CHAPTER 4

INSTRUCTIONAL PACKAGE

CHAPTER IV

INSTRUCTIONAL PACKAGE

4.0 Introduction

The present Chapter records all the Lesson Plans, Worksheets, and the Formative Assessments (Evaluation 1 & 2) for the content 'Real Numbers' of class IX (GSHSEB), weaved together to create the Instructional Package - which was implemented in the present Study. Each Lesson Plan includes the objectives, the teaching strategies used, the list of power point presentations [Appendix B (2)] and the teacher-student activities in the dialogue form. The list of tasks and questions in the Worksheets and the questions included in the Formative Assessments are also reported in the following section. Self-Learning Materials given to students for extra support is attached in [Appendix B (4)].

In the Lesson Plans detailed below, 'Teacher's dialogues' may seem to be more visible and prominent, as they involve - giving clear and in-depth explanations of concepts, providing clear and specific instructions to direct students for higher order thinking tasks; doing regular and higher order questioning and related probing; and finally consolidating the student-discovered mathematical generalizations. 'Student Dialogues' may not be prominently visible throughout the Plans, as they are directed to get engaged cognitively more for in-depth explorations; observing and identifying patterns; discovering mathematical facts - through the Worksheets and Classroom interactions- and responding to HOTS questions – through the Formative Assessment tasks.

Some of the abbreviations used in the Lesson plans are: **T E :** Teacher Explanation; **T Q :** Teacher Question; **T P :** Teacher Probe; **S A :** Student Answer; **T I :** Teacher Instruction; and **T C :** Teacher Consolidation.

NUMBERING SYSTEM: REAL NUMBERS

4.1 Lesson Plan 1

Unit : Real Numbers **Grade :** IX

Topic : Meaning of Real Numbers **Duration :** 40 min

- **General Objective:**

1. Students will be able to understand the relations between numbers of N, W, Z and Q

- **Specific Objectives:**

1. Students will be able to identify the use of numbers in real life.
2. Students will be able to categorize the uses of numbers – to represent, count d measure.
3. Students will be able to visualize and state the elements of the Numbering systems – W, N, Z and Q.
4. Students will be able to state the relation between the different Numbering Systems.

- **Teaching Aids:**

PPT Slide 2 & 3: Content Map

PPT Slide 4: Place of Real Numbers in the realm of Numbers

PPT Slide 5: Relation between different Numbering systems

PPT Slide 6: True & False questions on relation between different Numbering systems

- **Teacher's Activity and Student's Activity:**

Teacher shows the Content Chart and explains the topics to be covered in this unit.

T E : Students, you have been using numbers not only in your math classrooms but also in your real life…

T Q : Where in our real lives are Numbers used? [Teacher probes students for answers]

S A : Mobile numbers, Roll Nos., Date, Money like Rs. 5, length 38 cm, weight 20 kg, 10 people, 7 pens, etc. **[**Teacher notes on board the answers given by the groups.] Probes with:

1) Can you think of a large number being used in real life?
2) size of micro-organisms?
3) what about fractional and decimal nos.?
4) What about square roots, cube roots etc., aren't they numbers?

S A : (1) 10^9 kms, Area of a State, distance between Earth and Planets, a billion dollars etc. (2) 5×10^{-12} mm, size of an atom, … (3) 2/3 of the cake, 3.5 m of cloth (4) $\sqrt[2]{2}$, $\sqrt[3]{7}$

[Teacher writes these kinds of numbers on BB in separate columns.]

[Teacher probes the students to identify the categories.]

Students analyze the list written on the board and generalize the following:

S A : Numbers are used mainly for representing, counting and measuring different quantities.

T C : Numbers are used to *represent* objects (box nos., roll nos., mobile nos…..). Numbers are used to *count* living (man, animals, plants..) & non-living things (different objects, money, stars, atoms…), *measure quantities that can be seen* like solids (weights of things in kg, g etc., length-breadth-height, area-perimeter-volume of different things), liquids (quantity of water etc. in l, ml, etc.), distances (on road, air, space, etc.) .. and *measure quantities that cannot be seen* like amount of gas, time, temperature, force, pressure etc. Numbers can be very large and very small, they can be in the form of Fractions, Decimals, Roots etc.

T E : We can see a variety of 'numbers' on the blackboard, which we have been studying right from our childhood. We have been trying to understand these numbers in groups also called Numbering Systems.

We started with the group or set of *Natural numbers* i.e. all positive numbers starting from 1 followed by 2, 3, 4,…50, 51, … 5000, 5001, …..37 lakh,…….50 crore,……and goes on.

The next set is called *Whole numbers* which includes 0 and all the Natural numbers. Till fifth class you have been studying about all the properties of Whole and Natural numbers and their mathematical operations. In further classes you have studied about *Integers* i.e. the infinite set of all negative numbers, zero and positive numbers (negative numbers and Whole numbers) along with their properties and operations.

Besides you also have studied about fractions and decimals in the primary class; their meaning, properties and how mathematical operations are done on them. In your previous class you have studied in detail about *Rational numbers* which includes all the Integers and all the numbers lying between two Integers. In this Chapter we will study about one more Numbering system called Irrational numbers.

Rational nos. and Irrational nos. together form the Numbering system called *Real numbers*. In further classes you will be studying many other form of Numbers as shown in PPT slide 4.

T E : Let us understand the Real numbers with help of the Slide 5: [Points 1, 2, 3, etc….form set of Natural numbers. Along with zero, the set is called Whole numbers. With all negative numbers (-1), (-2), (-3)… the set is called Integers. The space between two Integers i.e. 0 and 1, 1 and 2, 2 and 3,…0 and (-1), (-1) and (-2), (-2) and (-3) etc. have uncountable Fractional/Decimal numbers. Thus this set of all points representing Integers and all points representing a Fractional number lying between them is called Rational Numbers. There are points in this space which cannot be represented as Fractions, which are called Irrational Numbers (we will study about them later)]. Let us concentrate on Rational numbers now.

T Q : State True or False with Reasons: (Questions in PPT Slide 6)

1. Every Whole number is an Integer

T I: Just visualize the set of Whole numbers…hold on the image of the first few numbers. Now visualize the set of Integers…hold on the image of the Integers around 0…now concentrate on the question….check in your mental image: Are the Whole numbers in the first image a part of the second image?...widen your image with more numbers…What is the answer? *Answer:* W = { 0, 1, 2, 3, 4,………..} and Z = { all negative nos., 0, 1, 2, ……}

Thus, all numbers of W $\in$ Z. Thus, W $\subset$ Z. So, the statement is true.

2. Every Integer is a Whole number
3. Not all Rational numbers are Integers
4. Every Rational number is an Integer
5. Every Integer is a Rational number
6. Not all Integers are Rational numbers
7. None of the Natural numbers are Rational numbers
8. All Real numbers are Rational numbers
9. Every Rational number is a Real number

T E: So, in the present Unit we are going to study the Numbering System called Real Numbers. With this we step into Secondary level Mathematics, we move on to the abstract zone where there are numbers that cannot be represented or made visible in concrete forms.

- **<u>Teaching Strategies used in Lesson Plan 1 :</u>**

Cognitive Strategies: Comparing the new to the already known concepts, classifying information, use of real world examples, providing visuals (PPT), naming critical features of Numbering system.

Visualization: Students explore their world (mentally) to find different kinds of numbers and relationship between different Numbering systems (W, N, Z, Q).

Generalization: Students are probed to generalize the use of numbers into categories.

Estimation: Students are given a scope to estimate while responding for very large and very small numbers.

Mathematical connections: Students explain the relation between the Numbering systems - N, W, Z and Q

Higher order questioning: Q- True/False with reasons. Use of open ended questions is done throughout the instructions.

