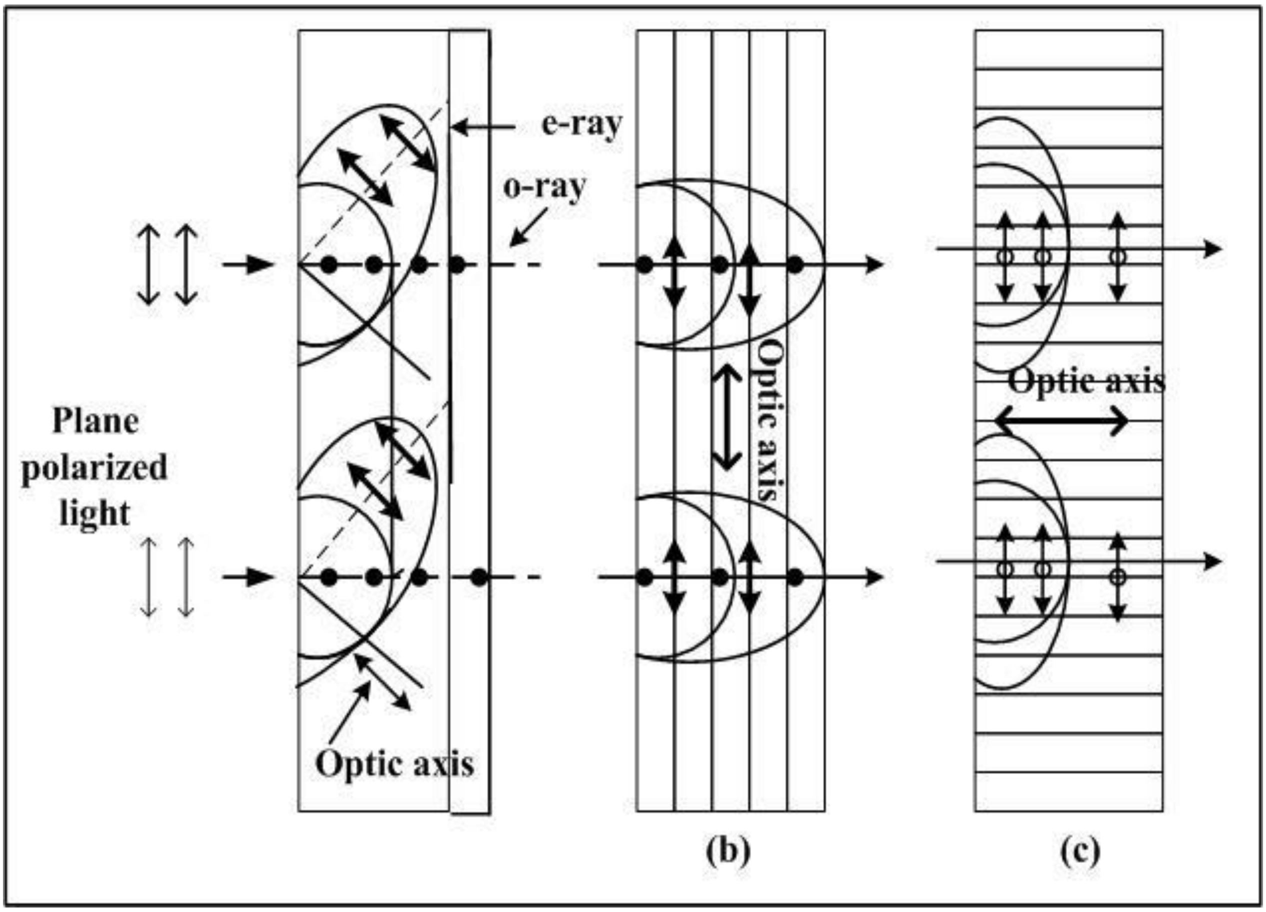

Figure 3.2: Propagation of e-ray and o-ray in a negative uniaxial crystal is depicted schematically. (a) The incident ray at any arbitrary angle with the crystal's optic axis, (b) Incident ray perpendicular to the crystal's optic axis or (c) Incident ray parallel to the crystal's optic axis.

3.1.2 Types of Crystals

Two types of uniaxial crystals are uniaxial positive crystals and uniaxial negative crystals. The difference in these crystals is due to the difference in refractive index values. The refractive index of the extraordinary ray (μ_e) is more than the refractive index of the ordinary ray (μ_o) for uniaxial positive crystals. As a result, o-rays travel quickly in the uniaxial crystal, and their polarization is directed in the direction of less density, whereas e-rays move slowly and have their polarization directed in the direction of the crystal that has more density. Since e-ray polarization is parallel to the optic axis, the optic axis acts as the slow axis for a uniaxial positive crystal and the fast axis for a uniaxial negative crystal.

An optical indicatrix is an efficient way to show variation of the refractive index with respect to direction in an anisotropic material.(Nelson, 2014c) It is a three-dimensional ellipsoid figure where the length of the vectors from the centre to the surface are proportional to the refractive index of the light, the polarization is parallel to the vectors from the central point. Optical indicatrix is extremely beneficial to study the vibration directions of light within the crystals.

For isotropic crystals, the refractive index (μ) is constant in all crystallographic orientations and the optical indicatrix is a sphere as shown in Figure 3.3a. Two extreme refractive indices are used to represent the optical indicatrices for uniaxial crystals(Nelson, 2014d). (μ_e or ε corresponding to the vertical axis of the ellipsoid) and (μ_o or ω corresponding to the horizontal axis of the ellipsoid). If $\varepsilon > \omega$ or $\mu_e > \mu_o$ for a crystal, it is called as uniaxial positive crystal. It is represented with prolate indicatrix as shown in Figure 3.3b and if $\varepsilon < \omega$ or $\mu_e < \mu_o$ it is called as uniaxial negative crystal represented with an oblate indicatrix as shown in Figure 3.3c.

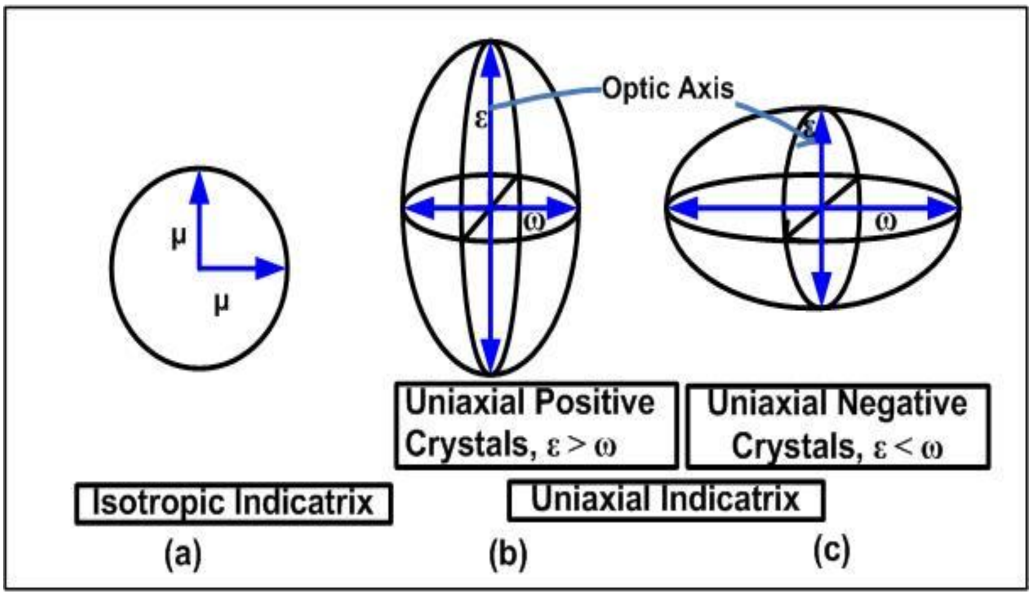

Figure 3.3: Optical indicatrix for (a) Isotropic, (b) Uniaxial positive, and (c) Uniaxial negative crystals are shown. Isotropic crystals have same refractive indices in all directions, and when it comes to uniaxial crystals, the refractive indices vary between two extreme values and form uniaxial prolate and uniaxial oblate indicatrix.

Biaxial anisotropic crystals have two optic axes, and when a ray refracts through them, they have two refracted rays. Both are extraordinary rays and defy Snell's law. Biaxial crystals have an intermediate refractive index β, as well as two extreme refractive indices (smallest, α and largest, γ). A crystal is referred to as a biaxial positive crystal if β is closer to, α and if β is closer to γ, crystal is referred biaxial negative crystal. The optical indicatrix of a biaxial crystal with a direction-dependent refractive index has three planes, with the β plane positioned between the α plane and the γ plane. As a result, there exist two circular β planes with the same radius. Optic axes are dark lines drawn perpendicular to the β circular sections shown in Figure 3.4. Biaxial crystals therefore have two optical axes that correspond to each β plane (Nelson, 2014a).

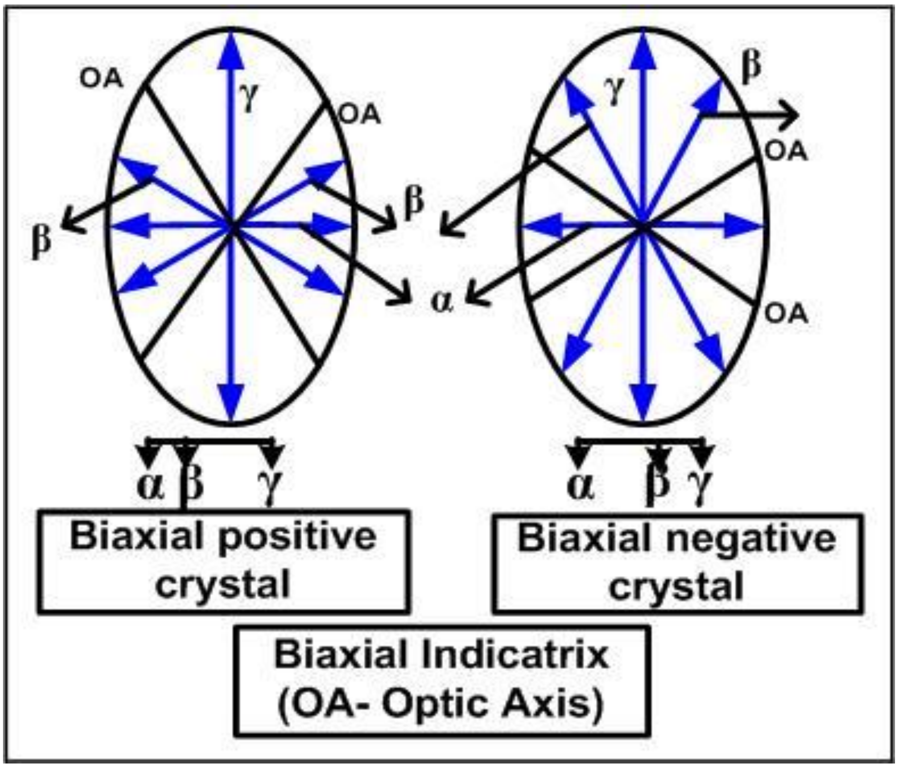

Figure 3.4: A biaxial positive and biaxial negative crystal optical indicatrix is shown. Biaxial crystals have one intermediate refractive index (β) that lie between two extreme refractive indices (α and γ).

