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Airy's stress Solution for Isotropic Rings with Eccentric Hole Subjected to Pressure

Payal Desai[a]* and Visaj Pandya[b]

[a] *Associate Professor, School of Science and Engineering, Navrachana University, Bhayli, Vadodara 391 410 India.*

[b] *b Graduate student, School of Science and Engineering, Navrachana University, Bhayli, Vadodara 391 410 India.*

**Corresponding author*

Abstract

Elasticity solution is obtained for stresses in rings with eccentric holes in bipolar coordinate system. The ring is subjected to internal/external pressure at inner and outer boundaries. Materials of the cylinder is assumed to be isotropic and homogeneous. Symmetrical deformations are included in the analysis, yet it is necessary to calculate stresses with various parameters such as angle between the foci and logarithmic ratio of the foci, both defines the coordinates of the bipolar system appeared in the final formulations. With the help of Airy's stress function approach in bipolar coordinate system, the chosen function satisfies the bi harmonic equations in bipolar coordinate system. The results obtained in this study shows the greater effect of eccentricity on the stresses in bipolar coordinates $\sigma\beta$ and $\sigma\alpha$, the analogous stresses in polar coordinates as radial and hoop stresses for cylinders and rings with concentric holes. Along the boundaries, the stresses $\sigma\beta$ coincides with the external applied pressure on the boundaries, shows the valid results in agreement with the formulations.

Keywords: Bipolar coordinate system, Eccentric rings and cylinders, Airy's stress function, Elasticity solution

1. INTRODUCTION

Rings or cylinders with hole are widely used in engineering structure. e.g. pressure vessels, missiles, aircraft, etc. Most of the solid elastic structures used in pressure vessels, missiles, aircraft, etc. have holes introduced either for reducing weight or for the purpose of inspection. These holes are usually causing the change of stresses, displacements and decrease the load carrying capacity. It is important to comprehend the associated effects in the analysis and structural/mechanical design or flight control of the structure.

Numerical Results for plate with concentric hole are common and available in much literature. The rings with eccentric hole or multiple holes have not been investigated by many for elastic body in the past. The main reason for this is that the problems formulated are in Cartesian and polar coordinates system. The Cartesian and polar coordinates systems have limited applications for tackling the problems having straight/circular shape and surface boundaries, respectively. The problems having eccentricity and geometric irregularities are occurred in practice and difficult to analyse, such as eccentric annulus, two parallel cylinders, cylinders with eccentric holes, plates with eccentric holes, elastic half space etc…Such problems are complex when attempt is made to analyse it analytically or computationally using Cartesian or Polar coordinate systems. The problems of eccentric holes, annulus rings, two cylinders etc. can better be treated in bipolar coordinates system which is a two dimensional orthogonal curvilinear coordinate. This is mainly because the shape of the boundary surface of certain bodies can be expressed most conveniently in certain curvilinear coordinate [1]. As a result, the boundary conditions can be expressed in simple forms. Some analytical work has been done in the past in bi polar coordinate system.

Identification of stresses in a homogeneous isotropic disc weakened with an eccentric circular hole is given in [2] using complex variable method. Stresses induced in an infinite plate with two unequal circular holes by remote uniform loadings and arbitrary internal pressures in the holes in [3] and stresses induced in an elastic and isotropic disk by an eccentric press-fitted circular inclusion is given in [4]. The inclusion is assumed to be of the same material as the annular disk and both elements are in a plane stress or plane strain state[4]. Green's functions of Laplace problems containing circular boundaries are solved by using analytical and semi-analytical approaches in [5].Mathematical formulations for elastic stress fields derived in bipolar coordinate systems are seen in [6].

The effect of variation in the eccentricity of the hole on the vibration characteristics of plates is investigated in [7] using finite element method. Using N. I. muskhelishvili's method, multi valued displacement problem is considered for an eccentric ring in [8], The method of separation of variables, an exact analytical solutions of 2D problems of elasticity is given [9]. Formulas have been obtained to connect the basis solutions of the lame vector equation in polar and bipolar coordinates in[10]. In Jeffery's paper [11], the bi harmonic equation is given in bi polar coordinates, for which the coordinate curves are coaxial circles, solution enables to treat the problems of an infinite plate containing two circular holes, a semi infinite plate bounded by a straight edge and containing one circular hole, and a circular disc with an eccentric hole. Okumura has derived stress and strain tensors, the equilibrium equations in orthogonal curvilinear coordinates. The solution of the Laplace equation is obtained [12] by using transformation in a transformed plane in the complex variable theory with focus on the connection between conformal mapping and curvilinear coordinates. The thermal stresses produced when a uniform heat flow is disturbed by two insulated unequal circular holes in an infinite elastic plate, are obtained in the paper [13]. General formulations of the Toupin–Mindlin strain gradient theory in orthogonal curvilinear coordinate systems are derived in [14].Natural frequencies and natural modes for circular plates with multiple circular holes by using the indirect formulation in conjunction with degenerate kernels and Fourier series is given by [15]. The problem of normal contact with friction of a rigid sphere with an elastic half-space is considered in the toroidal coordinates in [16]. Problem of adhesive contact of a rigid cylinder with an elastic half-space is considered in bipolar coordinates is given in [17]. Using the Wiener-Hopf technique which allowed for a detailed analysis of the contact stresses, strain, displacement, and relative slip zone sizes for the problem of indentation with friction of a rigid cylinder into an elastic half space in bipolar coordinates is given by [18]. Problem is reduced to a singular integral equation with respect to the unknown normal stress in the slipzones. There has been less analytical investigation on the effect of radial inhomogeneity upon the elastic field in non-axisymmetric problems other than plates see, e.g.[19].

The work cited in literature shows us the analytical solutions of problems in bipolar, toroidal and orthogonal curvilinear coordinates includes elastic half space, annulus cylinder, plate with eccentric hole, eccentric rings. In most of the work, complex variable method, conformal mappings, biharmonic equations, Laplace equations are used for isotropic and homogeneous materials.

Plane strain problems of elastic eccentric rings are studied in bipolar coordinates in the present work. Two dimensional elastic eccentric ring has been defined initially in bipolar coordinate system which is a coordinate system having two foci. Stress analysis of eccentric ring is carried out subjected to uniform internal and external pressure using Airy's stress function approach. Stresses are obtained with various parameters such as eccentricity, angles between two foci and natural logarithmic ratios of two foci. Effect of eccentricity is investigated on the stresses of the eccentric rings.

2. PROBLEM FORMULATION

There are several ways to define bipolar coordinates. In this document, the coordinates will be called β and α with following ranges [20],

$$-\infty \leq \beta \leq \infty\,, 0 \leq \alpha \leq 2\pi \tag{1}$$

The Relation between the Cartesian and bipolar systems are given by

$$x = \frac{a\sin\beta}{\cosh\alpha - \cos\beta} \qquad y = \frac{a\sinh\alpha}{\cosh\alpha - \cos\beta} \tag{2}$$

The problem of eccentric rings subjected to uniform internal and external pressure is considered and shown in Fig. 1. αandβ defined as bi-polar coordinates. We assume rings are made up of elastic material. The elastic ring has inner radius r, outer radius R and eccentricity e, also both member are subjected to the uniform pressure. The inner and outer contours of the eccentric ring are defined by $\beta_1$and β_2, respectively.

$$\beta_1 = \cos^{-1} h\frac{[(R^2) - (r^2) - (e^2)]}{2er}\beta_2 = \cos^{-1} h\frac{[(R^2) - (r^2) + (e^2)]}{2eR} \tag{3}$$

The length a and the thickness of the ligament λ are given by

$$a = \frac{1}{2e}\sqrt{(R^2 + r^2 - e^2)} - 4r^2R^2, \qquad \lambda = R - r - e \tag{4}$$

Where, a is a positive length and e is an centre to centre distance between outer circle and inner circle. Inner(r) and outer radius(R) of the circle can be defined by following equations,

$$r = \frac{a}{\sinh\beta},\ R = \frac{a}{sinh\beta} \tag{5}$$

The problem is formulated using an Airy stress functionφ, which satisfied the biharmonic equation [11],

$$\frac{\delta^4 \mathrm{hx}}{\delta\beta^4} + 2\frac{\delta^4 \mathrm{hx}}{\delta\alpha^2\delta\beta^2} + \frac{\delta^4 \mathrm{hx}}{\delta\alpha^4} + 2\frac{\delta^2 \mathrm{hx}}{\delta\beta^2} - 2\frac{\delta^2 \mathrm{hx}}{\delta\alpha^2} + \mathrm{hx} = 0 \tag{6}$$

Where,

$$hx = \frac{\varphi}{a^2(cosh\alpha - cos\beta)},$$

hx is the modified stress function, since the stress distribution is symmetric, with respect to α coordinate, the modified stress function can be chosen for the general formula of biharmonic equation (6).