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4.2 Lesson Plan 2

Unit : Real Numbers **Grade :** IX

Topic : Rational Numbers **Duration :** 40 min

- **General Objectives :**

1. Students will be able to understand the relationship between numbers and the Numbering systems they belong to.

- **Specific Objectives:**

1. Students will be able to distinguish between a Whole number, fraction and a decimal number.
2. Students will be able to identify the numbers that belong to the set of Rational Numbers.
3. Students will be able to define Rational number.
4. Students will be able to distinguish between different numbers belonging to different numbering systems.

- **Teaching Aids:**

8 (100-squared) graph papers

PPT (slides 7 to 11): Meaning of **2 ¾** th and 2.75

PPT Slide 12: Answers of Quiz

SLM 1: Self learning material on Natural, Whole, Integers, Fractions, Decimals (previous Knowledge)

WS 1: Relationship between different Numbering systems

- **Teacher's Activity and Student's Activity:**

T Q : In the previous class we discussed about Natural numbers, Whole numbers, Integers and how they are related to each other. Today we will understand Rational numbers realistically.

Questions :

1. What are Natural numbers, Whole numbers and Integers? Explain with examples.
2. I have a few 100 block-graph papers here. I want to give **2** papers to student A, **2 ¾** th of the papers to student B and **2.75** papers to student C. How would I do that?

[Teacher demonstrates the same with help of students' answers]. Also may take the help of PPT slides 7 to 11 for the same.

3. Based on this understanding about the numbers 2, $2\frac{3}{4}$ and 2.75; tell me how are they different? Or what do they represent?

[2 represents two full pages, whereas $2\frac{3}{4}$ and 2.75 represents two full pages and a part of the third page. Also $2\frac{3}{4} = 2.75$]

4. We have learnt the numbering systems N, W and Z. So, these numbers 2, $2\frac{3}{4}$, and 2.75 belong to which numbering systems?

[Teacher consolidates very clearly that only numbers which represent full quantity i.e. unbroken elements belong to N, W &Z; fractions and decimals do not belong to Z]

T E : So Fractions/Decimals do not belong to the set of Integers. Thus, a bigger set is considered which include both the Integers as well as the fractions and is called Rational numbers. Examples of Rational numbers are: 3, 6, -8, 0, 1243, -342, 1/3, 7/32, 24/8……..

T E : Now let us understand a very important difference between the concept of 'Fraction' and 'Rational number'.

[Teacher writes the statement on the board '$\frac{1}{2}$ is a Fraction but $\frac{-1}{2}$ is not a Fraction.' Prove this statement with a real example. You can use this paper.]

[One of the students fold and show one of the parts as $\frac{1}{2}$. But no one could explain $\frac{-1}{2}$ with the help of the part/whole model.]

T Q : Since there is no possible representation of $\frac{-1}{2}$, negative fractional numbers are not considered as Fractions. But on a Number line of Integers, where would $\frac{-1}{2}$ lie?

S A : Between -1 and 0.

T E : Very good and so we can say that though $\frac{-1}{2}$ is not a Fraction, it surely is a Number. But which Numbering system will it belong? Yes…they are included in the Rational Numbering system.

T Q : With this knowledge in mind, try to define Rational Numbers.

S A : We have studied in class VIII, Numbers that can be converted to fraction p/q form are Rational numbers. But now should we be using the word Fraction??

T P : Okay. Try to use $\frac{-1}{2}$ to define Rational numbers.

S A : A number of the form *p/q*….

T P : Good. In *p/q*, *p* belongs to which Numbering system?

S A : (-1) is an Integer. So *p* belongs to Integer.

T P : Good. What about *q*? That can be negative as well.

S A : Then *q* also belong to Integer.

T P : But can *q* be zero? Say a number like $\frac{-1}{0}$.

S A : We don't know.

T E : Any non-zero number divided by zero is considered undefined. Why? The justification is given in the self-learning material. Go through it and try to understand it on your own. Now try to define Rational numbers.

S A : Number that can be written in the *p/q* form, where *p* and *q* belong to Integers and $q \neq 0$ is a Rational number.

[Teacher writes down some numbers on the Board : $\frac{2}{3}$, $\frac{-7}{12}$, $3\frac{2}{3}$, 2.3, -3.12, 9, (-50) are Rational numbers.']

T Q : Each of the numbers are Rational numbers. You have to justify by checking each with the conditions given in the definition of Rational number.

[Teacher probes students to convert mixed number, decimals and Integers into fraction forms..]

Thus, all this discussion today leads us to understand the Rational Numbers. It would be advisable to see Rational numbers as numbers and play with the same, though yes they do represent physical quantities whole as well as broken - as fractions. The negative fractional form may be used to indicate measures of opposite directions, but let us not delve much into that. It is clear by now that the set of Rational numbers include all Integers and the (positive and negative) Fractional numbers.

T Q : Let us play a quiz. I will speak out some numbers, you have to say they belong to which Numbering systems with reasons.

- **Black Board Work**

Number	Numbering System	Reason
7		
-23/15		
0		
3.5		
-2×10^{7}		
2.3×10^{-5}		
0.00005		
1 crore		

(After getting answers from students teacher consolidates using PPT slide 12)

Home Task : Read the material (SLM 1) given to you carefully, solve Worksheet 1

WORKSHEET 1

Q1. Notice the number in bold in each of the following statements and identify the different Numbering Systems it belongs to. Put a tick mark in the respective columns. Also mention the reason of your answer in the 'Justify' column. (Note : N-Natural numbers, W-Whole numbers, Z-Integers, Q-Rational numbers)

Sr.	Statements	N	W	Z	Q	Justify your answer
1.	Size of the bedsheet is **2m × 3m.**					
2.	Radha took only one pencil from the box, so its cost turned out to be **₹ 4.50**.					
3.	The temperature in Kashmir in winters become as low as **-20°C**					
4.	The population of India has crossed the mark of **1 billion** long back.					
5.	The prime number **37** has only two factors.					
6.	The speed of light in vacuum is $\mathbf{3 \times 10^{8}}$ **m/s.**					
7.	The Earth takes $\mathbf{365\,{}^{1}/_{4}}$ **days** to complete one revolution around the Earth.					
8.	The approximate diameter of a Carbon atom is $\mathbf{9.97 \times 10^{-20}}$ **cm**.					
9.	The number **0** was introduced by Aryabhatta.					
10.	A cow has $\mathbf{\sqrt{9} + 1}$ legs.					

Q2. Based on the pattern of Tick marks in the above Table, observe carefully and answer:

a. Which Numbering System has the maximum number of 'Numbers'?

b. It includes which other Numbering systems?

Q3. Are the following statements True or False? Give reasons for your answers.

(1) Every Whole number is a Natural number. (   )

(2) Every Rational number is an Integer. (   )

(3) Every Natural number is a Whole number. (   )

(4) Every Integer is a Whole number. (   )

(5) The number 51/17 is a natural number. (   )

(6) (-2.5) is a Rational number but not an Integer. (   )

- **<u>Teaching Strategies used in Lesson Plan 2 :</u>**

Cognitive Strategies: Comparing the new to the already known concepts, classifying information, giving best examples and non-examples, providing visuals (PPT) and concrete demonstrations, naming critical features of Rational numbers.

Visualization: Students are shown concrete forms of numbers like 2, 2.75 and $2^3/_4$; helping them to distinguish between each; visualize their place in respective Numbering systems; and generalizing that fractional and decimal numbers belong to Q but not Z.

Mathematical Connection: Students are shown the specific connections that lead to the definition of Rational numbers. Multiple representations of numbers are also shown to help students understand the math connections.

Higher order questioning: Students are able to prove by giving proper reasons that a given number belong to which numbering system. Questions included in Worksheet 1.

Generalization: In Worksheet 1 students are given a scope to generalize 'the numbering systems N, W and Z are subsets of Q'.