Based on light transmission properties a categorization chart for optical media is shown in Figure 3.5. Chart bifurcates crystals in opaque, optically isotropic and optically anisotropic.

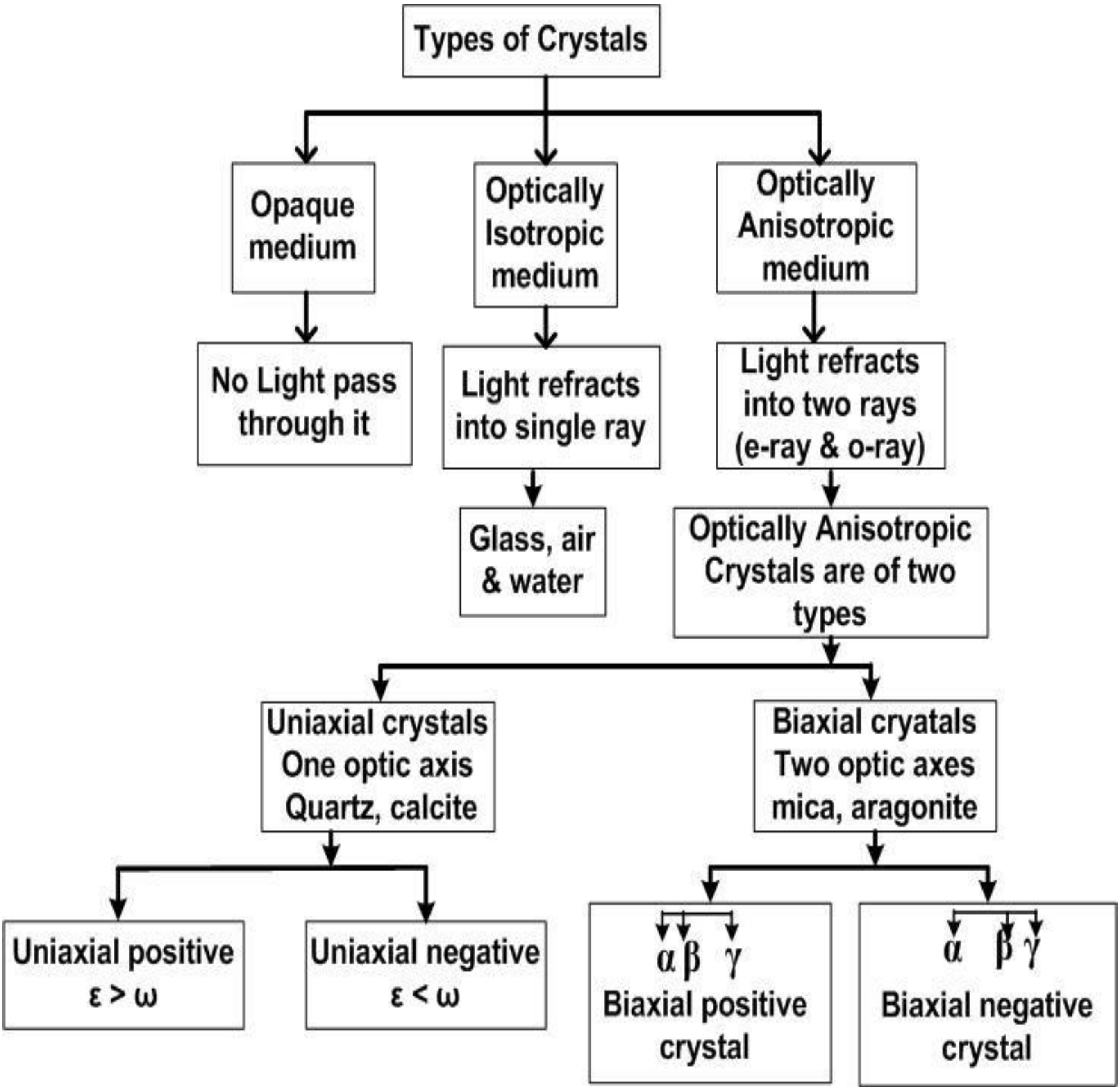

Figure 3.5: A flowchart illustrating how an optical medium is classified according to its light-propagation characteristics**.**

3.1.3 The Use of Birefringence in Wave Plates

Wave plates are optical elements that enable double refraction and are built of uniaxial and biaxial anisotropic materials. The e-ray or o-ray is referred to as a fast ray or a slow ray depending whether the ray will travel fast or slow within the crystal. After entering an anisotropic crystal of thickness "d," light exhibits double refraction by splitting into two rays: a fast ray and a slow ray are shown in Figure 3.6. In a specific amount of time the fast ray travels a distance 'd' with an additional distance 'Δ', which is known as retardation (Δ), whereas the slow ray travels a distance of 'd'. An anisotropic crystal's anisotropy causes retardation (Δ), by varying the speeds between the fast and slow rays.

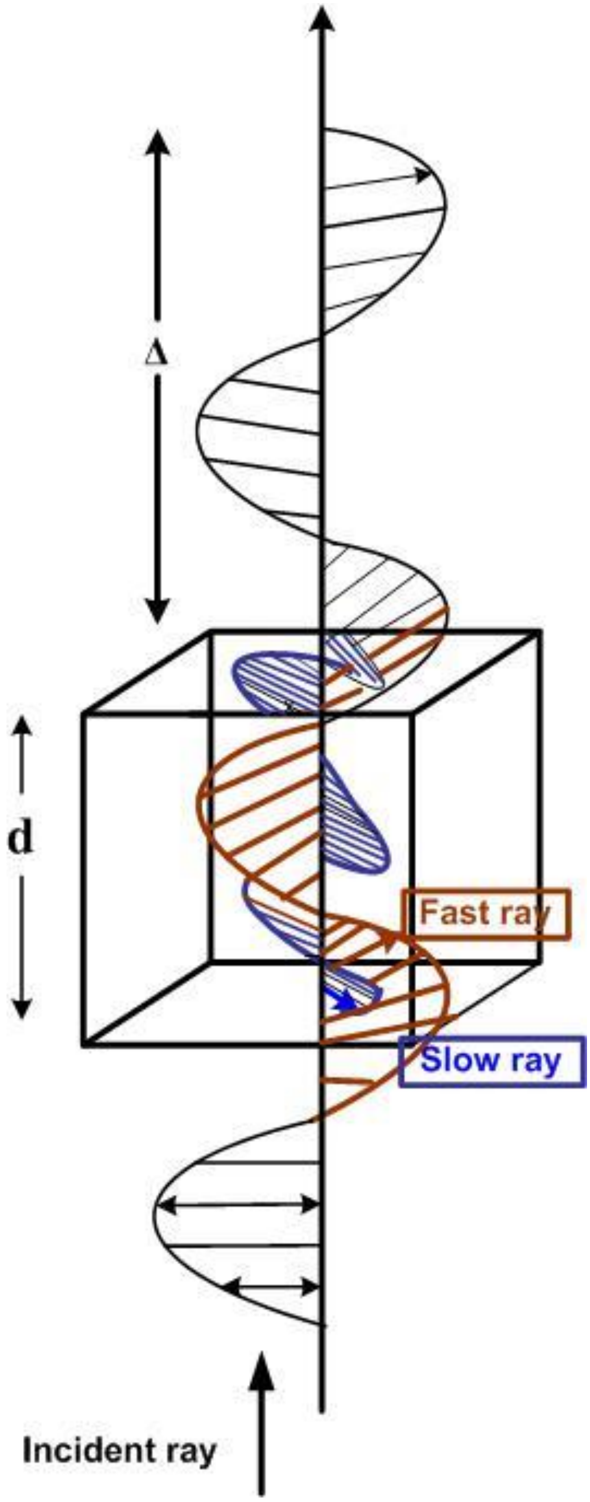

Figure 3.6: Light splits into fast and slow rays when it enters an anisotropic crystal with a thickness of 'd'. The slow ray is represented by a blue wave in the crystal, whereas the fast ray is represented by a brown wave. In comparison to the fast ray, the slow ray covers a shorter distance and develops retardation 'Δ'.

Some types of wave plates are quarter wave plates, half wave plates, and full wave plates. In all wave plates optic axis is kept parallel to the refracting edge and the incident plane and variation in their thickness develops a path and phase difference between two refracted rays. A quarter wave plate is made such that the two refracted rays are separated by a phase difference of $\pi /2$ and a path difference of $\lambda /4$. It is used to create elliptically and circularly polarized light, and it is also occasionally employed as an optical isolator to stop reflected light from entering a laser cavity. Thickness of a half wave plate is made such that it introduces a path difference of $\lambda/2$ and phase difference of π between the two refracted rays (e-ray and o-ray). These wave plates (half wave plate) rotate the plane of polarization of the incident plane polarized light through an angle 2θ because both refracting rays emerge with phase difference of 180°. After travelling through half wave plate both refracted rays combine and rotate plane of polarization by an angle 2θ° as shown in Figure 3.7. (Subrahmanyam et al., 2010) Hug.et al used half wave plate to make virtual enantiomers.(Hug, 2003) (Yamamoto & Watarai, 2010) A full wave plate makes a phase change of 1λ between two polarizations.

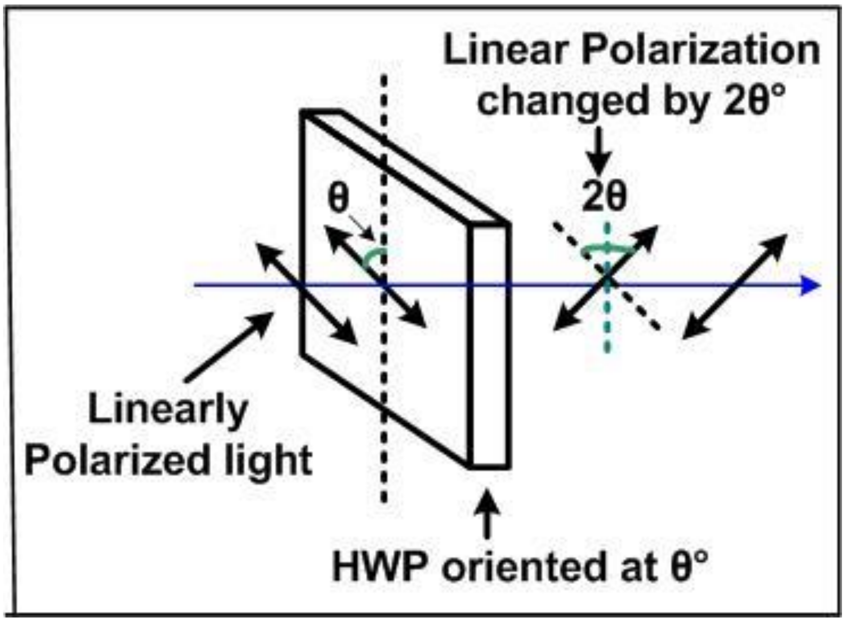

Figure 3.7: Plane of polarization rotated through an angle 2θ° by half wave plate.