Following stress function is chosen for the present work,

$$\varphi \ = A \ log\beta \ + \ B\beta^2 \ log\beta \ + \ C\beta^2 \ + \ D \tag{7}$$

The modified stress function will become,

$$h_x = \frac{A \ log\beta^2 cosh\alpha + C\beta^2 cosh\alpha - A \ log\beta^2 cos\beta - C\beta^2 cos\beta}{a^2} \tag{8}$$

Corresponding stress are given by the relations:

$$\sigma\beta \ = \frac{\partial^2 hx}{\partial\beta^2}(cosh\alpha \ - \ cos\beta) - \frac{\delta hx}{\delta\alpha} sinh\alpha - \frac{\partial hx}{\partial\beta}(sin\beta) + \ hx \ cosh\alpha \tag{9}$$

$$\sigma\alpha \ = \frac{\partial^2 hx}{\partial\alpha^2}(cosh\alpha \ - \ cos\beta) - \frac{\delta hx}{\delta\alpha} sinh\alpha - \frac{\partial hx}{\partial\beta}(sin\beta) + \ hx \ cos\beta$$

$$\tau\beta\alpha \ = \ -\frac{\delta^2 hx}{\delta\beta\delta\alpha}(cosh\alpha - cos\beta)$$

Two times derivation of stress function hx with respect to β,α and β, α are found in order to obtain normal stresses in β and α direction and shear stresses. Following stresses have been obtained after derivations with respect to corresponding bipolar coordinates;

From derivation stress function will become,

$$\frac{\partial hx}{\partial\beta} = \ A \ \left[\frac{cosh\alpha}{\beta} - \frac{cos\beta}{\beta} + \ sin\beta \ log\beta\right] + \ C \ [2\beta \ cosh\alpha \ - \ 2\beta \ cos\beta \ + \ sin\beta \ \beta^2] \tag{10}$$

$$\frac{\partial^2 hx}{\partial\beta^2} = \ A \ \left[-\frac{cosh\alpha}{\beta^2} + \frac{2sin\beta}{\beta} + \frac{cos\beta}{\beta^2} + \ cos\beta \ log\beta\right] + \ C \ [2cosh\alpha \ - \ 2cos\beta \ + \ 4\beta \ sin\beta \ + \ cos\beta \ \beta^2] \tag{11}$$

Stresses in β and αdirections have been found based on above derived stresses function hence, stresses in β direction will become,

$$\sigma\beta \;=\; A/a^2[\left(-\frac{cosh\alpha^2}{\beta^2}\right) + \;(log\beta\; cos\beta\; cosh\alpha) + \;\left(2sin\beta\,\frac{cosh\alpha}{\beta}\right)$$
$$+\;\left(cos\beta\,\frac{cosh\alpha}{\beta^2}\right) + \;\left(cosh\alpha\,\frac{cos\beta}{\beta^2}\right) - \;(log\beta)(cos\beta^2)$$
$$-\;(2\; sin\beta\; cos\beta/\beta) \;-\; (cos\beta^2/\beta^2) \;-(cosh\alpha)\;(sin\beta)/\beta$$
$$-\;(\log\beta)(sin\beta^2) \;+\; (cos\beta\; sin\beta/\beta) \;+\; log\beta\;(cosh\alpha^2)$$
$$-\;(\log\beta\; cos\beta\; cosh\alpha)] \;+\; C/a^2\;[2(cosh\alpha^2) \;-\; (2\; cos\beta\; cosh\alpha)$$
$$+\;(4\beta\; sin\beta\; cosh\alpha) \;+\; (\beta^2\; cos\beta\; cosh\alpha \;-\; (2\; cosh\alpha\; cos\beta)$$
$$+\;2(cos\beta^2) \;-\; (4\;\beta\; sin\beta\; cos\beta) \;-\; (\beta^2)\;(cos\beta^2)$$
$$-\;(2\beta)\;(cosh\alpha\; sin\beta) \;-\; (2\beta\; cos\beta\; sin\beta) \;+\; (\beta^2)\;(sin\beta^2)$$
$$+\;(\beta^2)\;(cosh\alpha^2) \;-\; (\beta^2)\; cos\beta\; cosh\alpha] \tag{12}$$

$$\sigma\alpha \;= \frac{A}{a^2}\Big[(log\beta)(cosh\alpha^2) \;-\; (log\beta)(cosh\alpha)(cos\beta) - \;\left(\frac{cosh\alpha}{\beta}\right)sin\beta$$
$$-\;(log\beta)(sin\beta^2) \;+ \frac{cos\beta\; sin\beta}{\beta} + \;(log\beta\; cosh\alpha\; cos\beta)$$
$$-\;(log\beta)(cos\beta^2)\Big]$$
$$+\frac{C}{a^2}[(\beta^2)(cosh\alpha^2) \;-\; (\beta^2)(cos\beta) - \;(2\beta)(cosh\alpha)sin\beta$$
$$+\;(2\beta)(cos\beta)(sin\beta) - \;(\beta^2)(cosh\alpha\; cos\beta) + \;(\beta^2)(cos\beta^2)] \tag{13}$$

$$\tau_{\beta\alpha} = 0 \tag{14}$$

Inner and outer contours of the elastic rings are defined as $\beta_1 and \beta_2$ respectively, Along the boundaries, the stresses σ_β coincides with the external applied pressure on the boundaries, the boundary conditions are,

$$(\sigma)_{\beta=\beta 1} = \;-P_i, (\sigma)_{\beta=\beta 2} = \;-P_0 \tag{15}$$

Shear stresses are assumed to be vanished on both boundaries,

$$\tau_{\alpha\beta}(\beta = \beta_1\,) = 0\;;\; \tau_{\beta\alpha}\,(\beta = \beta_2\,) = 0 \tag{16}$$

whereP_i and P_o are the uniformly internal and external pressures. Applying boundary condition (15) on equation (12) on inner and outer contours, following equations are obtained.

$$
\begin{aligned}
\sigma\beta_1 \;=\; A/a^2[&\left(-\frac{cosh\alpha^2}{{\beta_1}^2}\right)+\;(log\beta_1\,cos\beta_1\,cosh\alpha)+\;\left(2sin\beta_1\frac{cosh\alpha}{\beta_1}\right)\\
&+\;\left(cos\beta_1\frac{cosh\alpha}{{\beta_1}^2}\right)+\;\left(cosh\alpha\frac{cos\beta}{{\beta_1}^2}\right)-\;(log\beta_1)(cos{\beta_1}^2)\\
&-\;(2\,sin\beta_1\,cos\beta_1/\beta_1)\;-\;(cos{\beta_1}^2/{\beta_1}^2)\;-(cosh\alpha)\,(sin\beta_1)/\beta_1\\
&-\;(\log\beta_1)(sin{\beta_1}^2)\;+\;(cos\beta_1\,sin\beta_1/\beta_1)\;+\;log\beta_1\,(cosh\alpha^2)\\
&-\;(\log\beta_1\,cos\beta_1\,cosh\alpha)]\;+\;C/a^2\,[2(cosh\alpha^2)\;-\;(2\,cos\beta_1\,cosh\alpha)\\
&+\;(4\beta_1\,sin\beta_1\,cosh\alpha)\;+\;({\beta_1}^2\,cos\beta_1\,cosh\alpha\;-\;(2\,cosh\alpha\,cos\beta_1)\\
&+\;2(cos{\beta_1}^2)\;-\;(4\,\beta_1\,sin\beta_1\,cos\beta_1)\;-\;({\beta_1}^2)\,(cos{\beta_1}^2)\\
&-\;(2\beta_1)\,(cosh\alpha\,sin\beta_1)\;-\;(2\beta_1\,cos\beta_1\,sin\beta_1)\;+\;({\beta_1}^2)\,(sin{\beta_1}^2)\\
&+\;({\beta_1}^2)\,(cosh\alpha^2)\;-\;({\beta_1}^2)\,cos\beta_1\,cosh\alpha]
\end{aligned}
$$