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4.3 Lesson Plan 3

Unit : Real Numbers **Grade :** IX

Topic : Rational Numbers **Duration :** 40 min

- **General Objectives :**

1. Students will be able to understand the denseness of Rational numbers.

- **Specific Objectives:**

1. Students will be able find the number of Rational numbers between two given Rational numbers.
2. Students will be able to explain the converging and the diverging (denseness and continuum) property of the set of Rational numbers.

- **Teaching Aids:**

PPT slide 13: Object can be divided into smaller and smaller parts indefinitely

PPT slide 14: There are uncountable Rational numbers between any two Rational numbers or Integers

- **Teacher's Activity and Student's Activity:**

T E : Discussion of Worksheet 1 and solving Q3 of WS 1

T E : Let us now try to find the answer to the question 'How many Rational numbers are there between two given Integers and Rational numbers?'

For that let us take up the same Page-Activity. The whole page represents the Integer/Rational number **1**, once I start dividing it into parts, these parts are represented as fractional numbers.

[Teacher cuts into half, then one-fourth, then one-eighth along with PPT (slide 13). Teacher emphasizes the fact that like the page can be continuously divided into smaller parts, not only till it is visible but also when it is invisible like molecules – atoms – protons – quarks going on to sizes smaller than 0.8×10^{-15} or $8/10^{16}$m (radius of a proton)]

Now the question is where can we find these numbers among the set of the known visible numbers? Let us understand it with help of a Number Line (PPT slide 14)

- Imagine the space between 0 and 1, it will include which type of numbers? [Fractions/Decimals]
- How many numbers are there between the Integers 0 and 1 or between 3 and 4 or between any two Integers? [Not sure]
- Let us try to find out. Consider the space between 0 and 1 (Imagine the space is enlarged by using a magnifying glass.)

- Now remember the Paper activity, where we kept on dividing the paper into smaller and smaller parts. Likewise, we first divide the space between 0 and 1 into ten equal parts, then each part will be represented by 1/10 or 0.1, 2/10 or 0.2, 3/10 or 0.3…. giving nine Rational numbers between 0 and 1….. as shown in the slide 10 (Line 2)
- Again, if each part (0 and 1/10 or 1/10 and 2/10) is divided into ten equal parts, then the space between 0 and 1 has now hundred parts, size of each part being 1/100 or 0.01units (as shown by Line 3 in slide 10). Thus, the space between 0 and 1 has now ninety-nine Rational numbers in between 0 and 1.
- Further if each part say 4/100 and 5/100 is divided into 10 parts as shown by Line 4, then the space between 0 and 1 will have nine hundred and ninety nine Rational numbers.
- This division can continue on and on…….innumerable Rational numbers can be found between any two Integers like 0 and 1 or 1 and 2 or 50 and 51 or 999999 and 1000000.

T Q : So, what can you generalize or conclude from all these discussions about the set of Rational Numbers?

S A : There are uncountable Rational numbers between any two Integers.

T P : Similarly, how many Rational numbers are present between any two Rational numbers?

S A : It is also uncountable, because the space between two Rational numbers can be divided into uncountable number of divisions.

T E : We were trying to move inwards between two Rational numbers, likewise if we try to move outwards on two opposing directions from a specific Rational number, what would happen?

S A : There are uncountable numbers on both sides.

T Q : Consider a Rational number ‘5’. Moving in the forward direction, which will be the next Rational number? and moving backwards, which will be the next?

S A : Moving forward 5.1 and backward 4.9.

T P : Good. But if say 5.01 instead of 5.1 and 4.9999 instead of 4.9, am I wrong?

S A : That is also possible.

T P : What are the other possibilities?

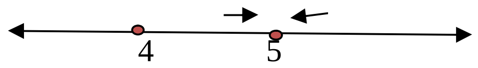
S A : 5.000001 and 4.9999999

T Q : Can we get one specific Rational number that can be considered to be the next to 5?

S A : No, we cannot.

T E : Right! So mathematically it can be stated that 'the Number line representing Rational numbers is infinite in both ways as we move outward from any Number or we try to move inwards towards a specific Number.

In other words, there is always a point which is infinitely smaller or greater than 5. You have to imagine that you will approach very very close to 5 but can never reach 5. It is like moving in the space which has no end or moving towards an imaginary point which does not exist.



This is a concept you will learn in detail in Class XI in Calculus.

- **Teaching Strategies used in Lesson Plan 3 :**

Cognitive Strategies: Comparing the new to the already known concepts, giving best concrete examples, providing visuals (PPT) and concrete demonstrations, naming critical features of Rational numbers.

Visualization: Use of 'visualization object' with the 'continuous and equal division of paper' example leading to the abstract concept of division of molecules, atoms, atomic particles etc.

Mathematical Connection: Students are shown the connection of the abstract math concept of 'continuity of Rational numbers' with real life.

Generalization: Students are led through the visual process of dividing the space between two Rational numbers to help them generalize the math concept that 'the Numbering system of Rational numbers is dense and infinite both in and out.

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4.4 Lesson Plan 4

Unit : Real Numbers **Grade :** IX

Topic : Rational Numbers **Duration :** 40 min

- **General Objectives :**

1. Students will be able to understand the denseness of Rational numbers.

- **Specific Objectives:**

1. Students will be able to deduce that there are countable Integers and uncountable Rational numbers between any two consecutive Integers or Rational numbers
2. Students will be able to find Rational nos. between any two Rational nos. (fractional nos.)

- **Teaching Aids:**

PPT Slide 15: Equivalent fractions

Worksheet 2: Exploring Rational numbers between different Numbering system

- **Teacher's Activity and Student's Activity:**

T Q : Consider the numbers -5 and 5. Answer the following questions with reasons.

1. How many Natural numbers will be there between the two Rational numbers (-5) and 5? [**SA:** four, they are 1, 2, 3, and 4]
2. How many Whole numbers will be there between the two Rational numbers (-5) and 5? [**SA:** five, they are 0, 1, 2, 3, and 4]
3. How many Integers will be there between the two Rational numbers (-5) and 5? [**SA:** nine, they are (-4), (-3), (-2), (-1), 0, 1, 2, 3, and 4]
4. How many Rational numbers will be there between the two Rational numbers (-5) and 5? [**SA:** Many numbers. All the Integers lying between (-5) and 5; all fractions that lie between (-5) and 5; and all decimal numbers lying between (-5) and 5.]

T Q : Okay, now let us consider the specific interval 4 and 5. Which Rational numbers can you visualize between them?

S A : $4\frac{1}{2}$, $4\frac{1}{3}$, $4\frac{1}{4}$, $4\frac{1}{5}$, $4\frac{1}{6}$, $4\frac{1}{7}$, $4\frac{1}{8}$, $4\frac{1}{9}$.

T P : So, exactly how many numbers between 4 and 5?

S A : Eight.

T P : Refer to what I taught in the last class and then reflect on your answer.

S A: Yes, then there are uncountable Rational numbers between 4 & 5.

T Q : Okay, if there are uncountable Rational numbers between 4 and 5. Give a few more numbers besides the one you mentioned earlier. Think on the fractional part.

S A : $4\frac{2}{3}$, $4\frac{4}{7}$, $4\frac{3}{5}$, $4\frac{8}{9}$, …..

T P : Good. In fact fractional numbers can also be written in a different form, so the same numbers you mentioned can also be written in that form.

S A : Decimal forms, like 4.1, 4.2, 4.3….4.9.

T Q : A few more…

S A : 4.24, 4.87, 4.476, 4.193,………

T Q : Similarly there must be uncountable Rational numbers between 2/7 and 6/7. Tell me any ten.

S A : 3/7, 4/7, 5/7 ????