Some studies only employ zero order quarter wave plates that are used for a wide range of frequencies, they are not sensitive to temperature variation, wavelength and angle of incidence. These wave plates (zero-order) are expensive. The majority of wave plates are multiple order wave plates as they are economical and are used for a limited range of frequencies. These wave plates are strongly affected with variation in temperature, wavelength and angle of incidence.

Recent study describes a technique for calculating the thickness and order of a zero-order quarter wave plate created by attaching several multiple order quarter wave plates according to the intensity observed. (Hu et al., 2013)

3.1.4 Methods for Determining the Fast and Slow Axes of a Quarter Wave Plate

The state of polarization of the light can be changed by changing the orientation of quarter wave plate with respect to the incident polarization. If the transmission axis of the polarizer is kept parallel to optic axis of the quarter wave plate, linearly polarized light is generated. If optic axis is oriented at 45° with respect to linear polarizer, circularly polarized light is generated as shown in Figure 3.8.

Circularly polarized light is the superposition of two coherent linearly polarized light waves of equal amplitude (E cos45° and E sin45°) oscillating in perpendicular planes and having a phase difference of 90°. When the optic axis of QWP is oriented at 45˚ with respect to the transmission axis of the polarizer, due to the birefringent nature of QWP, it splits the incident ray into two rays that are of the same amplitude, E cos45° and E sin45° (as QWP is oriented at 45˚) , vibrate in perpendicular planes, and travel within the QWP at different speeds. When both refracted rays emerge from the QWP, both rays are perpendicularly polarized and coherent, as a phase difference of $\pi/2$ (maintained) and a path difference of $\lambda/4$ are developed between the two refracted rays. Both coherent perpendicularly polarized components superimpose, and circularly polarized light is produced.

To produce left- or right-circularly polarized light, it is necessary to discriminate between the fast and slow axes because the fast and slow axes orientation decides the circularly polarized light’s rotation direction but to differentiate them is a practical challenge. Without appropriate knowledge, circularly polarized light with a known orientation cannot be generated. We give a brief overview of different approaches for determining the quarter wave plate's fast axis that have been mentioned in the literature.

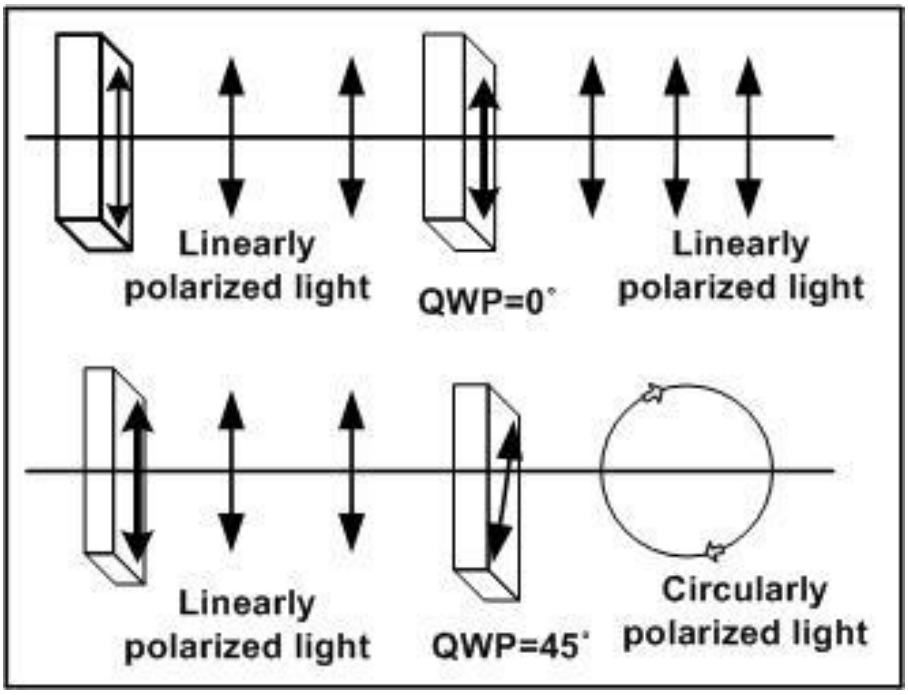

Figure 3.8: A quarter wave plate's (QWP) response to linearly polarized light is depicted visually. When optic axis of QWP is parallel to the polarizer transmission axis no change in polarization but when it is oriented at 45°, it changes the state of polarization and circularly polarized is generated.

If the fast axis of the quarter wave plate is oriented at +45° with respect to the transmission axis of the polarizer, right circularly polarized light (RCP) is generated, and if it (fast axis) is oriented at -45° left circularly polarized light is generated as shown in Figure 3.9.

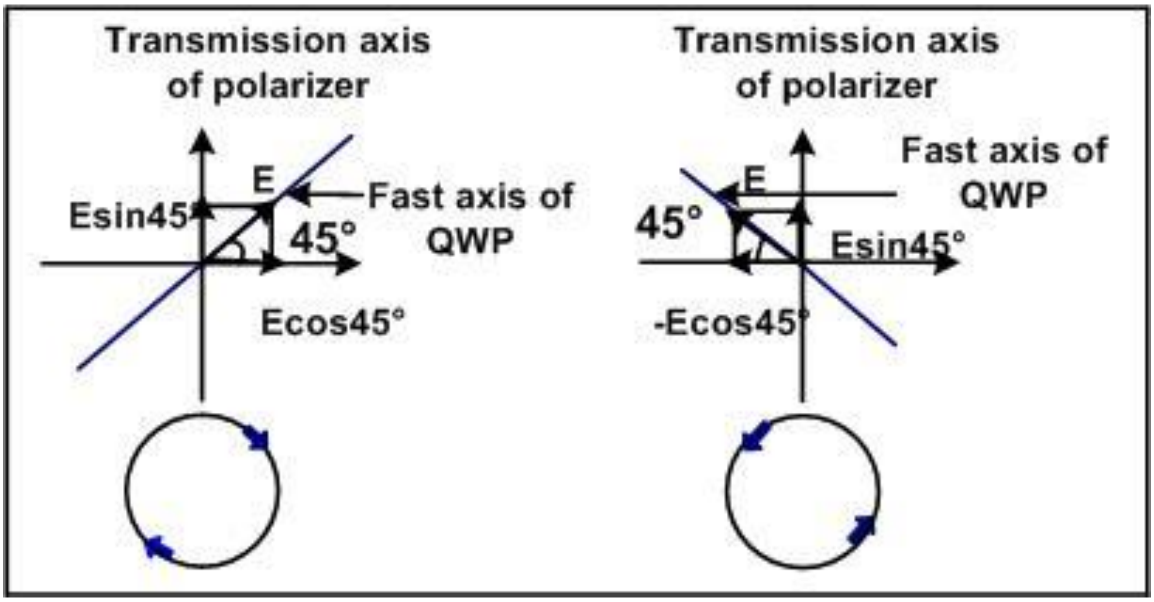

Figure 3.9: Generation of right/left circularly polarized light when fast axis of quarter wave plate is oriented at +45° and -45°. (Figures are drawn by observing optical geometry from the outside)

To identify the fast and slow axis, Li Guohua et al. used a two beam Shimadzu 460 infrared spectrometer.(Guohua et al., 1990) Their technique involves placing polarizers P1 and P2 in the spectrometer's sample and reference beams, respectively, with the polarization transmission axes pointing in the same directions. In the sample beam path, polarizer P1 and analyzer P3 are positioned by keeping their transmission axes parallel. A quarter wave plate is introduced and rotated between parallel polarizers (P1 and P3) and the transmission spectrum is recorded to observe maximum and minimum transmittance as shown in Figure 3.10a and Figure 3.10b.

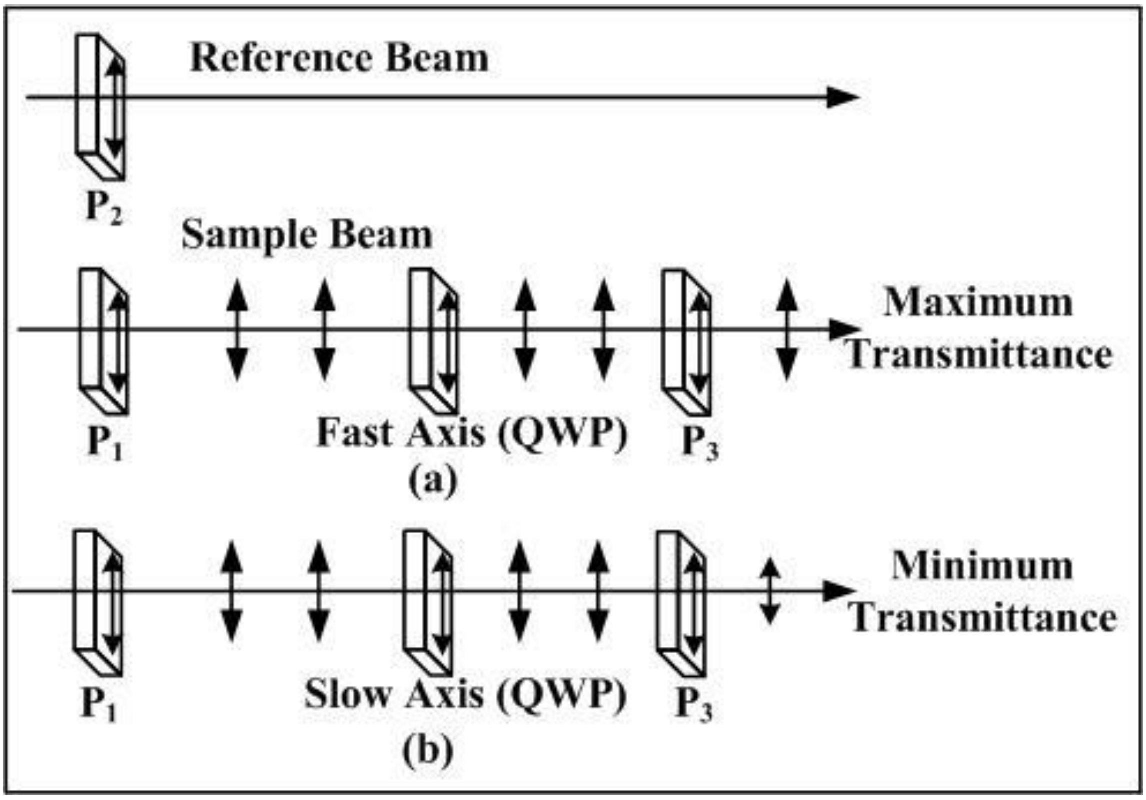

Figure 3.10: (a) A two-beam Shimadzu 460 infrared spectrometer schematic arrangement is shown. The transmittance spectra are recorded and detected by rotating the fast and slow axes of the quartz and mica wave plates. Maximum (minimum) transmittance values are obtained by keeping the fast axis (Figure 10a/(slow axis) Figure 3.10 (b) parallel to the polarizer and analyzer axes. (Guohua et al., 1990)

If the slow axis of the wave plate made up of quartz (a positive crystal) is kept parallel to the polarizer, the e-ray travels slower than the o-ray as in a positive crystal ($\mu_e > \mu_o$). The refractive index of the quarter wave plate material is more for slow axis. Polarization of the e-ray is parallel to the optic axis (slow axis for positive crystal) it travels slow and this results in more optical path length (μd) as it travels through analyzer while o-ray polarization is in perpendicular direction and it does not travel through the analyzer.