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(17)

$$
\begin{aligned}
\sigma\beta_2 \;=\; A/a^2[&\left(-\frac{cosh\alpha^2}{{\beta_2}^2}\right)+\;(log\beta_2\,cos\beta_2\,cosh\alpha)+\;\left(2sin\beta_2\frac{cosh\alpha}{\beta_2}\right)\\
&+\;\left(cos\beta_2\frac{cosh\alpha}{{\beta_2}^2}\right)+\;\left(cosh\alpha\frac{cos\beta}{{\beta_2}^2}\right)-\;(log\beta_2)(cos{\beta_2}^2)\\
&-\;(2\,sin\beta_2\,cos\beta_2/\beta_2)\;-\;(cos{\beta_2}^2/{\beta_2}^2)\;-(cosh\alpha)\,(sin\beta_2)/\beta_2\\
&-\;(\log\beta_2)(sin{\beta_2}^2)\;+\;(cos\beta_2\,sin\beta_2/\beta_2)\;+\;log\beta_2\,(cosh\alpha^2)\\
&-\;(\log\beta_2\,cos\beta_2\,cosh\alpha)]\;+\;C/a^2\,[2(cosh\alpha^2)\;-\;(2\,cos\beta_2\,cosh\alpha)\\
&+\;(4\beta_2\,sin\beta_2\,cosh\alpha)\;+\;({\beta_2}^2\,cos\beta_2\,cosh\alpha\;-\;(2\,cosh\alpha\,cos\beta_2)\\
&+\;2(cos{\beta_2}^2)\;-\;(4\,\beta_2\,sin\beta_2\,cos\beta_2)\;-\;({\beta_2}^2)\,(cos{\beta_2}^2)\\
&-\;(2\beta_2)\,(cosh\alpha\,sin\beta_2)\;-\;(2\beta_2\,cos\beta_2\,sin\beta_2)\;+\;({\beta_2}^2)\,(sin{\beta_2}^2)\\
&+\;({\beta_2}^2)\,(cosh\alpha^2)\;-\;({\beta_2}^2)\,cos\beta_2\,cosh\alpha]
\end{aligned}
$$

(18)

Unknown constants A and C are found from the boundary conditions and substituted into the equations (12) and (13). For the simplification following notations are assumed,

$$A_1 = [\left(-\frac{cosh\alpha^2}{{\beta_1}^2}\right) + (log\beta_1\, cos\beta_1\, cosh\alpha) + \left(2sin\beta_1 \frac{cosh\alpha}{\beta_1}\right) + \left(cos\beta_1 \frac{cosh\alpha}{{\beta_1}^2}\right) + \left(cosh\alpha \frac{cos\beta}{{\beta_1}^2}\right) - (log\beta_1)({cos\beta_1}^2) - (2\, sin\beta_1\, cos\beta_1/\beta_1) - ({cos\beta_1}^2/{\beta_1}^2) - (cosh\alpha)\, (sin\beta_1)/\beta_1 - (\log\beta_1)({sin\beta_1}^2) + (cos\beta_1\, sin\beta_1/\beta_1) + log\beta_1\, (cosh\alpha^2) - (\log\beta_1\, cos\beta_1\, cosh\alpha)] \tag{19}$$

$$A_2 = [\left(-\frac{cosh\alpha^2}{{\beta_2}^2}\right) + (log\beta_2\, cos\beta_2\, cosh\alpha) + \left(2sin\beta_2 \frac{cosh\alpha}{\beta_2}\right) + \left(cos\beta_2 \frac{cosh\alpha}{{\beta_2}^2}\right) + \left(cosh\alpha \frac{cos\beta}{{\beta_2}^2}\right) - (log\beta_2)({cos\beta_2}^2) - (2\, sin\beta_2\, cos\beta_2/\beta_2) - ({cos\beta_2}^2/{\beta_2}^2) - (cosh\alpha)\, (sin\beta_2)/\beta_2 - (\log\beta_2)({sin\beta_2}^2) + (cos\beta_2\, sin\beta_2/\beta_2) + log\beta_2\, (cosh\alpha^2) - (\log\beta_2\, cos\beta_2\, cosh\alpha)] \tag{20}$$

$$C_1 = [2(cosh\alpha^2) - (2\, cos\beta_1\, cosh\alpha) + (4\beta_1\, sin\beta_1\, cosh\alpha) + ({\beta_1}^2\, cos\beta_1\, cosh\alpha - (2\, cosh\alpha\, cos\beta_1) + 2({cos\beta_1}^2) - (4\,\beta_1\, sin\beta_1\, cos\beta_1) - ({\beta_1}^2)\, ({cos\beta_1}^2) - (2\beta_1)\, (cosh\alpha\, sin\beta_1) - (2\beta_1\, cos\beta_1\, sin\beta_1) + ({\beta_1}^2)\, ({sin\beta_1}^2) + ({\beta_1}^2)\, (cosh\alpha^2) - ({\beta_1}^2)\, cos\beta_1\, cosh\alpha] \tag{21}$$

$$C_2 = [2(cosh\alpha^2) - (2\, cos\beta_2\, cosh\alpha) + (4\beta_2\, sin\beta_2\, cosh\alpha) + ({\beta_2}^2\, cos\beta_2\, cosh\alpha - (2\, cosh\alpha\, cos\beta_2) + 2({cos\beta_2}^2) - (4\,\beta_2\, sin\beta_2\, cos\beta_2) - ({\beta_2}^2)\, ({cos\beta_2}^2) - (2\beta_2)\, (cosh\alpha\, sin\beta_2) - (2\beta_2\, cos\beta_2\, sin\beta_2) + ({\beta_2}^2)\, ({sin\beta_2}^2) + ({\beta_2}^2)\, (cosh\alpha^2) - ({\beta_2}^2)\, cos\beta_2\, cosh\alpha] \tag{22}$$

After the simplifications equation becomes,

$$P_i = \frac{A}{a^2}(A_1) + \frac{C}{a^2}(C_1)$$

$$P_o = \frac{A}{a^2}(A_2) + \frac{C}{a^2}(C_2) \tag{23}$$

Stresses can be found out using equations (12) and (13) with the help of constant value A and C substituted back in to the equations.

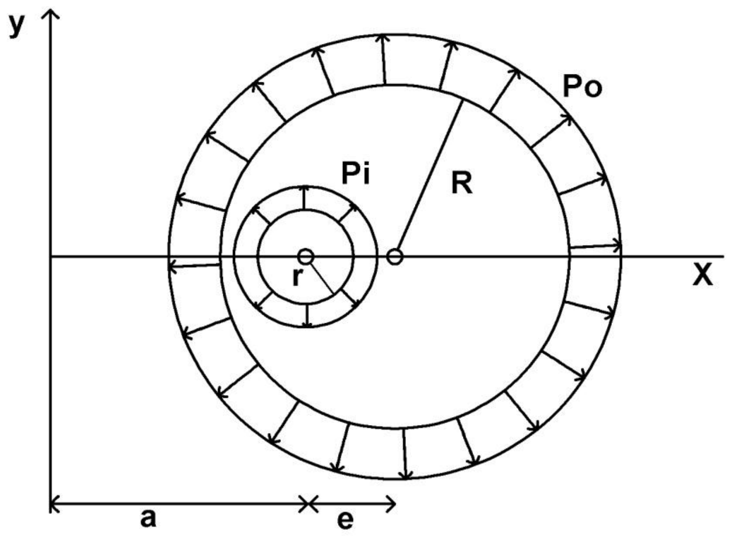

Fig. 1.1 Bipolar coordinates systems for problem of a disk/ring of radius R containing an eccentric circular hole of radius r in a disk.

3 RESULTS AND DISCUSSIONS

Numerical results are obtained for a ring under pressure applied on inner and outer boundaries for three cases varies with different r/R ratios gives thick, medium thick and thin elastic rings. Non dimensional parameters for are taken as σ_β/p_o, σ_α/p_o , β/R for plotting the curves.