T E : Take the help of Equivalent Fractions. What are Equivalent Fraction?... when we multiply or divide the same number to the Numerator and Denominator of a Fraction the value of the original Fraction does not change and that is called the Equivalent fraction of the Original fraction. (PPT slide 15..optional)

S A : So, we can find equivalent fractions of 2/7 and 6/7. That is 4/14 and 12/14. So the numbers in between are 5/14, 6/14, 7/14, 8/14, 9/14, 10/14, 11/14. But we get only seven numbers. Maybe if we find numbers between 6/21 and 18/21, then we will get the answer.

T Q : Very good, that is correct. But I want you to find an easier method to solve this kind of problem instead of doing this trial and error.

S A : We can keep on finding the middle number by finding the average of the given numbers, and then go ahead with the same process for every consecutive number to get as many in-between numbers as needed.

T P : Very good, come and show that on the BB.

[Student shows the lengthy work-out on the BB]

T P : Great! So all of you now know two methods to find Rational numbers between two given numbers. But both the methods are quite lengthy. Is there a easier and a smaller method that we can use? Try to use the concept of equivalent fractions again.

S A : Maybe if we multiply the fractions by ten in the numerator and denominator. So for the above problem we find the equivalent fractions of 2/7 and 6/7 as 20/70 and 60/70 and then we can easily write ten numbers between them : 221/70, 22/70, 23/70, 24/70, 25/70, 30/70, 35/70, 46/70, 53/70, 69/70.

T Q : Find ten numbers between 1/3 and 2/3. Think, if you multiply by 10, will you get the required ten numbers……if no then what is the other easier option?

[Students convert the problem as 100/300 and 200/300 and find the solution]

T Q : Find ten numbers between 1/5 and 1/3. Can we use the above method right away?

S A : No, they are unlike fractions, so we need to convert them into like fractions first.

T E : Very correct. How do you that?

S A : By finding the LCM of the Denominators and then finding equivalent fractions with the LCM as the Denominators.

T E : The LCM of 3 and 5 is 15, so 1/5 = 3/15 and 1/3 = 5/15

Now we need to find ten numbers between 3/15 and 5/15 by again finding equivalent fractions appropriately. So now we know exact methods that can be used to write down the Rational numbers that lie between two given Rational numbers.

Home Task : Worksheet 2

WORKSHEET 2

Q1. Write five Rational numbers belonging to the different Numbering systems as mentioned below : (i) Natural Numbers (ii) Whole numbers (iii) Integers (iv) Fractions Numbers (v) Decimals

Q2. Write down five Rational numbers that lie between the given two numbers in each of the following case: (i) 10000 and 10050 (ii) (-3) and 3 (iii) 1 and 2 (iv) 2.5 and 3.2 (v) $\frac{1}{7}$ and $\frac{7}{7}$ or 1 (vi) $5\frac{2}{9}$ and 6 (vii) $\frac{43}{8}$ and 6

Q3. Write any ten Rational numbers between the given Numbers using the equivalent fraction concept: (i) $\frac{3}{7}$ and $\frac{1}{2}$ (ii) $\frac{3}{5}$ and $\frac{5}{3}$ (iii) 1.2 and 1.3 (iv) $\frac{3}{12}$ and $\frac{5}{6}$ (v) 7 & 8

Q4. Given any two Integers, the number of Rational numbers lying between them is uncountable. Is this statement true or false? Justify your answer with examples.

Q5. Refer Q1 and Q2 and make a concluding statement regarding the number of Natural numbers, Whole numbers, Integers and Rational numbers between two given Rational numbers.

- **Teaching Strategies used in Lesson Plan 4 :**

Visualization: Students visualize the different processes that can be used to find Rational numbers between two given numbers.

Generalization: Students observe the patterns and conclude that ‘there are countable number of N, W and Z between any two given Rational numbers’ and ‘there are uncountable numbers of Rational numbers between two given Rational numbers’.

Mathematical connections: Students are shown the mathematical connection of the concept ‘finding Rational numbers between two given Rational numbers (fractional forms)’ with the previously learnt concepts of ‘equivalent fractions’, ‘operations on fractional numbers’

Higher order questioning: Used to tap cognitions to find varied solutions and ultimately reach the best one to find as many Rational numbers as possible between two given numbers.

4.5 Lesson Plan 5

Unit : Real Numbers **Grade :** IX

Topic : Relation between Fractions & Decimals **Duration :** 40 min

- **General Objective:**

1. Students will be able to understand the relationship between Fractions and Decimals

- **Specific Objectives:**

1. Students will be able to identify ten-based and non-ten-based fractional numbers
2. Students will be able to convert ten-based and non-ten-based fractional numbers into decimal numbers
3. Students will be able to differentiate between the decimal numbers that are obtained from ten-based fractional numbers and those from non-ten-based fractional numbers
4. Students will be able to deduce the relationship between fractional and decimal numbers

- **Teaching Aids:**

PPT Slide 16: Relation between fractions and decimals

Worksheet 3: Understanding ten-based and non-ten-based fractions and the type of decimals they get converted into.

- **Teacher's Activity and Student's Activity:**

T E : In the previous classes we tried to understand the concept of Rational numbers and have also defined Rational numbers.

T Q : Define Rational numbers

S A : Rational Numbers are fractional numbers of the form p/q, where $p \in Z$; $q \in Z$; but $q \neq 0$.

T E : Thus all the fractional numbers are Rational numbers. What about decimal numbers? Is there any relationship between fractions and decimals? Teacher shows PPT (slide 16) (Explanation : See Figure and explain how the figures represent the given fractions and decimals. In each case decimals are related to fractions, how?)

S A : Decimals are 'fractions having denominators 10, 100, 1000 etc'.

T Q : Give examples to explain your answer.

S A : 3/10 = 0.3 or 35/100 = 0.35 or 1234/ 1000 = 1.234 etc.

T E : You are correct. Let us call these fractions like 3/10, 35/100 etc, i.e. Fractions with denominators 10, 100, 1000 etc. as Ten-Based Fractions.

Now solve Q1 of WS 3 in which you convert the Ten-Based Fractions in their Decimal forms. [Students are given 3 minutes to write the answers. Teacher discusses the answers.]

T I : Solve Q2 of WS 3 by the Equivalent Fraction method. Example : $\frac{1 \times 5}{2 \times 5} = \frac{5}{10} = 0.5$

[Students solve the sums. Teacher helps students. Answers are discussed.]

T E : As indicated after Q2, let us consider that there are two types of Fractional numbers: (1) Ten-based fractions and (2) Non-ten-based fractions.

T Q : Which are the different methods that we can use to convert a fractional number into a decimal number?

S A : Directly if the denominators are exponents of ten, otherwise by converting the given fractions into equivalent fractions with denominators as exponents of ten.

T I : Good. Now try to Solve Q3 of WS 3.

T Q : Observe the Fractions given. Are they ten-based-fractions?

S A : No. The denominators cannot be converted into 10, 100, 1000 etc.

T E : So, let us call such Fractions 'Non-Ten-Based-Fractions.

T Q : How can you convert these Fractions into decimals? Can you use the Equivalent Fraction method?

S A : No, we have to divide the numerator with the denominator.

[Teacher tells students to convert the given fractions in Q3 into decimals using the division method.]

Home Task :Q 4 of Worksheet 3 has to be completed at home. You cannot use a Calculator to convert the Fractions into Decimals. Use the division method and write around 9 to 10 decimal places in each case (wherever possible).

WORKSHEET 3

Q1. Convert the following Ten-Based Fractions into Decimals :

(1) $\frac{5}{10}$ (2) $\frac{(-17)}{100}$ (3) $\frac{2}{1000}$ (4) $\frac{457}{1000}$ (5) $\frac{(-37)}{10}$ (6) $\frac{(-2345)}{10000}$

Q2. Use the Equivalent Fraction method and convert the following Fractional numbers to Decimal numbers :(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{2}{5}$ (4) $\frac{3}{8}$ (5) $\frac{6}{25}$ (6) $\frac{7}{125}$

Let us call these Fractions – ½, ¼, 2/5, 3/8, 6/25, 7/125 also as 'Ten-Based Fractions' as each can be converted to fractions with denominators 10, 100, etc. Fractions like 2/3, 7/15, 3/7, 5/6, 1/9, 4/17 etc. whose denominators cannot be converted to 10, 100, 1000 etc. can be termed as 'Non-Ten-Based Fractions'.