Different is the case when fast axis of quartz (a positive crystal) is aligned parallel to the polarizer axis. Polarization of the o-ray is parallel to the optic axis (fast axis for positive crystal) due to which it travels fast and this results in less optical path length (μd) while e-ray cannot pass through the analyser. However, the explanation for a negative crystal will be the exact reverse of a positive crystal. The quarter wave plate's fast axis and slow axis are correlated to the maximum and minimum transmittance values, respectively.

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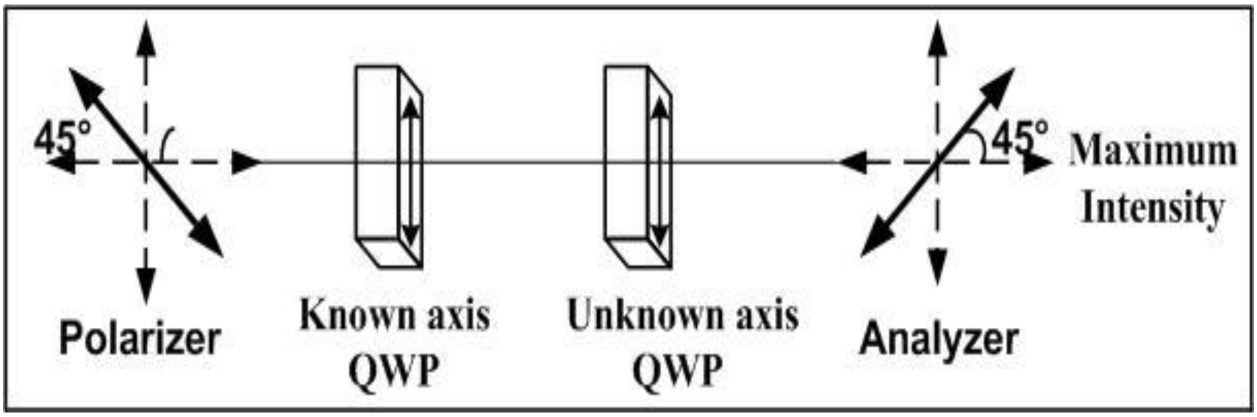

Figure 3.11: A quarter wave plate with a known axis is used to determine fast axis of a quarter wave plate whose fast axis is unknown. (Galgano & Henriques, 2005)

Using a quarter wave plate with a known fast axis is one of the well-known ways to determine the unknown fast axis of a quarter wave plate. Two linear polarizers are placed in a crossing position for the purpose, with their transmission axes 45° to the left and right of the vertical transmission axis. Then, between two crossed polarizers, a quarter wave plate with a known fast axis is inserted and its fast axis is kept vertically as shown in Figure 3.11. Unknown fast axis quarter wave plate is positioned next to the known fast axis quarter wave plate and rotated until maximum transmission is observed. The unknown quarter wave plate's fast axis will also be in a vertical position. (Galgano & Henriques, 2005) This approach is simple, however a quarter wave plate with a known axis is required to carry out this test.

P.C. Logofatu et al. gave another method for determining fast and slow axes that is based on a metal's ability to change the phase of the incident beam by introducing a phase shift to the p and s polarization components of reflected light from its surface, where p and s polarization components represent the electric field polarization that is parallel and perpendicular to the plane of incidence, respectively. In this experiment He-Ne laser (633 nm) and He-Cd laser (442 nm) both were used, along with steel alloy as the metal surface. A linear polarizer is positioned with its axis at 45˚ with respect to the vertical axis. After the linear polarizer quarter wave plate is placed and either its fast or slow axis is aligned vertically, the light beam strikes the metal surface, and the reflected light is examined by an analyzer with its axis kept at 90˚ degrees to the linear polarizer. A photo detector is placed after the analyzer. Only the metallic surface is rotated to change the angle of incidence but distances between all the optical components are kept fixed as shown in Figure 3.12. Both experimental and theoretical Fresnel curves of reflectance versus incidence angle for the fast and slow axes of the quarter wave plate are compared for the interpretation of the axes.(Logofatu, 2002)

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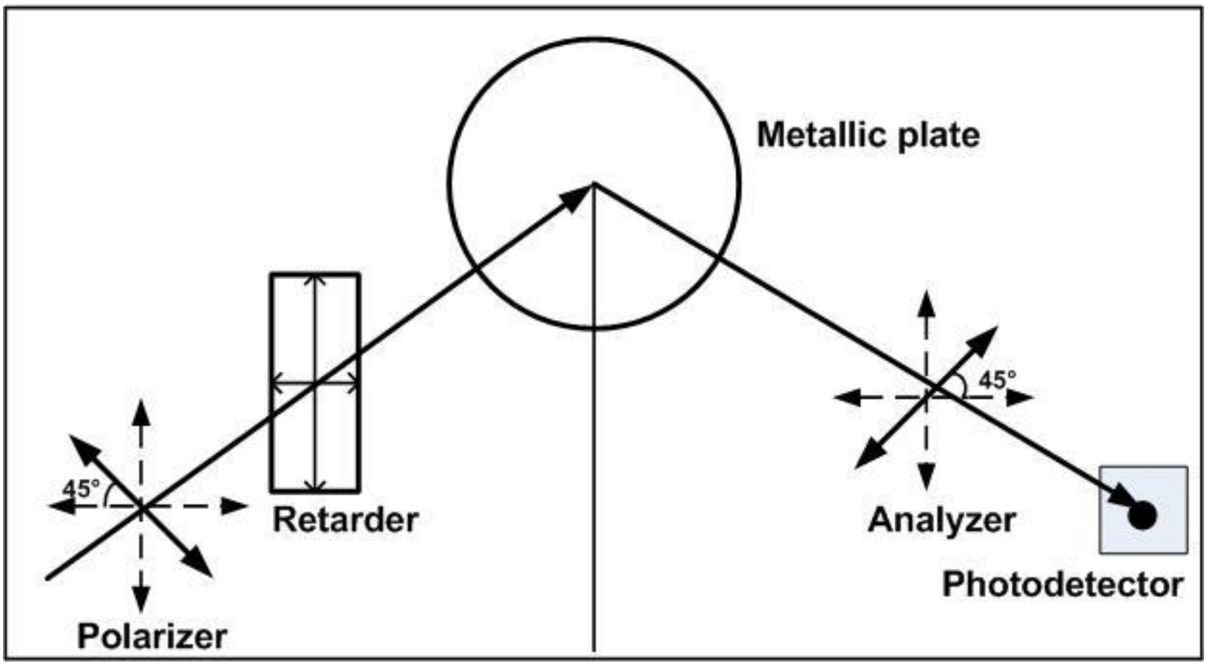


Figure 3.12: Schematic arrangement shows how metal's ability to modify phase is used to determine fast axis of a quarter wave plate. The transmission axes of two linear polarizers are crossed at 45° to the left and right of the vertical direction, respectively. (Galgano & Henriques, 2005; Logofatu, 2002)

Likewise, Galgano et al. (Galgano & Henriques, 2005) used a semiconductor diode laser with a wavelength of 635 nm to observe the intensity of light reflected from a metallic surface by alternately maintaining the wave plate's fast and slow axes in vertical direction. The optical setup is similar to that of the experiment by Logofatu et al. Then, Galgano et al. used Fresnel equations to determine the reflection coefficients. (Logofatu, 2002). When the fast axis was vertical, the intensity detected by the photo detector was larger, and when it was kept horizontal, the intensity detected was lower.

Using two laser beams of different wavelengths and a heterodyne interferometric method, M.H. Chiu et al. (Chiu et al., 1996) determined the fast axis and phase retardation. He-Ne laser of wavelength (λ_1) 632.8 nm is used as Laser 1 and He-Cd laser of wavelength (λ_2) 441.6 nm is used as Laser 2. The fast axis of an electro optic modulator (EOM) is orientated at +45˚ with respect to the transmission axis of the polarizer, which is oriented along the x-axis. From EOM the beam is incident on a beam splitter, where it split into reflected and transmitted light. The transmitted light travels through a quarter wave plate (WP), while the reflected beam travels through an analyzer (A_r). The orientation of the quarter wave plate with respect to the x-axis is at θ angle. Phase retardation (δ) is introduced by the wave plate before the light enters the analyzer (A_t). The transmission axes of both analyzers are positioned along the y-axis, followed by photo detectors D_t & D_r. To measure the phase difference (ϕ) between the two signals, a phase meter is placed as shown in Figure 3.13.