For case I, Numerical value and variation of stresses σ_α and σ_β are shown in Fig.2.1 -2.7. For the case when a = 0.25 m, β= 0.246 m for inner circle and β= 1 m for outer circle. Result are obtained for R = 1.008 m, r = 0.213 m, β_1= 1.51 m,β_2 = 0.436 m and and$\alpha = \frac{\pi}{3}, \frac{\pi}{6}, \frac{\pi}{2}$,…radian. Value of inner and outer pressure is 1 psi or 6894.75 N/m^2. It is found from Figures that value of σ_α increases and σ_β decreases with β (coordinate which is a logarithmic ratio between the two focal length of two circles), [20] near the boundaries of the eccentric rings. Along the boundaries, the stresses σ_β coincides with the external applied pressure on the boundaries, shows the valid results in agreement with the formulations. When, β = β_2, Eq. (12) gives $\sigma_\beta = p_o$. The stresses for concentric rings under pressure is calculated [21, 22] to check the effect of eccentricity on the stresses as shown in Fig. 2.2, 3.2, 4.2. For the case 2, Numerical value and variation of stresses σ_α and σ_β are shown in Fig.3.1. - 3.7. For the case when a = 0.25 m, β= 0.246 m for inner circle and β= 0.7 m for outer circle. Result are obtained for R = 1.008 m, r = 0.32 m, β_1= 0.9 m,β_2 = 0.324 m. Value of inner and outer pressure is 1 psi or 6894.75 N/m^2. It is found from Figuers that value of σ_α

increases and σ_β decreases with β (coordinate which is a logarithmic ratio between the two focal length of two circles), [20] near the boundaries of the eccentric rings. When, β = β_2, Eq. (12) gives $\sigma_\beta = p_o$. For case 3, Numerical value and variation of stresses σ_α and σ_β are shown in Fig.4.1 – 4.7. For the case when a = 0.25 m, β= 0.246 m for inner circle and β= 0.6 m for outer circle. Result are obtained for R = 1.008 m, r = 0.4 m, β_1= 0.258 m,β_2 = 0.1 m.Value of inner and outer pressure is 1 psi or 6894.75 N/m^2. It is found from Figures that value of σ_α increases and σ_β decreases with β (coordinate which is a logarithmic ratio between the two focal length of two circles), [20] near the boundaries of the eccentric rings. When, β = β_2, Eq. (12) gives $\sigma_\beta = p_o$. Following observation is seen from the variations of stresses. When α = 0, σ_α approached maximum value at the outer boundaries, as α value increases, σ_β starts with increased value at the inner boundaries, and at some values of α attains maximum value at the inner boundary. Similar pattern is observed for all other cases of thin and medium thick rings, When, α = π/3, results with concentric rings (for r = r inner in case of concentric rings) are also plotted along with eccentric rings. It is observed that at α = π/3, values of $\sigma_r$and σ_θ is more at inner boundary as compared to analogous stresses σ_β and σ_α in bipolar coordinates for thin and medium thick cases as shown in Fig. 3.2 and 4.2. It is observed that at α = π/3, values of σ_r is less at inner boundary as compared to analogous stresses σ_β in bipolar coordinates for thick cases and reverse is the case at the outer boundary as shown in Fig. 2.2. Same is the case for σ_θ and σ_α. Minimum value of stresses are obtained at the inner boundary when $\alpha = 0, \pi/6$. Maximum values of stresses are obtained at the inner boundaries from $\alpha = \frac{\pi}{2}$ onwards.It is also seen, the numerical values of the stresses are very large for all values of α greater than $= \frac{\pi}{2}$, and there is a stress concentration especially for the case 2 and 3 when ring is thin. This is the case of stress singularity near to the inner boundaries as observed in fig. 3.6 – 3.7 and fig. 4.6 – 4.7.

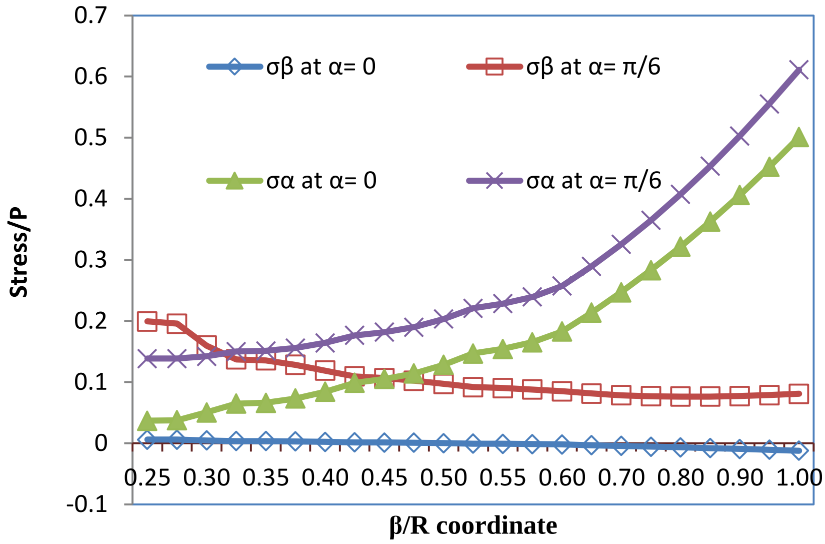

Fig.2.1 Variation of stresses σ_α and σ_β through the thickness and the bipolar coordinate β for $\alpha = 0, \frac{\pi}{6}$

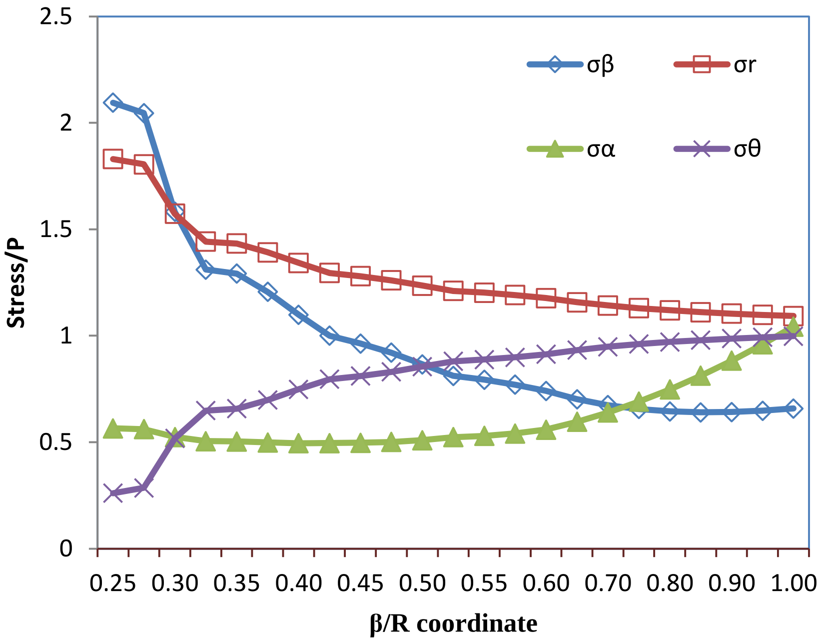

Fig.2.2 Variation of stresses σ_α and σ_β through the thickness and the bipolar coordinate β for $\alpha = \frac{\pi}{3}$

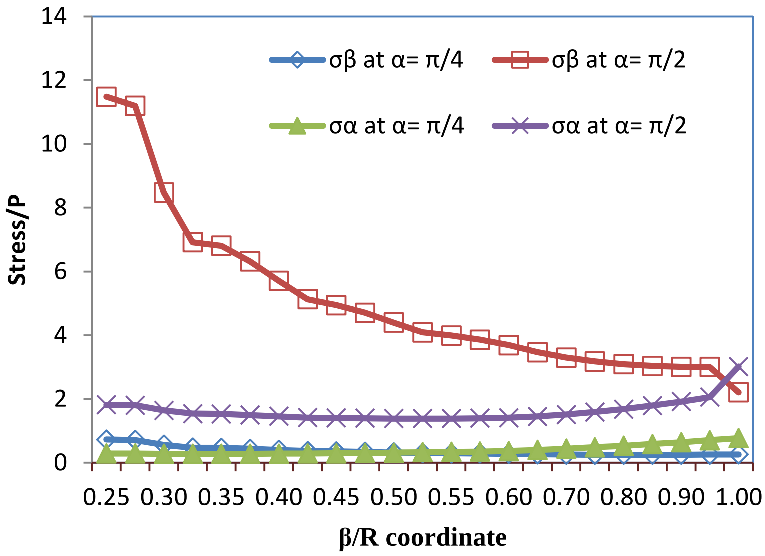

Fig.2.3 Variation of stresses σ_α and σ_β through the thickness and the bipolar coordinate β for $\alpha = \frac{\pi}{4}, \frac{\pi}{2}$

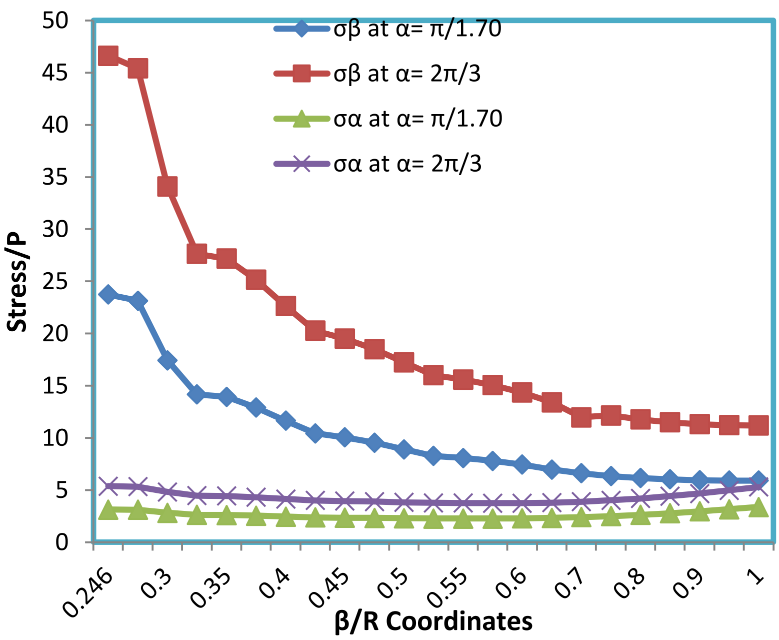

Fig.2.4 Variation of stresses σ_α and σ_β through the thickness and the bipolar coordinate β for $\alpha = \frac{\pi}{1.70}, \frac{2\pi}{3}$