Q3. Observe the following Fractions : $\frac{1}{3}, \frac{4}{9}, \frac{3}{7}, \frac{1}{6}, \frac{8}{11}, \frac{-5}{13}, \frac{-7}{14}$ ………

(1) Is it possible to convert the Denominators into exponents of tens (10, 100, 1000, etc.)

(2) If no, then can we use the Equivalent Fraction method to convert these Fractions into Decimal numbers?

(3) How can we convert these Non-Ten-Based-Fractions into Decimal numbers?

Q4. Do as directed:

(1) Classify the Fractions into Ten-Based Fractions and Non-Ten-based Fractions :

2/5, 1/3, 6/25, 7/20, 6/11, 3/125, 7/8, 4/9, 71/31, 5/4, 7/2, 7/21, 3/25, 11/23, 15/6, 3/10, 234/1000

(2) Take up all the Ten-Based Fractions from (1) and convert them into Decimals (by Division)

Ten-Based Fractions	**Decimal Form**	**Remainder**

(3) Take all the Non-Ten-Based Fractions from (1) and convert them to Decimals (Use division) (Can use a calculator and write till 10 decimal places).

Non-Ten-Based Fractions	**Decimal Form**	**Remainder**

Q5. Refer to Q4 and Write the main differences that you can observe in the Decimal numbers obtained from Ten-Based Fractions and Non-Ten-Based Fractions.

Q6. Overall what can you say about the relationship between Fractions and Decimals?

- **<u>Teaching Strategies used in Lesson Plan 5 :</u>**

Visualization: (1) Students are given a scope to refresh their previous knowledge on Decimal numbers realistically and mathematically through visual aid. (2) Students get a scope to visualize in concrete forms transformation of ten-based fractions and non-ten-based fractions into different decimal forms (terminating and non-terminating from the worksheet 3).

Generalization: Based on a sequence of logically connected steps students infer the relation between fractions and decimals.

Mathematical connections: Students are given scope to connect their knowledge of 'converting fractions into decimals' from previous class to deduce that 'all types of fractions can be converted into decimals'.

Cognitivist Strategies: Classifying fractional numbers into ten-based and non-ten-based fractions.

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4.6 Lesson Plan 6

Unit: Real Numbers **Grade :** IX

Topic : Types of Decimal Numbers **Duration :** 40 min

- **General Objective:**

1. Students will be able to understand the types of decimals numbers.
2. Students will be able to understand the relation between terminating decimal numbers and Rational numbers

- **Specific Objectives:**

1. Students will be able to deduce the relation between fractions and decimals
2. Students will be able to differentiate between terminating decimals, recurring decimals and non-terminating-non-recurring decimals
3. Students will be able to prove that terminating decimals belong to the set of Rational Nos.

- **Teaching Aids:**

Worksheet 3: Discussed and consolidated

Worksheet 4: Proving that terminating decimals can be converted into fraction and thus are Rational numbers (Question 1 and 2)

- **Teacher's Activity and Student's Activity:**

T E : The previous class and WS 3 gave us an idea as to what decimals are and that every fraction can be converted into decimals. What are your conclusions of Q4 (WS 3) which you recorded as your answer in Q5?

S A : 10-based fractions give decimals with finite decimal places/ the remainder becomes zero after point of time but non-10 based fractions give decimals with infinite decimal places/ the remainder never becomes zero.

T E : Correct… the decimals obtained from ten-based fractions are called 'Terminating Decimals' and the decimals obtained from non-ten-based fractions are called 'Non-Terminating Decimals'.

Like 5/10, 2/5, ¼, 1/8, 3/25, 7/125 get converted to terminating decimals and 1/3, 5/7, 8/9, 4/31 etc. get converted to non-terminating decimals.

T Q : Further, observe the decimal places of the non-terminating decimals. Can you observe any pattern?

S A : In some of the decimals, the digit in the decimal place keeps on repeating and in some a set of digits is repeated.

T E : Very Good, like in 1/3 = 0.6666… can also be written as $0.\overline{6}$.

Like we discussed earlier that, we try to reach a point (say 0.66…), but can never reach it. We can approach very very close to it.

Similarly 1/7 = 0.142857142857…. ($0.\overline{142857}$ here the block 142857 keeps on repeating).

Such decimals ($0.\overline{6}$ and $0.\overline{142857}$) are called 'Non-Terminating Recurring Decimals' or only 'Recurring Decimals'.

Thus, with all this discussion we can conclude that every fraction can be converted into decimals, which may be 'terminating' or 'recurring'.

Based on this discussion …Answer Q6 of WS 3

NOTE : Every fraction converts into either 'terminating' or 'non-terminating recurring' decimals and never to 'non-terminating non-recurring' – decimal nos. whose decimal part is never ending and there is no repetition of digits (we will learn about this later) like a number like 3.130113300111333000………

T E : For that let us consider two cases :

Every RATIONAL NUMBER is a FRACTION….
Every FRACTION can be converted into TERMINATING or NON-TERMINATING-RECURRING DECIMALS….
We want to prove that every TERMINATING and NON-TERMINATING-RECURRING DECIMALS are RATIONAL NUMBERS.
For that we need to check whether TERMINATING and NON-TERMINATING-RECURRING DECIMALS can be written as FRACTIONS.

Case 1: Conversion of TERMINATING DECIMALS into FRACTIONS – Solve Q1of WS 4

T Act : Discusses the answers with students after they finish working.

T Q : Can terminating decimals be converted into fractions? --Yes--- So can we state that Terminating Decimals are Rational numbers? ---Yes---

Case 2 : Conversion of Recurring Decimals into Fractions. Try to solve Q2 of WS 4

[Teacher allows the students to try out their methods, notes a few on the board and points out why they are not correct. (expected answer : 0.24…= 24/100 ….]

T E : The number 0.242424…. is a never ending number and since this number has uncountable decimal places, how can we decide whether the denominator is 10, 100, 1000…) In the next class we will try to get the solution to this problem.

WORKSHEET 4

Q1. Convert the Terminating Decimals into Fractions and State whether it is possible or not:

a) 0.24 b) 324.25 c) 75.0 d) (-0.008) e) 0.287

f) (-124.0) g) 0.00001 h) 255.552

Q2. Try to convert the following Recurring Decimals into Fractions and State whether it is possible or not: a) $0.\overline{24}$ b) $4.\overline{5}$

Q3. Prove that the following Recurring Decimals are Rational Numbers.

Convert 7.4343… or $7.\overline{43}$ into Fraction. (Remember, your aim is to eliminate the decimal part ($\overline{43}$) by taking a 'new no.' having the same decimal part as the 'given no.' ($7.\overline{43}$). So should the 'new no.' be 10x, where x =7.4343…?)

-YES / NO? Work below to find out.

-If NO, what should be the New no. (should it be 10x or 100x or 1000x….)? Work out on your own.

Q4. Convert the following Recurring Decimals into Fractions to prove that they are Rational numbers.

1) 1.257257….	2) 47.1212….	3) 34.777….	4) 9. 568568…..
5) 0.099099099…..	6) 0.0505…..	7) 1.477777…..	8) 3.1232323…….

Thus, based on the solutions obtained from Q1 and Q4, what can you deduce about Terminating and Recurring Decimals…….

Q5. Which numbering system do they belong? Justify your answer.

- **<u>Teaching Strategies used in Lesson Plan 6 :</u>**

Visualization & Generalization: Concept clarity through well-reasoned and connected steps wherein students work out, observe patterns and conclude that 'All ten-based fractions are terminating decimal numbers and the non-ten-based fractions are recurring decimal numbers.'