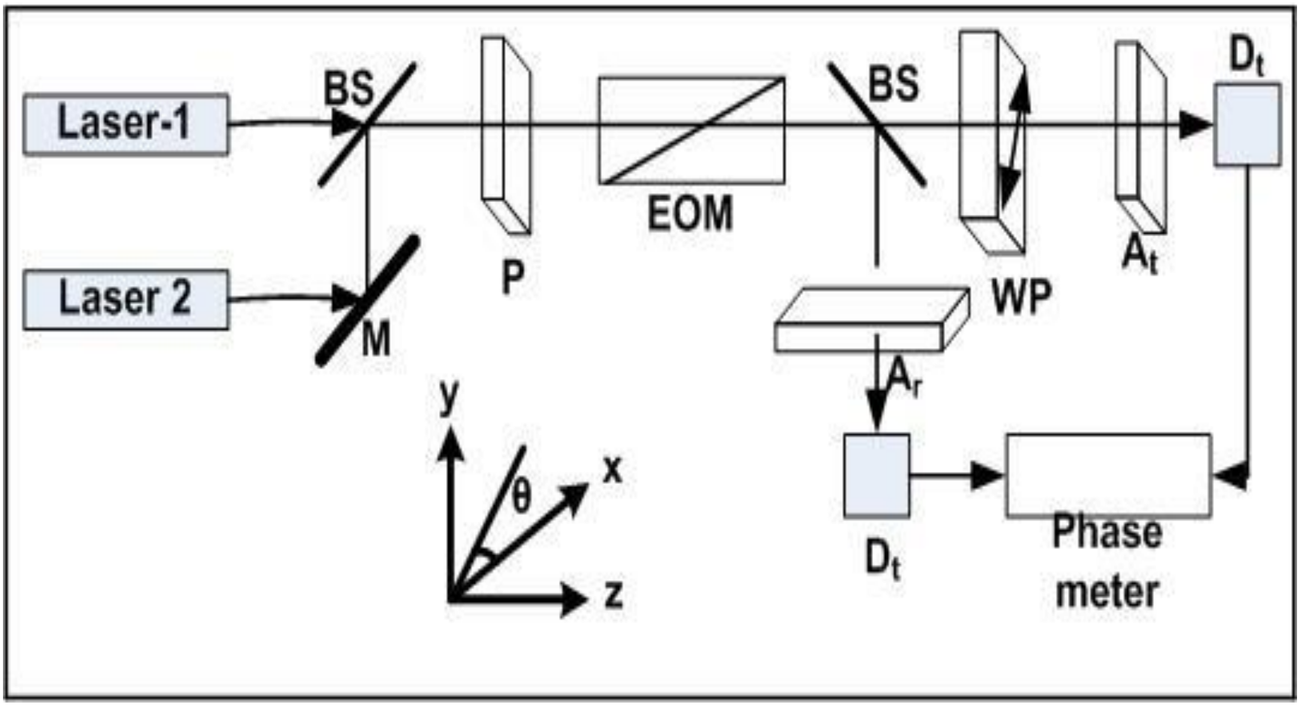


Figure 3.13: An overview of the setup used by M.H. Chiu et al. to calculate the fast axis and phase retardation using heterodyne interferometry. (BS- beam splitter; M- Mirror; P- Polarizer; EOM- Electric optic modulator; A_t and A_r -analyzer; W- wave plate; D_t and D_r ,-photodetector. (Chiu et al., 1996)

To test the sensitivity of the apparatus (ϕ versus θ) the wave plate is rotated with respect to Laser 1 (He-Ne laser). For sensitive arrangement ϕ = 0 and for insensitive arrangement ϕ ≠ 0. If setup is not sensitive, Laser 2 (He-Cd laser) is introduced and the wave plate is rotated to make it sensitive (ϕ = 0). In this configuration, the wave plate's fast axis can be found either along the x-axis where $\theta = 0°$ or along the y-axis where $\theta = 90°$. When the sensitivity requirement is met, the wave plate is rotated at a 45° angle.

To obtain ϕ_1 and ϕ_2 two different sources of wavelength λ_1 and λ_2 are used. The experimental results were checked and correlated to the reference equation (Eq. 1), for the calculated intensities of transmitted and reflected light. The axis at +45˚ is the fast axis if the fast axis equation is satisfied; else, it is the slow axis.

$$\phi_2 \cong \frac{\lambda_1}{\lambda_2}\,\phi_1 \qquad (1)$$

The methods for determining an axis discussed above(Chiu et al., 1996; Galgano & Henriques, 2005; Guohua et al., 1990; Logofatu, 2002) require significant instruments like, lasers, quarter wave plate with known fast axis, infrared spectrometer, electro optic modulator, phase meter, etc, thus increasing the cost of the techniques. However, several companies that make optical components recommend a different straightforward process that uses less equipment. However, the majority of them do not adequately describe the process, and it is still unclear what physical principles underlie the optical phenomena. We go into great detail to explain the method and phenomena involved in it.

3.2 Materials and Methods

3.2.1 Interference Colours

This section describes the experimental conditions under which interference between the doubly refracted rays takes place. When light is transmitted through an anisotropic crystal, distinct colours are observed. The state of polarization of the incident ray does not change when an anisotropic crystal is placed between two cross polarizers with its optic axis parallel to the transmission axis of the polarizer; simply linearly polarized light parallel to the polarizer axis is transmitted through the quarter wave plate, and as the analyzer is kept at the cross position, no light comes out of the analyzer. If the optic axis of an anisotropic crystal is kept at some angle with respect to the linearly polarized light and the analyzer is kept in a crossed position, In this arrangement, some light can travel through the analyzer and different colours can be observed. Observation of light (colour) is either due to the e-rays or o-rays or due to components of the e and o-rays'.

Which colour will be observed entirely depends upon the orientation of the optic axis of the crystal and phase difference developed between the e and o rays. When anisotropic crystals optic axis is perpendicular to the incident ray the refracted e and o rays travel in the same direction but at different speeds. Therefore, the fast ray travels a longer distance than the slow ray, as discussed in the introduction of the present chapter.

Consider an anisotropic crystal of thickness d, the fast ray's speed is v_F , the slow ray's speed is v_S and the duration to traverse through the crystal for the fast ray is T***fast*** and for slow ray the duration is T***slow***. Mathematically traverse time of fast ray to travel distance "d" is calculated with Eq. 2. and the amount of time taken by the slow ray to travel distance "d" inside the crystal is calculated with Eq. 3.

$$T_{fast} = \frac{d}{v_F} \tag{2}$$

$$T_{slow} = \frac{d}{v_S} \tag{3}$$

The slow ray lags the fast ray because of the difference in velocities. By the time the slow ray covers a distance "d," the fast ray has travelled distance "d," and an additional distance equal to the retardation between the fast and slow rays. Equating fast and the slow rays meeting time.

$$\frac{d}{v_S} = \frac{d}{v_F} + \frac{\Delta}{c} \tag{4}$$

$$\Delta = d\left(\frac{c}{v_S} - \frac{c}{v_F}\right) \tag{5}$$

$$\Delta = d\left(\mu_S - \mu_F\right) \tag{6}$$

Retardation = (Thickness of the material) x (Birefringence)

Where Δ is the amount of retardation, d is the material's thickness, and $(\mu_S - \mu_F)$ is the material's birefringence. (Nesse, 1991) Thus, Eq. 6. Provides the relationship between retardation, thickness of the material and birefringence of the crystal.

3.2.2 Transmission of Monochromatic and Polychromatic Lght through an Anisotropic Crystal

In this section transmission of (i) monochromatic and (ii) polychromatic light through an anisotropic crystal is discussed. Anisotropic crystal of arbitrary thickness is placed between two crossed polarizers. In the first case, ***monochromatic light*** after passing through a linear polarizer falls on an anisotropic crystal of random thickness whose optic axis is parallel to its refracting edge and oriented at +45° with respect to the polarizer axis As a ray enters an anisotropic crystal, it splits into fast and slow rays that oscillate in perpendicular planes and travel in the same direction but at different speeds. Due to this difference the fast ray leads the slow ray. Resolved components of fast and slow rays generated from an anisotropic crystal are passed through the analyzer. These resolved components that are allowed to pass through the analyzer interfere to produce an interference phenomenon. Rays coming out of the analyzer will interfere destructively or constructively, depending entirely upon whether the resolved components are in phase or out of phase.

When the optical path difference between refracted rays is an integral multiple of wavelength (in phase) and oscillates in perpendicular planes, they interfere destructively. The resolved components of the fast ray and slow ray are of the same amplitude but oscillate in opposite directions; they cancel each other, and no light passes through the analyzer (components parallel to the transmission axis of the analyzer). The components perpendicular to the analyzer transmission axis are anyway not allowed to pass through it. Resolved components interfere destructively, and the analyzer is not able to pass any light through it, and complete darkness is observed. Figure 3.14 show that the cancellation of resolved components is the reason behind destructive interference.

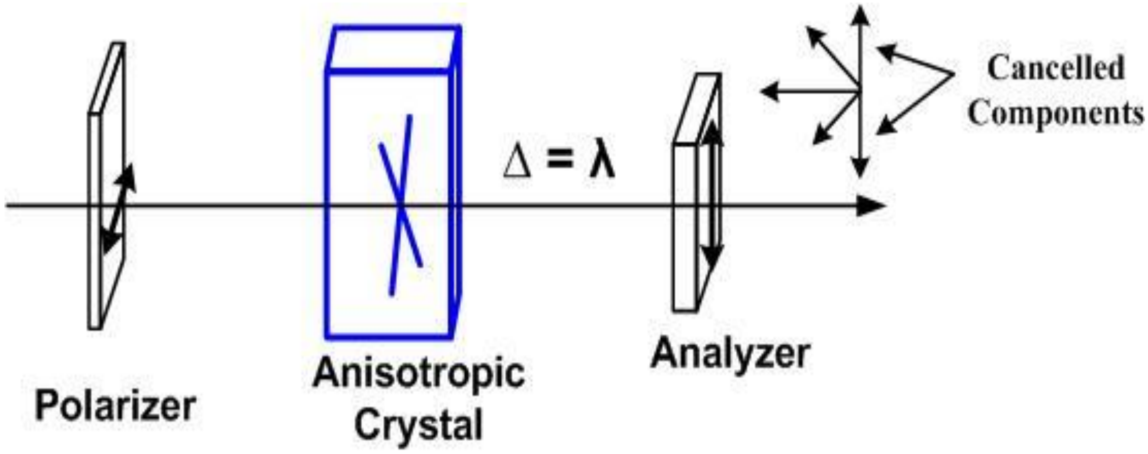


Figure 3.14: Destructive interference is observed as resolved components from an anisotropic crystal are cancelled along the transmission direction of analyzer.

When optical path difference (retardation) is not an integral multiple of wavelength (out of phase), rays interfere constructively because resolved components are of different amplitude and rays are allowed to pass through the crossed analyzer (kept perpendicular to the polarizer transmission axis). Rays coming out of the analyzer interfere, and in output brightness is observed. Figure 3.15 shows that all resolved components are added and allowed to pass through the analyzer, and due to it constructive interference is observed.