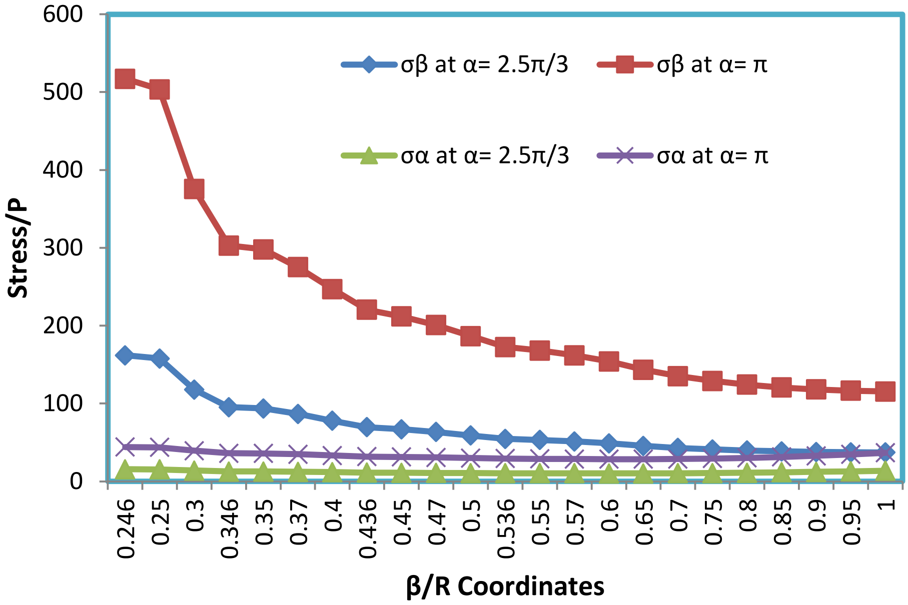

Fig.2.5 Variation of stresses σ_α and σ_β through the thickness and the bipolar coordinate β for $\alpha = \frac{2.5\pi}{3}, \pi$

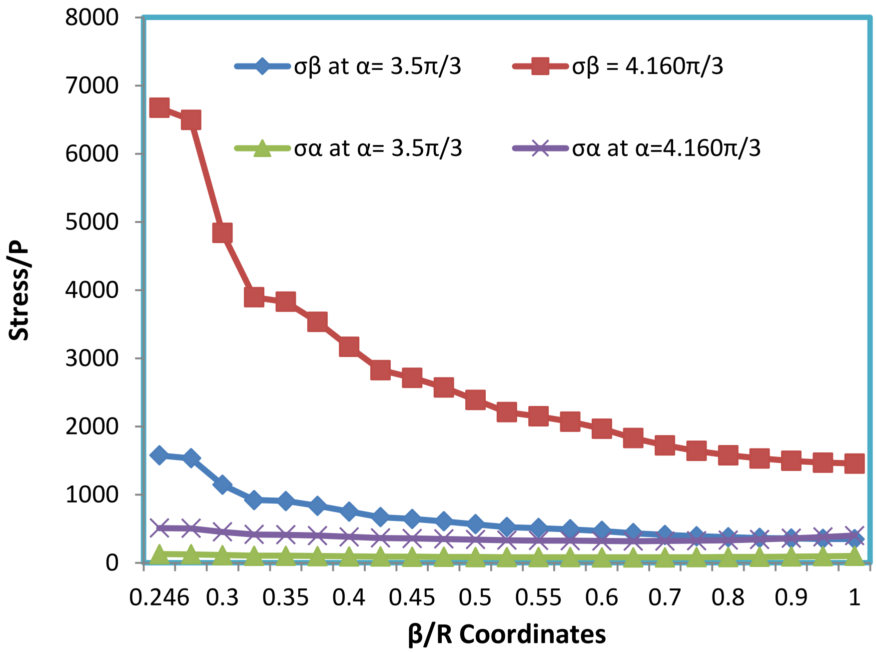

Fig.2.6 Variation of stresses σ_α and σ_β through the thickness and the bipolar coordinate β for $\alpha = \frac{3.5\pi}{3}, \frac{4.160\pi}{3}$

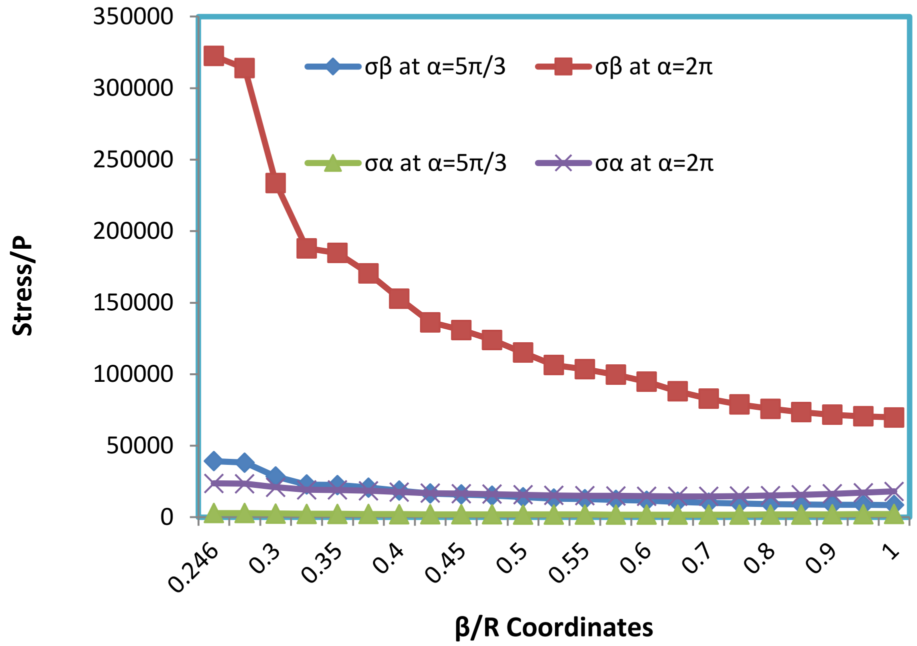

Fig.2.7 Variation of stresses σ_α and σ_β through the thickness and the bipolar coordinate β for $\alpha = \frac{5\pi}{3}, 2\pi$

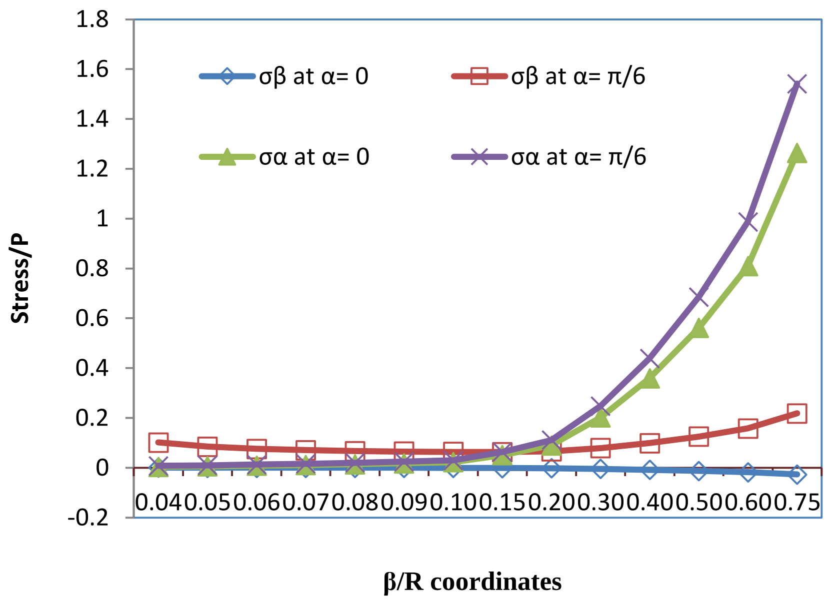

Fig.3.1 Variation of stresses σ_α and σ_β through the thickness and the bipolar coordinate β for $\alpha = \pi, \frac{\pi}{6}$