Estimation: Posing question with known background and encouraging students to guess the answer..(In WS 4, leading students towards known concept of 'converting terminating decimals into fractions' but when they encounter 'recurring decimals' they realize the hurdle and wrongly guess that recurring decimals cannot be converted into fractions.)

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4.7 Lesson Plan 7

Unit : Real Numbers **Grade :** IX

Topic : Decimal numbers are Rational numbers **Duration :** 40 min

- **General Objective:**

1. Students will understand that all decimal numbers belong to the Numbering system 'Rational Numbers'

- **Specific Objectives:**

1. Students will be able to prove that recurring decimals belong to the set of Rational Numbers.

- **Teaching Aids:**

Worksheet 4: Converting recurring decimal numbers into fractional numbers (Practice)

- **Teacher's Activity and Student's Activity:**

T E : In the previous class we have seen that – all fractions can be converted into decimals, which can be either terminating or non-terminating-recurring in nature. You worked out Q1 in WS 4 to prove that terminating decimals can be easily written in fractional forms, clearly proving that they can be Rational nos. (as per definition of Q)

You have tried to work out with recurring decimals (like $0.\overline{6}$ or $1.\overline{234}$) yesterday and as expected most of you found it difficult to convert them into fractions and unless we do that we cannot prove that recurring decimals also belong to the set of Rational numbers.

Thus, today's topic is: 'Conversion of Recurring Decimals into Fractions'

T E : Let us try to convert 1.33…. into fraction (1.33…. means $1.\overline{3}$)

In the last class we have seen why we cannot write 1.33…. as 133/100. Think on the following :

1. Since there is a problem with the decimal part (which is never ending), we need to find a way to eliminate the decimal part….. maybe we need a new number which has the same decimal part i.e. 333…. From which if our given number 1.33… is subtracted, we can get away with the decimal part.

So, how to find that *new number* with the decimal part as in 1.33….?

Consider the following process.

Let the *given no.* (which is to be converted to a fraction)**,** 1.33… = x

Let the *new no.* (which should have the same decimal part 33…) be 10 × x or 10x

So, the *new no.* is 10x = 10 × 1.33… = 13.33….

Subtracting the *given no.* from the *new no.*, so that the decimal part gets eliminated, we get

$10x = 13.33.....$

$\underline{-x = -1.33.....}$

$9x = 12$

$x = \frac{12}{9} = \frac{4}{3}$ which is in the fractional form. Thus, $1.\overline{3} = \frac{4}{3}$

Now, Divide 4 by 3 and check whether 4/3 = 1.33… or not.

[Students do the division in rough and verify the above result.]

T E : You might question as to why should we take 10x only as the new number and nothing else? So, let us try out some other number as the new number.

Here the problem is 'Converting the recurring number 1.333…. into a fraction'

Suppose, we take any number with the decimal part 33… as, say 7.333…

Let the *new no.*7.33…. be 'y' and the *given no.* 1.33… be 'x'

Now, if we subtract : y – x = 7.33… - 1.33… = 6.0

So we got y – x = 6 or

x = y - 6

Though the decimal part got eliminated, did this new number 'y' help us to get the fractional form of 'x' i.e. 1.33…. NO.

So this explains as to why we adopt the first method to convert non-terminating-recurring decimals into fractions and prove that they belong to Q

Thus, this proves that a recurring decimal no. like 1.333… is a Rational number.

Can we say that for all recurring decimals?

T I : Solve Q3 of WS 4 and find on your own.

Follow the instructions given below the question and discuss your finding.

[Students solve Q3 of WS 4.]

T Q : So, what is the New no. in Q3 of WS 4?

T P : Teacher probes till students realize that – the new no. would be multiples of 10, 100, 1000 etc. depending upon the repeating blocks.

Eg. If x = 2.777… (one digit recurring), the new no. would be 10x

If x = 2.3535… (2 digits recurring), the new no. would be 100x and so on.

Home Task : Q4 and Q5 of WS 4 **…**This would provide enough practice to the students with the method to be used to convert recurring decimal numbers into fractional numbers and prove that all decimal numbers are Rational Numbers.

- **<u>Teaching Strategies used in Lesson Plan 7 :</u>**

Generalization: Students were engaged into inductive reasoning i.e. to observe or work with given set of data, analyze it in the process and identify the pattern or the relationship that exists within the components and synthesize them to infer a mathematical rule, property, law, formula or definition : the same was used by the teacher guide students to internalize the method used to convert recurring decimals into fractional numbers.

Higher order questioning: Questions used to lead students discover the logic of multiplying exponents of ten to strategize the conversion.

Cognitivist Strategies: Naming critical, additional, false features of the concept to justify the algorithm to be used to convert recurring decimals into fractions.

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4.8 Formative Assessment - Evaluation 1

Total Marks : 60 **Time : 2 ½ hours**

1. Show that $7.3\overline{45}$ (7.3454545…..) is a Rational number.

2. Do as Directed :

(1)Write five Rational numbers which are Natural numbers but not Integers.

(2)Write five Integers which are not Whole numbers.

(3)Write five Rational numbers which are also Integers.

(4)Write five Rational numbers which are not Integers.

3. Write four points of difference between Rational Numbers and Integers.

4. Write five Rational numbers between the given Numbers. (Not in Decimal form)

(1) 4.007 and 4.6 : (2) 1 and $1\frac{1}{2}$:

5. Find which of the following numbers represent a Rational number (Q) but is not an Integer (Z). Give reasons for your answers :(1) (-2.3) (2) $\frac{24}{2}$

6. Write maximum Five numbers between the 7 and 8 as per the specifications:

(1) Integers (2) Rational numbers (3) Non-Ten-based Fractions

[*Non-Ten-based Fractions are those fractions whose denominators are not 10, 100, 1000,…. and neither can be converted to 10, 100, 1000, etc.]*

7. State whether the following statements are 'True' or 'False', with reasons.

(1) Not all Rational Numbers are Natural Numbers.

(2) 'The Fraction $\frac{x}{y}$ is a Rational Number', where 'x' is an Integer and 'y' is a number obtained by subtracting the set of Natural Numbers from the set of Whole numbers.

8. In a mathematical game, Student A gives a problem to Student B*: "A frog who is at number 1 has to reach number 2 after crossing each and every Rational number that comes on his way. Will the frog be able to reach his destination?"* Help Student B to find the answer with proper reasons.

9. In the given set of numbers, identify the numbers that lie between $\frac{2}{3}$ and $\frac{3}{2}$. Show working.

Numbers	Answer : Yes/No	Reason/Working
$\frac{13}{12}$		
$\frac{3.6}{5.4}$		
$\frac{51}{60}$		
$\frac{2^2 + 1}{2^2 + 2}$		

10. Divide the solution of $(3.38 \times 10^{-1} \times \frac{500}{169})$ by the Integer that lies between $\frac{53}{9}$ and $\frac{36}{5}$. What kind of Decimal will be obtained? Will it be a Rational number or no?

11. $2 \times 5 =$ ____

$4 \times 25 =$ _____

$8 \times 125 =$ _____

Go ahead along the same pattern and find six Ten-based-Fractions, whose denominators are not 2, 4, 8, 5, 25, 125, and multiples of ten.

12. Convert the following Fractions into Decimals : $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \frac{8}{9}$. Observe the pattern and infer from that the relationship between Fractions and Decimal

13. Observe the three Decimal numbers given below carefully and answer the questions that follow: (i) 3.5000000…. (ii) 3.5005005….. (iii) 3.5055005…..

(1) Are all the above Decimal numbers similar or different? Explain your answer.

(2) Do they belong to the Numbering system Q or no? Write proper reasons for each.