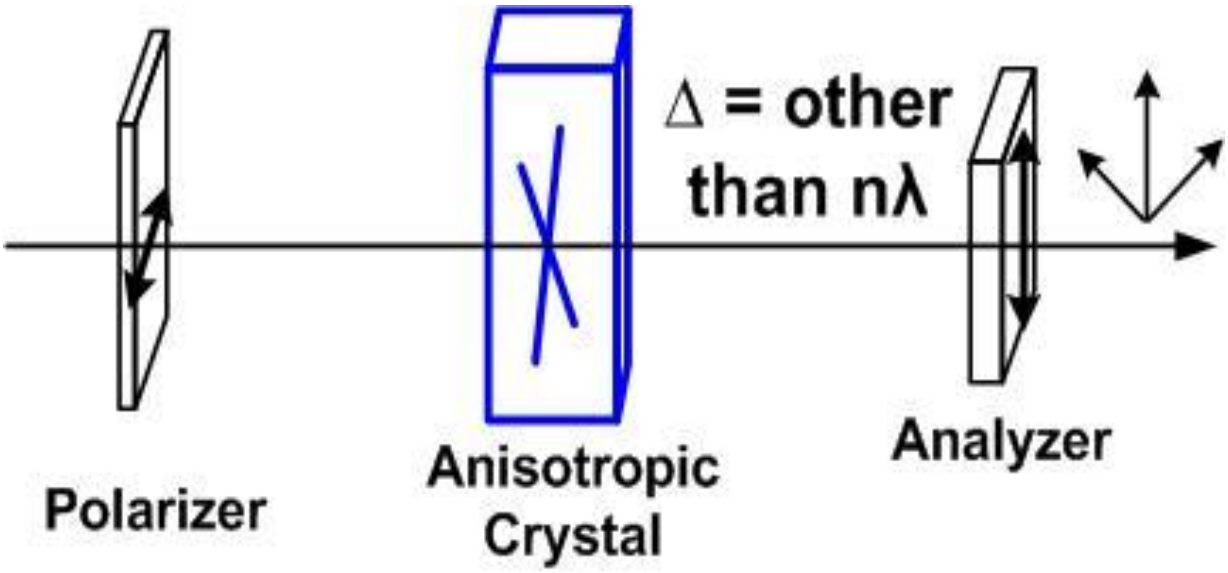


Figure 3.15: Constructive interference is observed as resolved components from an anisotropic crystal are added and allowed to pass through the analyzer.

In monochromatic light is allowed to pass through the quartz wedge kept between crossed polarizers. An interference pattern is observed with bright and dark bands as shown in Figure 3.16. When the retardation is an integer number of wavelengths, the fast and slow rays interact destructively at the analyzer. The two rays constructively interact at the analyzer when the retardation is other than integer number of wavelengths, and the light is transmitted.

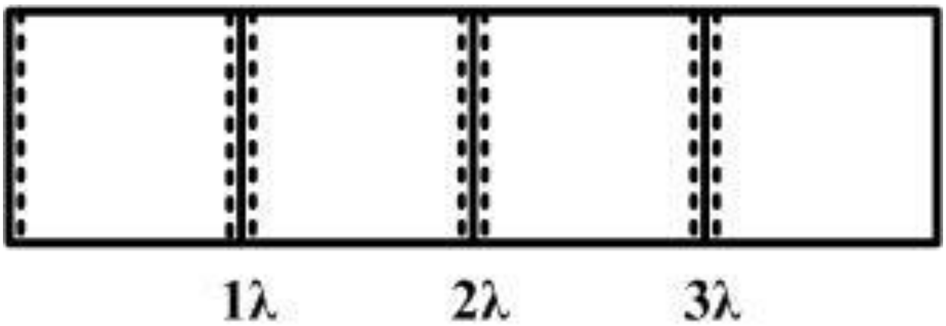


Figure 3.16: When monochromatic light is allowed to pass through the quartz wedge placed between the crossed polarizer and analyzer. Bright and dark bands appear due to interference of resolved components of fast and slow rays.

When the polarizer and the analyzer transmission axes are kept parallel to each other and an anisotropic crystal is placed between them. Constructive interference takes place when the optical path difference is an integral multiple of wavelength and destructive interference when the optical path difference is other than an integral multiple of wavelength. Despite of having a path difference of integral multiples of λ, it should be noted that when the anisotropic crystal is rotated at any angle other than 45˚, light output is observed due to partial destructive interference. The e and o waves' polarizations are unequal and are in opposite directions because of the elliptical polarization. But, the crystal's thickness, d, limits the amount of the path difference. The thickness of a quarter wave plate is made such that it generates maximum path difference of λ/4 and due to partial destructive interference, some light is always visible after passing through the analyzer. When linearly polarized polychromatic light strikes an anisotropic crystal, some wavelengths of light are in phase (have a path difference of λ) with one another, and their resolved polarization components do not pass through the analyzer due to destructive interference because they are equal and opposite to one another. Constructive interference is observed when the polarization components of certain light wavelengths are resolved in the analyzer's transmission direction, they contribute to a non-zero resultant because of specific phase differences in the light. As a result, the colours observed correspond to wavelengths that cause constructive interference. (Nelson, 2014b)

Therefore, based on the path difference formed by the anisotropic crystal held between two cross polarizers, monochromatic light sources are responsible for the observation of a single colour, whereas polychromatic light sources are responsible for the observation of different colours. Phase retardation and birefringence of mica are reported to be temperature and wavelength dependent in the literature. Through the use of a spectroscopic ellipsometer and conventional transmission ellipsometry, its phase retardation with respect to temperature is investigated. These investigations demonstrated how birefringence and phase retardation diminish as temperature increases. (Zhang et al., 2014) The value of birefringence is measured with respect to various wavelengths. According to their findings, birefringence decreases as wavelength increases.(El-Bahrawi et al., 1998)

With the help of the Michel Levy Interference colours chart, the observed interference colours are analyzed. (John Gustav Delly, 2017) Analytical microscopists frequently use it to correlate the thickness, retardation, and birefringence of anisotropic minerals. ***The Michel Levy Interference colour chart*** was first published in 1888 in Paris in Auguste Michel Levy's book "Les Minéraux des Roches," which refers to "The Minerals of the Rocks," and was used to identify unidentified minerals. A wedge-shaped anisotropic crystal sandwiched between cross polarizers can create a range of colours when white light with wavelengths (400 and 700 nm) travels through it, while when monochromatic light travels through it, bright and dark bands are observed as shown in Figure 3.16. With polychromatic light, different wavelengths provide maximum intensity and zones of darkness at various positions along the crystal wedge. These boundaries are divided into different orders.(Kerr, 1977) The wedge's thin end piece exhibits a colour transition from first order grey to white, yellow, orange, and finally red. The first order spectrum's red colour exhibits the maximum retardation, which corresponds to a 560 nm retardation. The second order spectrum is clearly divided into its constituent colours, with violet, indigo, blue, green, yellow, and finally red. Indigo, green-blue, yellow, and red are observed in third order spectrum while some colours are missing from it. Pale colours are exhibited in the fourth, fifth, and higher orders, due to more birefringence caused by more crystal thickness. White light is observed when components of white light are combined. Each order's end is represented by red colour , which represents a full retardation.(Gribble & Hall, 1993)

The x-axis of the Michel Levy interference colour chart is shown in Figure 3.17. displays the amount of retardation. It represents the optical path difference between the slow ray and fast ray. The thickness of the material is displayed on the chart's y-axis in millimetres. The amount of retardation that the crystal causes increases as the crystal thickness increases. We observe different interference colours due to different retardations in the crystal. The chart's top axis displays the birefringence values. The chart's right top corner and right side exhibit high birefringence values, while the left top corner displays low birefringence values and the mid-top shows medium birefringence values. (J. G Delly, 2008)

3.2.3 How to study Michel Levy Interference Colour Chart?

The interference colours observed depend on the wavelengths of light that are allowed to pass through the analyzer. Higher colour orders are observed as the thickness of the anisotropic crystal increases. For example, we performed an experiment on a quarter wave plate made up of mica (birefringence of muscovite mica .036) (Nesse, 1991) and used a white light source to perform the experiment. When mica QWP was inserted between cross polarizers, white light was observed in the output. Once interference colour was observed through the analyzer, that colour is located in the chart in order to determine the retardation from the x-axis of the chart. Cross cutting lines of birefringence and retardation indicate the thickness of the mica wave plate, which can be read from the y-axis of the chart. We observed a thickness of mica QWP of .015mm.

Same experiment was performed on a QWP made up of quartz (birefringence is.009), (Nesse, 1991)when it was inserted between cross polarizers, the pale-yellow interference colour was observed. Cross cutting lines of birefringence and retardation indicate the thickness of .03 mm. Hence, if the mineral's birefringence is known and interference colour is observed, retardation and thickness of the mineral can also be calculated by Michel Levy Interference Colour Chart. (Robinson & Davidson)

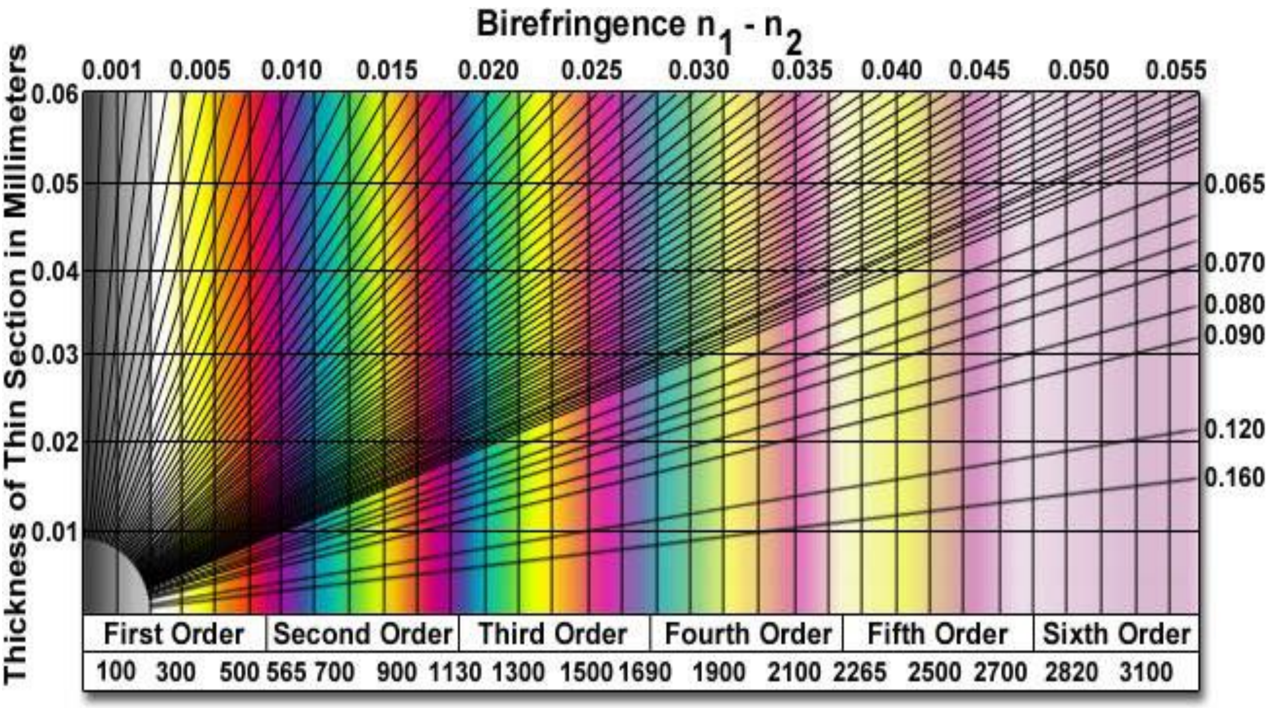
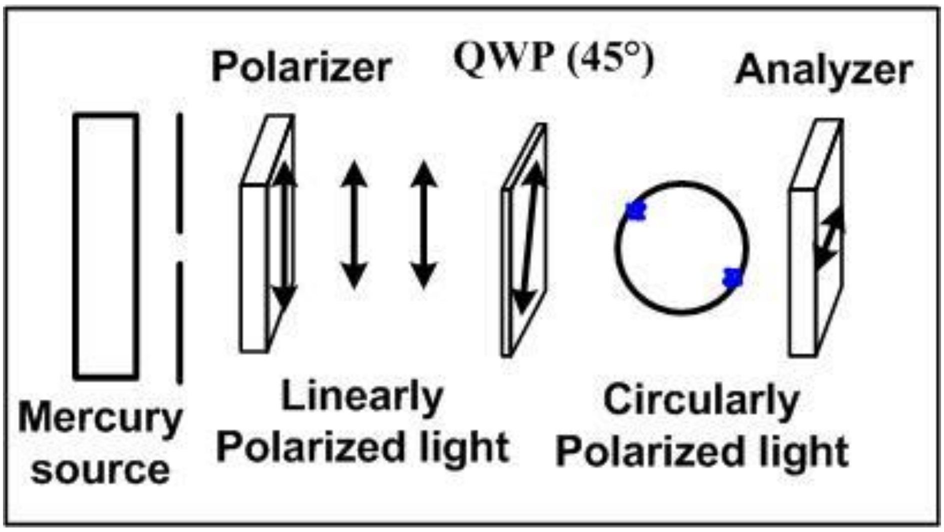