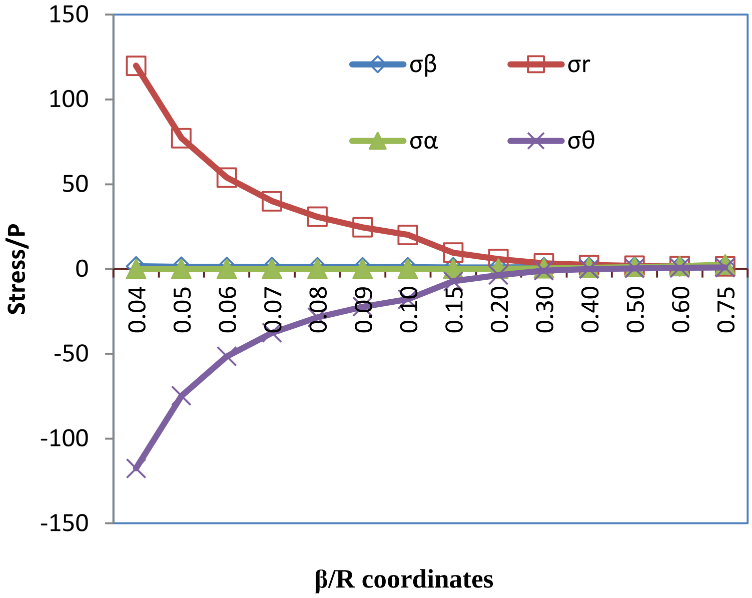

Fig.3.2 Variation of stresses σ_α and σ_β through the thickness and the bipolar coordinate β for $\alpha = \pi/3$

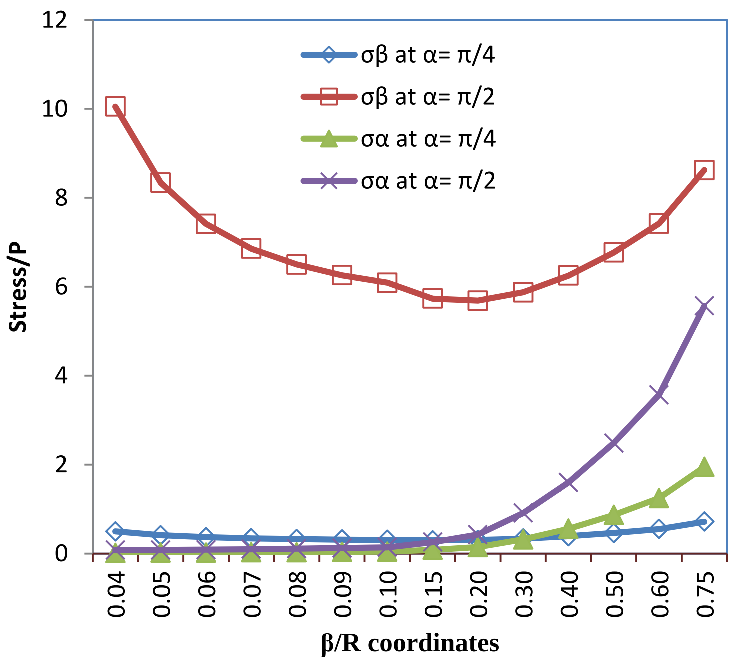

Fig.3.3 Variation of stresses σ_α and σ_β through the thickness and the bipolar coordinate β for $\alpha = \frac{\pi}{4}, \frac{\pi}{2}$

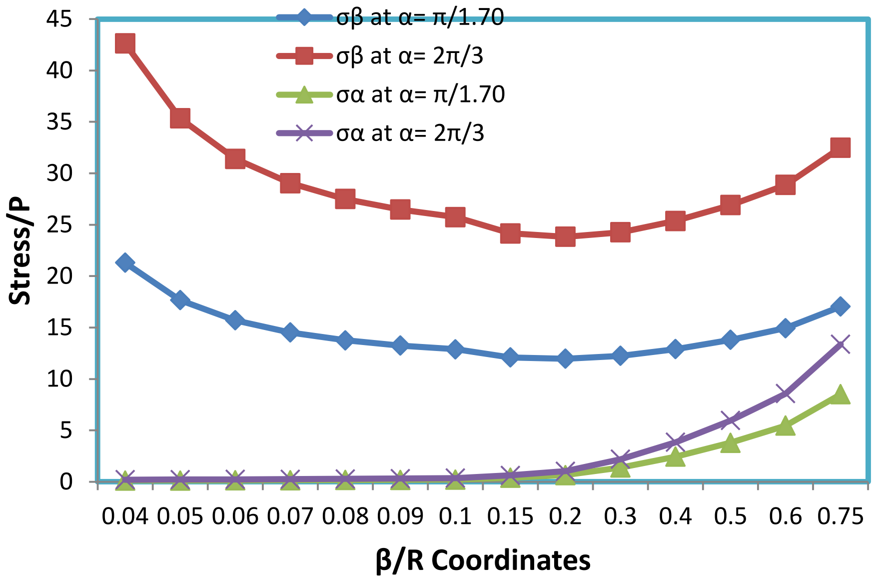

Fig.3.4 Variation of stresses σ_α and σ_β through the thickness and the bipolar coordinate β for $\alpha = \frac{\pi}{1.70}, \frac{2\pi}{3}$

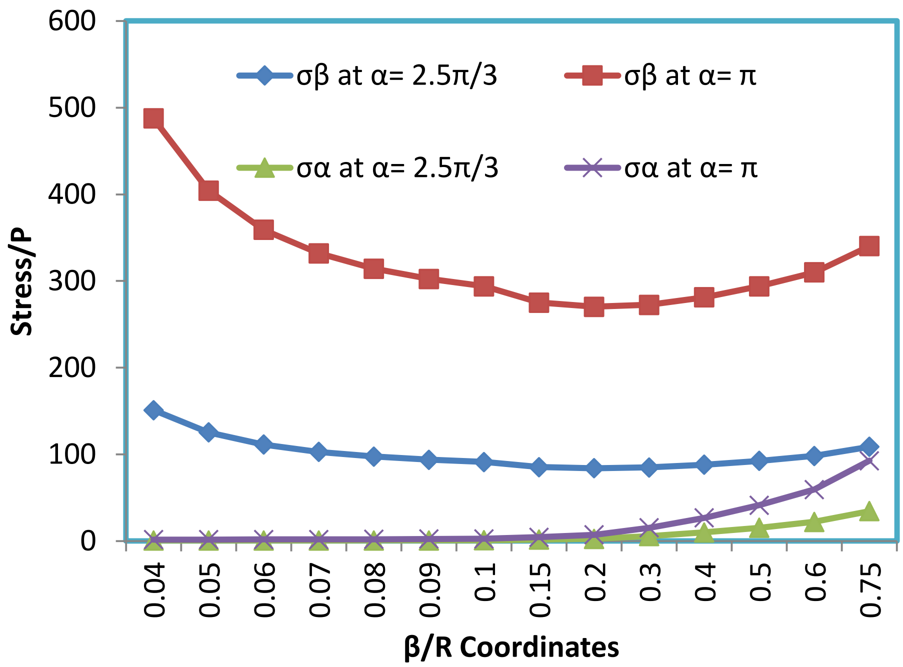

Fig.3.5 Variation of stresses σ_α and σ_β through the thickness and the bipolar coordinate β for $\alpha = \frac{2.5\pi}{3}, \pi$