14. "The Fractions $\frac{7}{12}$ and $\frac{3}{12}$ have same denominators." Does that mean they get converted to similar type of Decimal Numbers? Yes or No. Justify your answer.

15. Observe the pattern of the sums shown below. What can you conclude from that?

1) $\frac{5}{0.1} = 50$ 2) $\frac{5}{0.001} = 5000$ 3) $\frac{5}{0.00001} = 500000$ 4) …… …….

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4.9 Lesson Plan 8

Unit : Real Numbers **Grade :** IX

Topic : Irrational Numbers **Duration :** 40 min

- **General Objective:**

1. Students will be able to understand the meaning of Irrational Numbers.

- **Specific Objectives:**

1. Students will be able to identify and write decimal numbers that are Irrational numbers.
2. Students will be able to find square root of Whole numbers using long division method.
3. Students will be able to find that square root of 2, 3 and 5 are Irrational numbers.

- **Teaching Aids:**

PPT Slide 17: Long division method to find the square root of 2

Worksheet 5: Understanding Irrational numbers

- **Teacher's Activity and Student's Activity:**

[Teacher shows the content-chart and recapitulates the following: We are in the process of studying Real Numbers. Real Nos. include two sets, one that of Rational nos. (Q) and second Irrational nos. (I). Rational numbers include all the Integers and all the uncountable fractional (terminating and recurring decimal numbers) that lie between every two consecutive Integers. This numbering set is an infinite set inwards as well as outwards.]

Today we try to understand the second set of numbers belonging to the set of Real Numbers i.e. Irrational Numbers (I). Which type of numbers should belong to I? or Which numbers are called Irrational numbers?

The only kind of numbers that are left out from the set of Q are of this type : Non-terminating-non-recurring decimal numbers

Eg. 0.123112233…… or 345.050055000555…. or 23.438473672……. Note : Here the decimal part neither ends nor repeats itself.

All these non-terminating non-recurring decimals form a separate set or numbering system called the set of Irrational numbers (I).

T Q : Consider this number : 3.1248392……..Is this number an Irrational number?

S A : Yes

T Q : Are you sure that the decimal part of this number will never repeat?

S A : It may repeat after 10 places eg. 3.124839209124839209……

T Q : So how do we write non-terminating-non-recurring decimals that are Irrational numbers? Observe the examples (a) 0.123112233…… (b) 345.050055000555……. (c) 3.22122211122221111…… Are the decimal places repeating?

[After a series of wrong answers, students realize that the sequencing of the numbers in the decimal places is such that there is no scope of repetition.]

T Q : Good, now write down five Irrational numbers in the decimal form.

TERMINATING and NON-TERMINATING-RECURRING DECIMALS are attained from FRACTIONS.

Mathematically, how Irrational nos. can be calculated?

Students you already have learnt about 'square roots', 'cube roots', 'fifth roots' and so on.

T Q : So what is the square root of 4, i.e. $\sqrt{4}$?

S A : 2

T Q : What is the square root of 2 i.e. $\sqrt{2}$ = ? What should be the approximate value?

T P : Like $\sqrt{4}$ = 2, because 2 × 2 = 4; $\sqrt{16}$ = 4, because 4 × 4 = 16.

Similarly to find the value of $\sqrt{2}$, we need to find 'a' such that a × a = 2. So what should be the value of 'a'

S A : 'a' should be (1.some decimal no.) × (1.some decimal no.)

T E : Correct … so to find the value of 'a' let us use the Long Division method to find Square roots, which you have already studied in your earlier class.

[Teacher helps students to find the same using the long division method. (Reference 1 PPT slide 17)]

$\sqrt{2}$ = 1.41421356237… So, 1.41421356237… × 1.41421356237… = 2

[Students recall their previous knowledge and find the square roots of 3 and 5 on their own in WS 5.]

T Q : Now notice the answers, what kind of decimals do we get ?

S A : Non-terminating non-recurring.

T E : Exactly. Although these decimals do not look like Irrational numbers, but if we continue with the long division method the decimal will be non-recurring. Thus, $\sqrt{2}$ = 1.41421356237… is an Irrational number.

T E : And there is a complete different set of numbers (like $\sqrt{2}$, $\sqrt[4]{13}$, $\sqrt[3]{5}$ etc.), which are actually non-terminating- non-recurring decimals, and they are called Irrational numbers.

T Q : What do you think you will get as answers when you try to find the values of $\sqrt[3]{2}$, $\sqrt[4]{13}$, $\sqrt[3]{5}$ etc.?

TP: Will you get Whole numbers as answer?

SA: No. We will get non terminating-non-recurring decimals as answer.

T Q: Why? Explain.

SA: Like for $\sqrt{2}$ = **a** ; where **a** × **a** = 2, **a** is non-recurring decimal no.; similarly if $\sqrt[3]{2}$ = **b,** then **b** × **b** × **b** = **2,** here also **b** should be non-recurring decimal no..

T E: Great! Now whatever we discussed today was to give you an idea about **Irrational numbers**. I want you to find the perfect definition of Irrational Numbers (unlike the one given in textbook i.e. 'The numbers which are not Rational numbers are called Irrational numbers'). For that solve Worksheet 5 (Q 2), connect the different ideas and come down with the required conclusion.

Home Task : Worksheet 5 (Q2 A) to be completed by students using a calculator.

WORKSHEET 5

Q1. Find square roots of 2, 3, 5 by the Long Division Method

Q2. A) Find the square roots of all the numbers from 2 to 20 using a calculator. Write the answers till ten decimal places wherever possible. Place $\sqrt{2}$, $\sqrt{3}$, ….$\sqrt{20}$ in Column A and their values in decimal forms in Column B.

Q2. B) Observe the values of all the above square roots and answer the following:

1. Square roots of some of the 'numbers' are Whole numbers (like square root of 4 is 2). Write those 'numbers' as in Column A : ____________________
2. What are those numbers inside the square roots (like 4) in above answer (1), called?
3. Write the 'numbers' (from the table from column A) whose square roots (in column B) are in decimal form:
   (i) What are these kinds of Numbers (in answer 3) called? ________________________
   (ii) What kind of Decimal Numbers are they? (refer column B)
   (iii) Are they terminating? _____ Are they recurring? _____
   (iv) Thus, these decimals are ________________________________ in form.

Thus, the numbers written by you as answer to question Q2.B (3) above, are called Irrational Numbers.

Q3. Use all the above information to explain Irrational Numbers.

- **<u>Teaching Strategies used in Lesson Plan 8 :</u>**

Visualization: Visualization object: the slide which helps students to learns the long division method to find square roots. Introspective visualization …. Imagining the value of $\sqrt{2}$ on the basis of the values of $\sqrt{4}$ and $\sqrt{16}$.

Generalization: Students are allowed to observe the pattern of the Irrational numbers in the decimal form to understand the logic and applying the same to write more such numbers.

Mathematical Connections: Students are guided to establish the logical connections that lead to the deduction of Irrational numbers**.**

Estimation: Students are given scope to estimate the decimal values of different roots like $\sqrt{2}$ and $\sqrt[3]{2}$.

Cognitivist Strategies: Direct instruction to teach the algorithm used to find square roots of Whole numbers. Comparing the values of perfect square roots to estimate the value of $\sqrt{2}$.

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4.10 Lesson Plan 9

Unit : Real Numbers **Grade :** IX

Topic : Irrational Numbers **Duration :** 40 min

- **General Objective:**

1. Students will be able to understand the meaning of Real numbers.

- **Specific Objectives:**

1. Students will be able to define Irrational numbers.
2. Students will be able to define Real numbers

- **Teaching Aids:**

PPT Slide 18 : The decimal values of the square roots of all the numbers from 2 to 20.

Worksheet 5 : Understanding Irrational numbers

- **Teacher's Activity and Student's Activity:**

[Students solve WS 5 (Q2 B, C and D) (30 min), followed by discussion on the same (10min). The answer of WS 5 (Q2 A) put up on PPT slide 18.]