Figure 3.17: Michel Levy interference colour chart (Adapted with permission from "Molecular Expressions at Florida State University" (Robinson & Davidson)

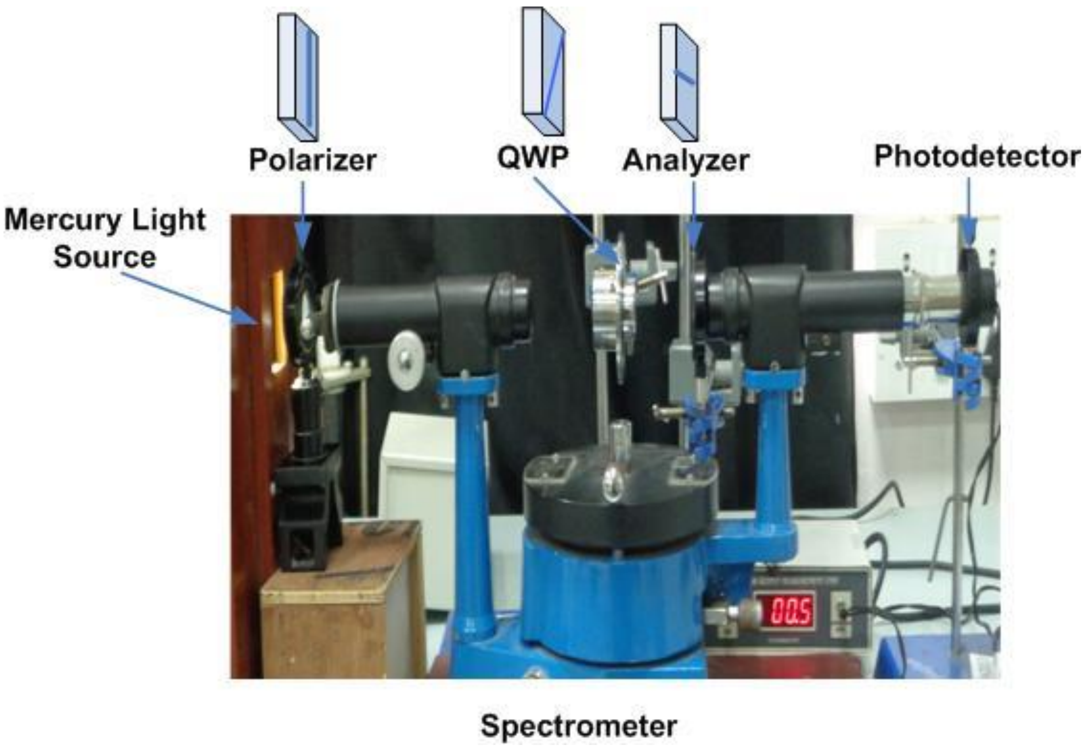
In an experimental setup, mercury vapour lamp was used as a polychromatic light source. Experiment was performed on prism table spectrometer and slit was kept widely open. Right after the telescope's eyepiece a photodetector is placed. Photodetector is a 5 mm NPN phototransistor (T-1, PT333-3C) made up of photosensitive silicon material purchased from Holmarc Opto Mechatronics Pvt. Ltd. (Model No: HO-ED-PHPD-01. The detector's spectrum sensitivity ranges from 150 nm to 1200 nm at ambient temperature, with the mercury spectral region between ~50-70% and having a sensitivity of 75% with sodium emission at 589.3 nm. (Shah & Ghalsasi, 2019)

A polarizer is placed in front of the slit of the collimator to polarize the white light coming from the mercury source and analyzer is placed in front of telescope objective the polarizer and the analyzer transmission axis are kept perpendicular to each other and complete extinction is observed .A quarter wave plate composed of mica is inserted between the crossed polarizers, just after the spectrometer's collimator optics. Some amount of light enters the analyzer once the quarter wave plate enters in the path of the light it change the state of polarization.

Now one complete rotation is imparted consistently to the mica quarter wave plate. During one complete rotation, four positions exhibit maximum intensities at intervals of 90˚. Two positions across 180° exhibiting maximum intensity correspond to the fast axis and the other two correspond to the slow axis. Circularly polarized light is generated when the axes indicated by these angles are positioned at 45° with respect to the polarizer axis as shown in Figure. 3.18a. The maximum intensity through the photodetector is measured in a precision mount holding the QWP. Photodetector recorded maximum intensity at angles of 55° and 145°. Experimental setup is shown in Figure 3.18b.



(a)



(b)

Figure 3.18:(a) Schematic of the experiment's setup (b) Our experimental actual setup. The light source is a mercury vapour lamp. Axes are represented as straight lines to indicate how each optical device is oriented.

The mica quarter wave plate's one of the two axes are oriented at +45° and tilted with respect to prism table parallel to the transmission axis of the polarizer (keeping its vertical axis perpendicular to the prism table) to observe interference colour.

3.3 Results and Discussions

Polarizer and analyzer transmission axis are kept perpendicular to each other. Quarter wave plate is inserted between the cross polarizer-analyzer arrangements and is oriented at +45˚, so maximum transmission can be observed in photodetector. To observe interference colour, photodetector is removed from the path and observations are made directly by eye. Which interference colour will be observed entirely depends upon two factors: the thickness of the QWP and the material of which it is made. Then QWP is tilted with respect to the vertical axis (parallel to the transmission axis of polarizer).

As QWP is tilted with respect to the vertical axis (parallel to the transmission axis of polarizer), it changes the plane of the wave plate with respect to the incident wave and due to this change apparent thickness of the quarter wave plate has changed. Change in apparent thickness directly affect the path difference between e-ray and o-ray. QWP is oriented at maximum transmission condition hence it is either tilted with respect to the fast or slow axis. When the quarter wave plate is tilted about the fast axis its constructive interference condition is changed to shorter wavelengths due to less retardation but when tilted about the slow axis it is changed to longer wavelengths due to more retardation.

White light is observed when a quarter wave plate made up of mica is oriented at +45˚ and kept between cross polarizers. When the quarter wave plate is tilted by keeping the vertical axis perpendicular to the prism table (parallel to the transmission axis of polarizer) white light turns grey and then turns black, if it is the fast axis of QWP, and it turns light yellow, if it is the slow axis. Interference colours observed are shown in Figure 3.19. Interference colours observed are present in the first order of the Michel Levy interference colour chart. Grey and black interference colours are present in first order and show less retardation, axis corresponds to the fast axis of the quarter wave plate. The yellow interference colour is present in the first order but show more retardation and corresponds to the slow axis. Therefore, the fast (slow) axis of the quarter wave plate is the axis that corresponds to less (more) retardation.

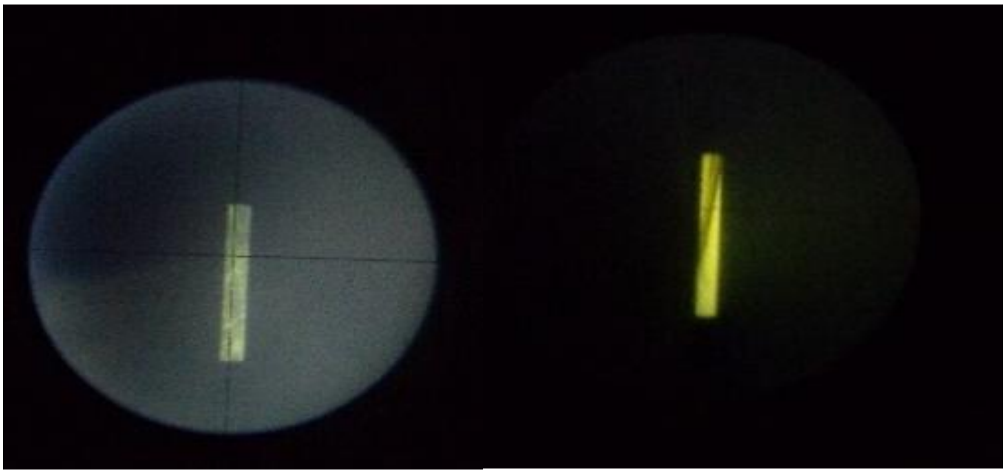


Figure 3.19: Figures depict colour as they appear when a quarter-wave plate composed of mica is tilted around one of the axes and seen through the telescope of the spectrometer. On the left, rotation around the fast axis causes white light to turn grey, and on the right, rotation around the slow axis causes white light to turn yellow.