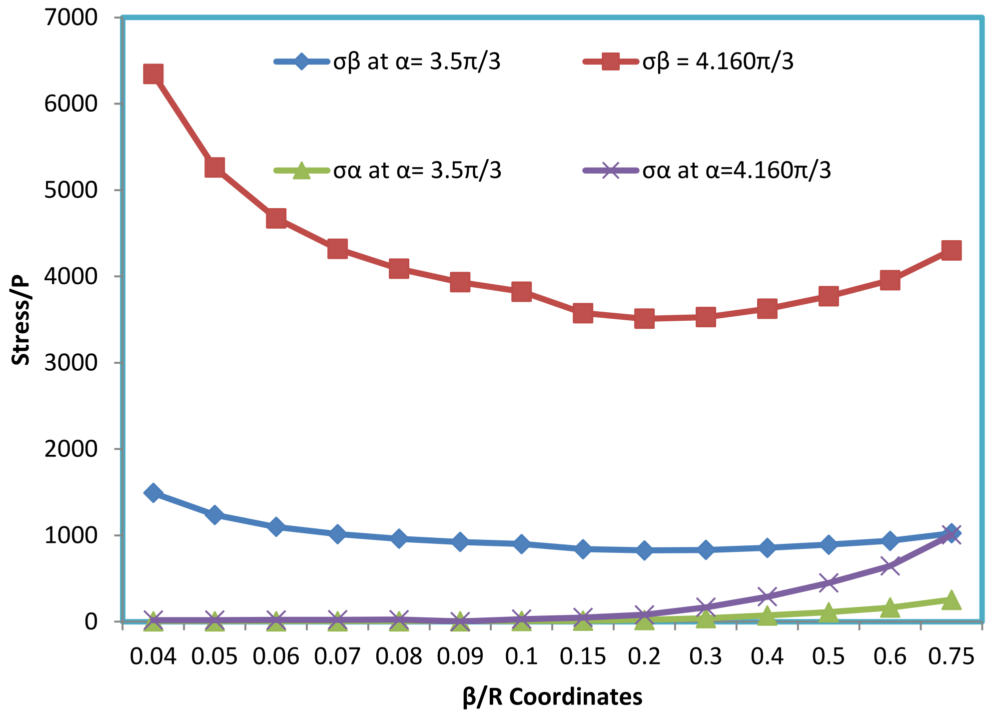

Fig.3.6 Variation of stresses σ_α and σ_β through the thickness and the bipolar coordinate β for $\alpha = \frac{3.5\pi}{3}, \frac{4.160\pi}{3}$

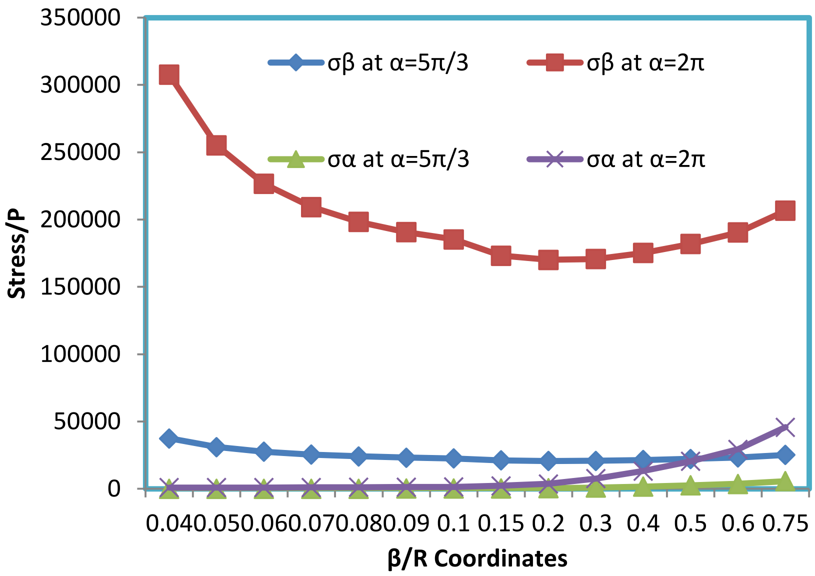

Fig.3.7 Variation of stresses σ_α and σ_β through the thickness and the bipolar coordinate β for $\alpha = \frac{5\pi}{3}, 2\pi$

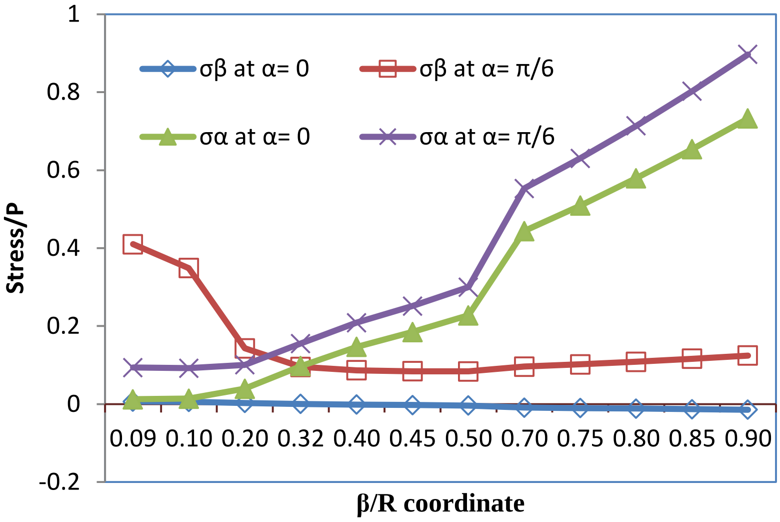

Fig.4.1 Variation of stresses σ_α and σ_β through the thickness and the bipolar coordinate β for $\alpha = 0, \frac{\pi}{6}$

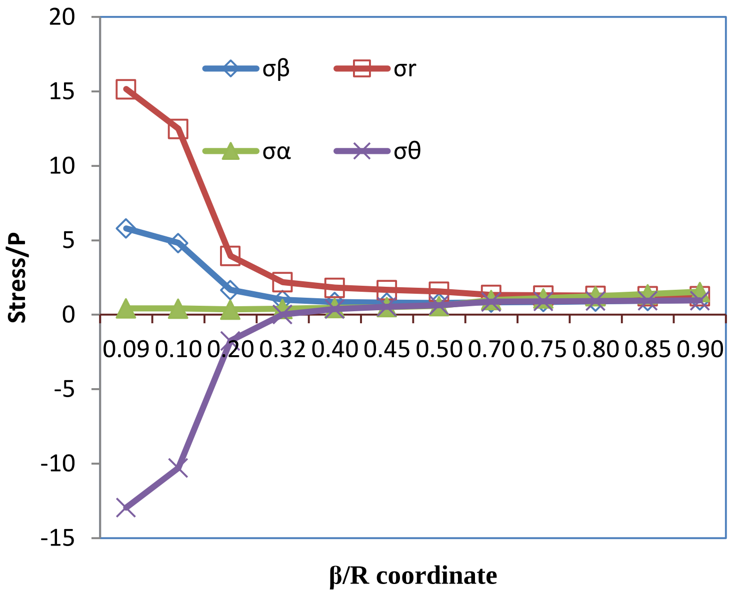

Fig.4.2 Variation of stresses σ_α and σ_β through the thickness and the bipolar coordinate β for $\alpha = \frac{\pi}{3}$

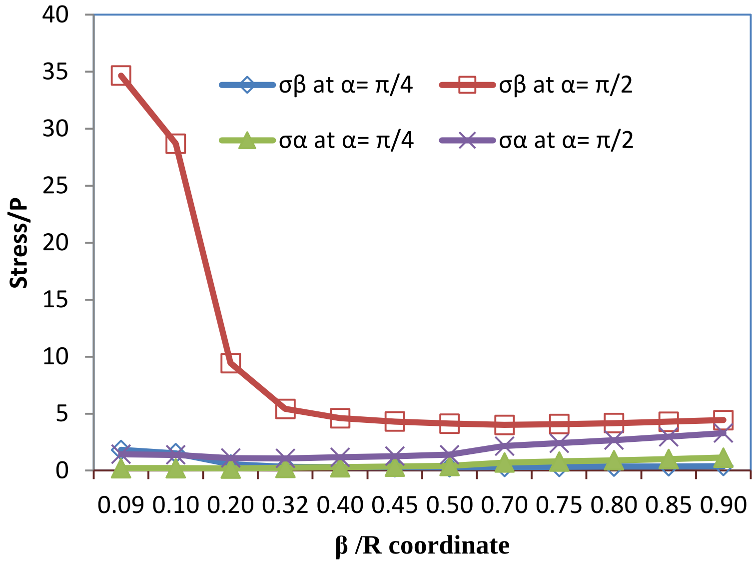

Fig.4.3 Variation of stresses σ_α and σ_β through the thickness and the bipolar coordinate β for $\alpha = \frac{\pi}{4}, \frac{\pi}{2}$

