Study of Effect of Perturbation Due to Oblateness of Earth on Satellites

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Abstract

The equation governing the motion of an artificial satellite under oblateness of the Earth are modelled and solved using Cowell's method. The effect of different initial velocities of satellite on the perturbation have been studied and compared.

Keywords

1 Introduction

The actual shape of the Earth is not sphere, this is one of the causes which effects the motion of the satellite. To find the position of an artificial satellite, it is necessary to consider the perturbation effects, one of the most important perturbation effect is oblateness of earth.

We consider the motion of an artifical satellite under the perturbation of oblateness of Earth. The oblateness potential of Earth can be modelled using the spherical harmonics function. The spherical harmonics function contains the effect of J_2, J_3, J_4, ... terms. We consider only the effect of J_2 in the perturbation effect and find the motion of an artificial satellite for initial displacement and different initial velocities. The study have been carried out for eight different cases with different initial velocities. The equations of motion are solved using Cowell's method. The numerical calculations are done for 900 days with the step size of 2 minutes. The paper is organized as follows:
In section 2, the equation of motion are modelled, the solution of equation of motion using Cowell's method are discussed in section 3 and section 4 contains conclusion.

2 Equation of Motion

The equation of motion for two body problem without perturbation is given by:

$$\ddot{\vec{r}} + \frac{\mu}{r^3}\vec{r} = 0, \tag{1}$$

where r is distance of satellite from earth and μ is Earth's gravitation constant, given by $\mu = GM$, where G is Newton's gravitational constant and M is the mass of the earth. The equation of motion with perturbation for two body problem takes the form:

$$\ddot{\vec{r}} + \frac{\mu}{r^3}\vec{r} = \vec{a_p}, \tag{2}$$

where a_p is the perturbed accelaration. This second order differential equation can be converted to two