The same experiment with the identical set-up was performed on a quartz wave plate with a known axis purchased from Holmarc. When quartz quarter wave plate is held between cross-polarized light at 45° without tilt, a pale-yellow colour is observed. When it is tilted about the slow axis, the pale-yellow colour changes to pink, and when it is tilted about the fast axis, the colour changes to cyan green. Observed colours are shown in Figure 3.20. In the Michel Levy Chart's second order, both colours are present where cyan green interference colour indicates less retardation while pink interference colour indicates more retardation. We might thus speculate that the axis corresponding to the cyan green interference colour is the fast axis and the axis corresponding to the pink interference colour is the slow axis.(Shah & Ghalsasi, 2019)

Figure 3.20: Figure depicts the colours as they appear through the telescope eyepiece of the spectrometer when a quarter-wave plate made of quartz is tilted around one of the axes. Due to rotation on the fast axis (reduced retardation), on the left, the light-yellow colour changes to cyan green, while on the right, the pale-yellow colour changes to pink (more retardation).

We used an LED light source (a white LED) to do the same axis determination test. Put the polarizer and the analyzer in a crossed position and inserted QWP made up of mica between them, white light was observed in the output. When wave plate was tilted about its fast axis white light changes to dull white light while when tilted about its slow axis white light changes to pale yellow colour. However, there was no colour change observed in the QWP made up of quartz. Therefore, one may conclude that white light LEDs do not discriminate between the fast and slow axes because the colour variations are minimal when the QWPs are rotated around either axis.

To observe photocurrent fluctuation with rotation angle, quarter wave plate made up of mica was kept between cross polarizers and was rotated uniformly. Experiments were performed by using mercury and sodium light Source. Photocurrent was recorded with respect to the rotation angle for one complete rotation. Four positions were registered with maximum intensity at the interval of 90˚ for one complete rotation. Two positions in the interval of 180˚ correspond to the fast axis and other two positions correspond to the slow axis. The maximum current that we notice in the photo detector was due to the production of the circularly polarized light, when either of two axes is oriented at 45˚ with respect to the polarizer axis. In photodetector we observe maximum current 2.9 μA for mercury sources and 4 μA for sodium light sources as shown in Figure 3.21. (Shah & Ghalsasi, 2019)

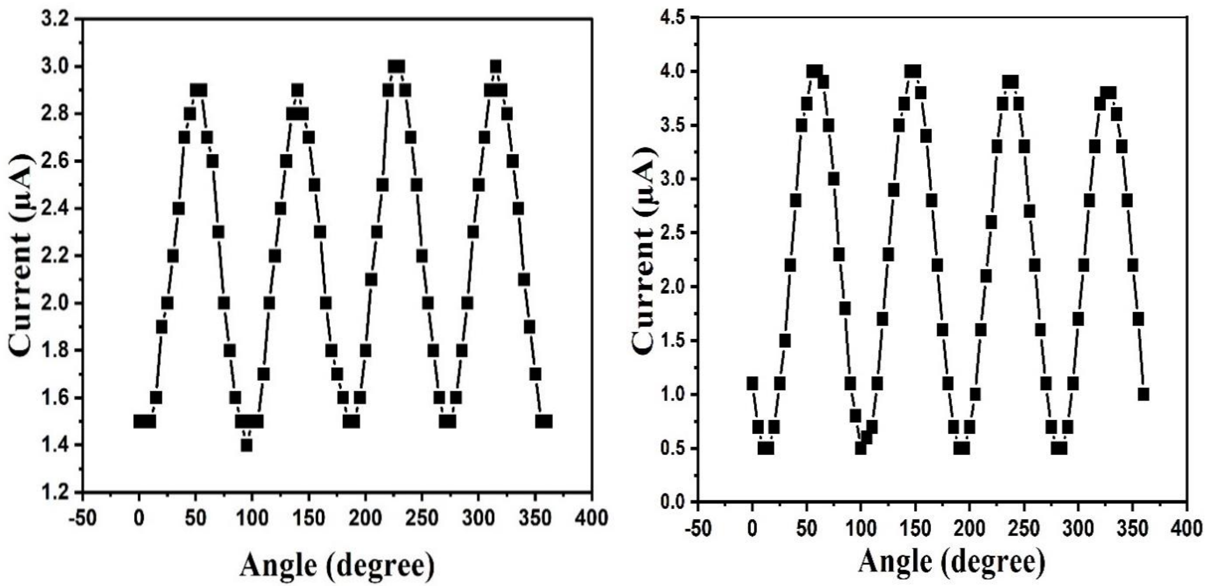


Figure 3.21: The left plot relates to photocurrent fluctuation with rotation angle caused by Mercury source, whereas the right plot corresponds to photocurrent variation caused by Sodium source when quarter mica wave plate is placed between cross polarizers. The graph's peaks

represent the angles when the intensity is at its maximum. At maximal intensity, the fast and slow axes' positions may be determined.

Additionally, in the second attempt, we performed an experiment by using sodium light and mercury light sources and kept the transmission axes of the polarizer and analyzer parallel to each other as shown in Figure 3.22. Quarter wave plate made of mica was used in the experiment. The quarter wave plate was inserted between parallel polarizers and rotated to observe the intensity variation with rotation angle.

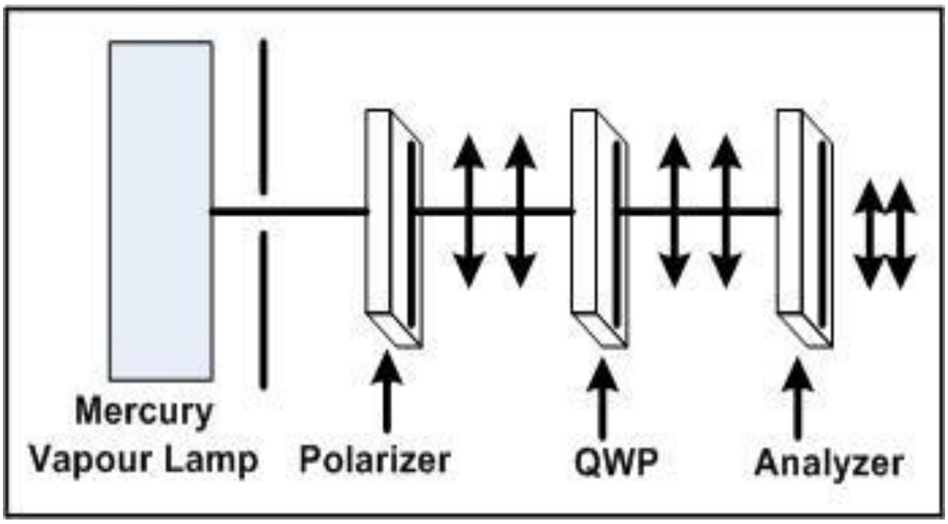


Figure 3.22: A quarter wave plate is placed between a polarizer and an analyzer in a parallel arrangement and rotated to monitor intensity fluctuations.

Maximum transmission is observed when the quarter wave plate's fast or slow axis is aligned parallel to the polarizers' transmission axes. The intensity of the transmitted light is maximum at 90° intervals, alternate intensity peaks correspond to the fast and slow axes. Our experiment setup that includes a mercury source shows a minor current difference between two peaks that correspond to the fast and slow axes. The error of the photocurrent meter used in this experiment is 0.05 μA. The first peak current value is less (2.5 μA), corresponds to the slow axis, while the second peak value, 2.6 μA, corresponds to the fast axis. When a sodium source is used, the slow axis shows 6 μA and the fast axis shows 6.9 μA current. Maximum current values can be observed in Figure 3.23. Since the fast axis has a lower refractive index than the slow axis, there is higher transmission in the fast axis and thus higher photocurrent. Even very small differences in currents were observed, the results still support the findings of our initial experiment. The purpose of this method is just to confirm the outcomes of the first method.

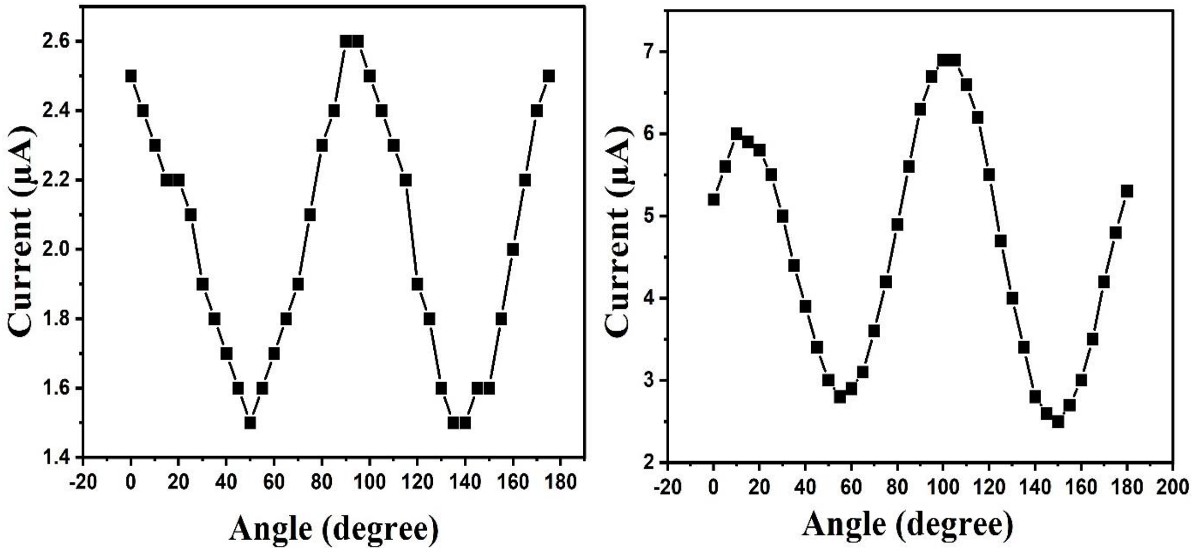


Figure 3.23: QWP made up of mica is placed between parallel polarizers and rotated, and intensity variation as a function of rotation is plotted. The left graph corresponds to a Mercury source, and the right graph corresponds to a Sodium source.

This method cannot be used first to identify the fast and slow axes of a quarter wave plate as differences in currents are within the error limit. This method can be used by using a more sensitive digital meter with high accuracy so that large current differences can be observed.

3.4 Conclusion

The current study describes an experimental method to find the fast and slow axes of a quarter wave plate comprised of mica and quartz using the minimum resources. A quarter wave plate is positioned to get the maximum amount of light when kept between two cross polarizers. Then it (QWP) is rotated along a vertical axis parallel to the polarizer's transmission axis. Then the colours that are observed are related to the optical path difference of the interacting o and e-rays. The optical path difference corresponding to the fast axis is less, while there is a greater path difference for the slow axis. This is because the refractive index is lower (or higher) along the fast (or slow) axis. With the help of the Michel Levy interference colour chart, it is possible to explain the observed colours and correlate it to the optical path difference.