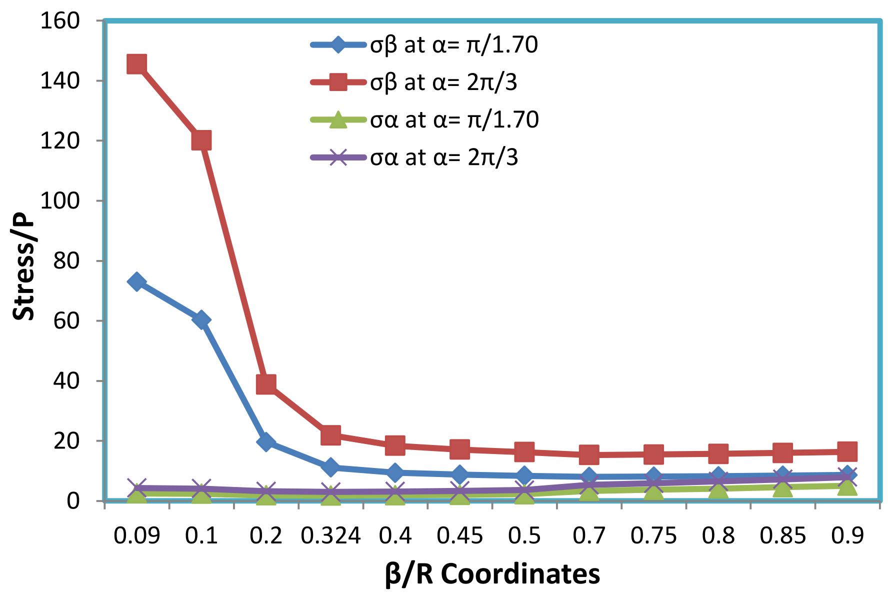

Fig.4.4 Variation of stresses σ_α and σ_β through the thickness and the bipolar coordinate β for $\alpha = \frac{\pi}{1.70}, \frac{2\pi}{3}$

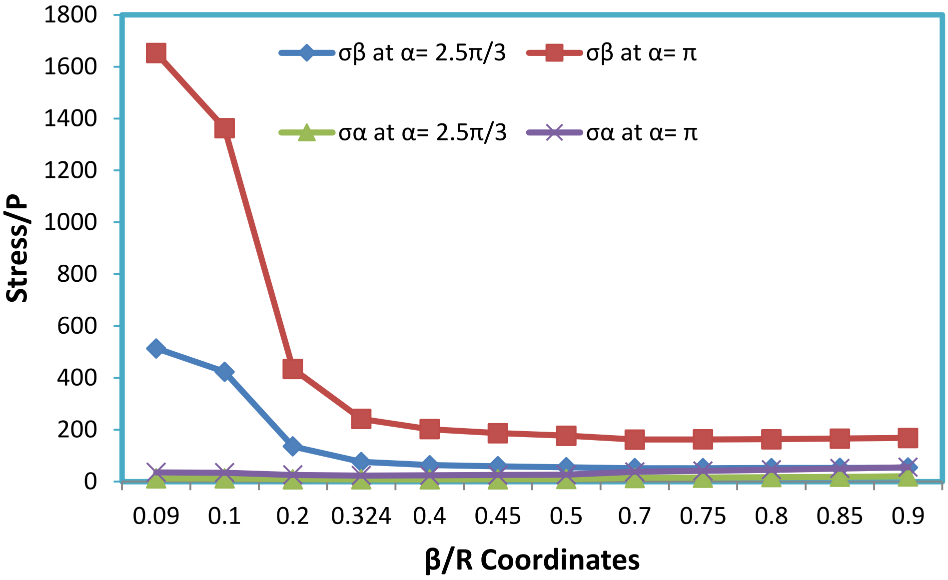

Fig.4.5 Variation of stresses σ_α and σ_β through the thickness and the bipolar coordinate β for $\alpha = \frac{2.5\pi}{3}, \pi$

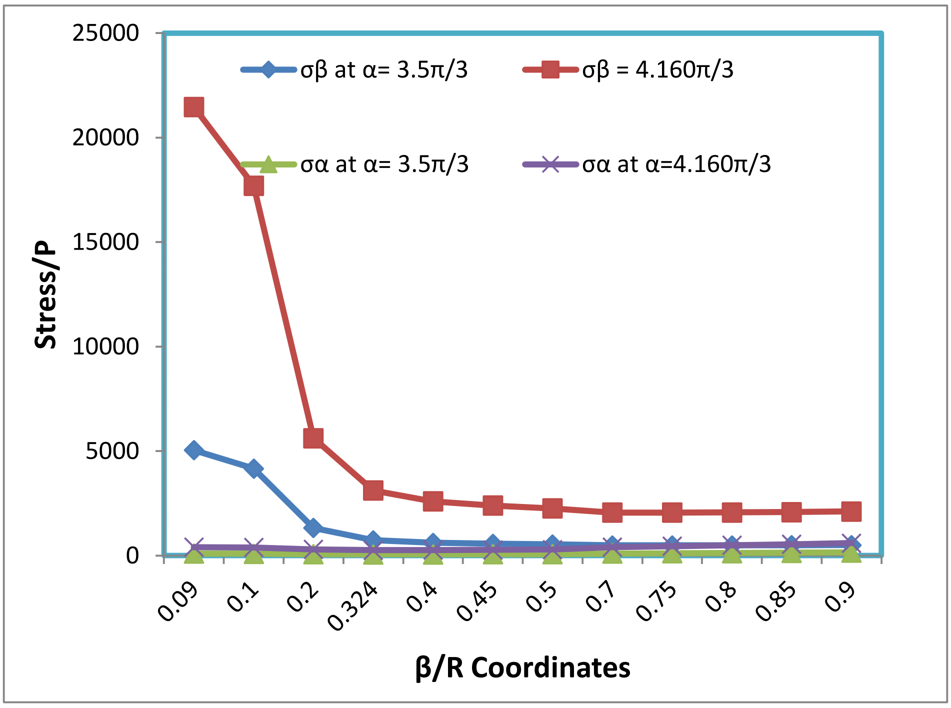

Fig.4.6 Variation of stresses σ_α and σ_β through the thickness and the bipolar coordinate β for $\alpha = \frac{3.5\pi}{3}, \frac{4.160\pi}{3}$

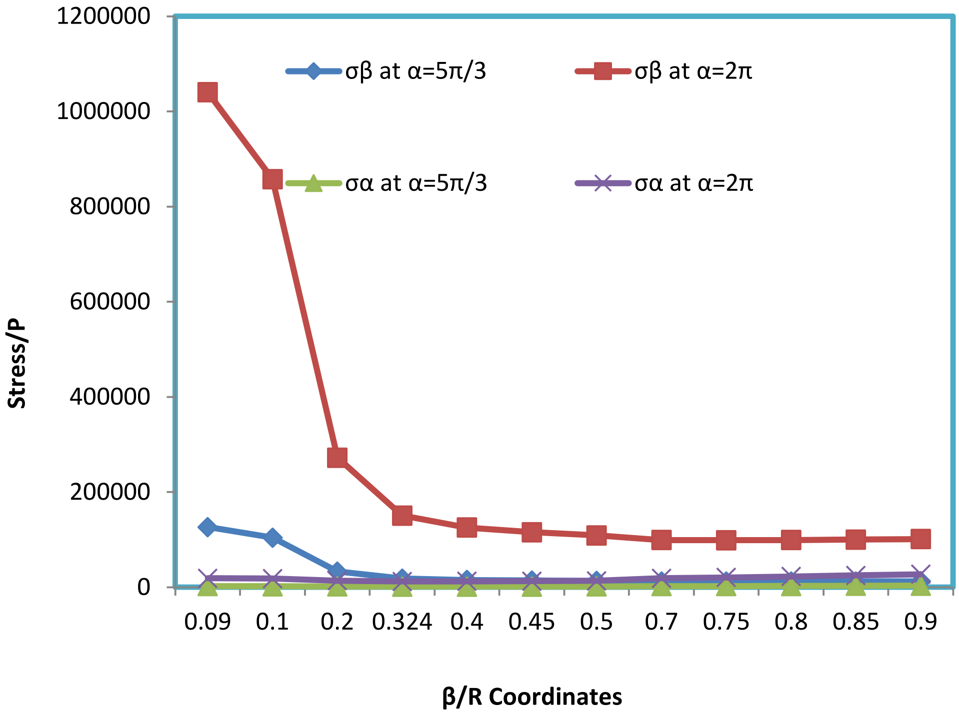

Fig.4.7 Variation of stresses σ_α and σ_β through the thickness and the bipolar coordinate β for $\alpha = \frac{5\pi}{3}, 2\pi$

4. CONCLUSION

Stress analysis is carried out to investigate stresses in eccentric rings in bipolar coordinates subjected to uniform internal and external pressure. With the help of Airy's stress function in bipolar coordinate system, an analytical solution is obtained for stresses in rings, where Airy's stress function satisfies the biharmonic equations and boundary conditions in bipolar coordinates. Stresses with various parameters such as eccentricity, natural logarithmic ratio of foci lengths and angle between the foci lengths are obtained. There is a greater effect of eccentricity on the stresses when compared the stresses with concentric ring. Present work assumes the analytical solution of isotropic materials and deformations are symmetric. The work presented here in the article is a basic study. This work can also be used to expand for solution of the problems of anisotropic cylinders and rings, layered cylinders and rings, functionally graded materials and including the effect of non-axisymmetric deformations.

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