Discussions:

T Q : Let us focus on the Q2 B of WS 5, what can you conclude ?

S A : Square roots of *perfect squares* are Whole numbers whereas square roots of *non-perfect squares* are non-recurring decimals.

T Q : Based on your understanding define Irrational numbers.

T P : It is a Numbering System containing which kind of Decimal Numbers?

S A : The Numbering System which includes those Numbers that represent Non-Recurring Decimal numbers is called Irrational Numbers.

T Q : Where do we get these Non-recurring decimal numbers from?

S A : Square roots of Non-perfect squares

T E : Exactly, also from cube roots of Non-perfect cubes and all other numbers of this kind.

T Q : Students, use these information to define Irrational Numbers

S A : Numbers that are not Rational are called Irrational Numbers.

T E : That is correct but make it more specific.

S A : The non-recurring decimals that are obtained from square roots of non-perfect squares or cube roots of non-perfect cubes are called Irrational Numbers.

T E : There are few other Numbers like 'π' 'e' etc. are also Irrational Numbers. How can we say that pi is Irrational? What is the value of pi?

S A : 22/7 or 3.14

T E : What kind of numbers are they?

S A : 22/7 is a fractional number and 3.14 is a decimal number.

T P : Elaborate further. They belong to which Numbering system?

S A : Rational Numbers…because 22/7 is a fractional no. and 3.14 is a terminating decimal number.

T E : But, in Mathematics π is considered to be an Irrational number. If you want to know why, watch the video using the link 'https://www.youtube.com/watch?v=HSuqbqENIek'.

T E : The video makes it very clear that the value of π is approximately and not exactly 22/7 or 3.14.

T E : Thus, with this we know about all the possible observable numbers that exists.

This whole big set of numbers which includes all whole positive and negative numbers, fractions, all types of decimal numbers is called *Real Numbers.*

Broadly the set of Real Numbers is made up of two different sets (1) Rational numbers (2) Irrational numbers; which are disjoint i.e. no number can be Rational as well as Irrational.

T Q : So, how are Rational numbers different than Irrational numbers?

[Students randomly answer based on the definitions, decimal place structures, physical appearances, numbering systems.]

Thus in Arithmetic, we study about these observable numbers called Real numbers. In Algebra we deal with unknown numbers to go beyond the concrete and frame formulae, rules, properties, thus moving from specific to general.

- **<u>Teaching Strategies used in Lesson Plan 9 :</u>**

Visualization: Visual aid used to make students visualize in concrete real terms as to why π is an Irrational number.

Generalization: Students observe the solutions (decimal forms of square roots of all numbers from 2 to 20), observe the patterns and define of Irrational Numbers.

Estimation: Concept of approximate values of decimal numbers invariably used throughout the lesson; also the description of the value of π needs students.

Mathematical connections: Students get a scope to see the connections between different mathematical concepts like square roots, perfect squares, decimal numbers and Irrational nos.

Higher order questioning: Students self-work to respond to the worksheet 5 questions which needed students to see the patterns, analyze, evaluate and then define Irrational nos. Thus, questions 2 and 3 if worksheet 5 align to this characteristic of higher order questioning.

Cognitivist Strategies: Providing visuals to aid comprehension using ppt slides, comparing new to the already known concept (difference between Rational and Irrational numbers).

4.11 Lesson Plan 10

Unit : Real Numbers **Grade :** IX

Topic : Representation of Rational numbers on Number line **Duration :** 40 min

- **General Objective:**

1. Students will be able to understand how Rational numbers are placed on Number line.

- **Specific Objectives:**

1. Students will be able to represent proper and improper fractions on Number Line.
2. Students will be able to represent 'numbers having more than one decimal places' on Number Line.
3. Students will be able to estimate the positions of Rational numbers on Number line with respect to Integers.

- **Teaching Aids:**

Worksheet 6: Representation of Fractions and Decimals on Number line (Practice sheet)

SLM 2: Representation of Fractions and Decimals on Number line (Appendix)

- **Teacher's Activity and Student's Activity:**

Teacher recapitulates with help of the Content-chart.

T E : So far we have discussed in detail about the members or the numbers that belong to the Numbering system called Real Numbers.

T Q : 1. Which two major numbering sets together form the set of Real Numbers?

2. The set of Rational numbers has which type of numbers?

3. The set of Irrational numbers has which type of numbers?

S A : (1) Rational and Irrational numbers. (2) Integers and all fractional numbers i.e. terminating and recurring decimal numbers. (3) Non-recurring decimal numbers or square roots of non-perfect squares.

T E : We refer to the Content-chart to check which topics has been covered till now in the Chapter 'Real Numbers'(Teacher specifies the same).

The topic that we go ahead with is 'Representation of Real Numbers on the Number Line'.

T Q : But why do we want to represent numbers on a Number Line?

S A : Not sure ???

T E : Representing Numbers on a Number line helps us to understand :

1) Every point on the Number line has a unique number associated with it, so we can find that point corresponding to that given number.
2) We can understand the positions of each number with respect to the other numbers.

3) A proper understanding of the Number line helps us to imagine the position of numbers which might not be practically possible to show on a Number line.

T E : For the next few classes our aim will be to learn how to place Real Numbers on the Number Line. Since Real Numbers comprises of Rational and Irrational numbers, we will first learn how to place Rational nos. on Number line and then we will go for Irrational nos.

So, today's topic is 'Representation of Rational Numbers on Number Line'

Now, Rational Numbers comprises of (1) Integers (2) Fractions/Decimals. You have already studied in your earlier classes how to represent Integers and Fractions on Number Line. Let us refresh our knowledge.

[Teacher draws a Number Line with Integers on Board and points out basic facts like – use of scale and pencil to draw Number lines, – arrows on both sides, - numbers clearly written and should be equidistant from each other.]

T E : In order to understand Representation of Rational Numbers (specially Fractions and Decimals) on Number Line, read carefully SLM 2, it very clearly shows the steps that can be used to represent fractional (proper and improper) numbers and decimal numbers with more than one decimal places on the Number line.

T Q : Draw a Number line in your note books, represent 3/8 on it. What process did you use?

[Students explain the process in accordance to the SLM.]

[Teacher draws a Number line on BB with 0, 1, 2, 3 marked]

T Q : On this Number line, without using the process, try to estimate the position of 7/8.

T P : Refer to the position of 3/8 in your notebook.

S A : Somewhere very near to 1, because 8/8 will be at 1.

T Q : Where will be 5/6……2/9,……3/13, ……1/100 on this Number line?

[Students get a scope to estimate and visualize]

T Q : Where will be 7/2, 10/3, 9/5 approximately placed on the Number line?

[Students convert them into mixed fraction form and answer]

T Q : Similarly use the method shown in SLM to mark the place of 2.6; 2.65; 2.658; on the Number line.

T Q : How many divisions we need to make between 2 and 3 on the number line in the first case,….in the second case ,….and in the third case.

S A : ten for first, second ???

T P : each of these ten parts is again divided into how many parts?

S A : each is divided into ten parts so, there are total ten parts and so in second case we make 100 divisions; similarly in third case thousand total divisions.

T Q : Without doing the process estimate the positions of 1.64 and 0.234 on the Number line. [Students point out the approximate positions on the Number line on the BB]

Home Task : Teacher instructs students to complete WS 6.

WORKSHEET 6

1. Draw Number Lines in each case and represent the following Rational numbers on them :

1) $\frac{6}{7}$ (2) $\frac{(-13)}{5}$ (3) $\frac{13}{4}$ (4) (-1.7) (5) 0.4

(6) 2.34 (Draw two Number lines as shown in SLM 2)

(7) (-1.367) (Draw three Number lines)

- **Teaching Strategies used in Lesson Plan 10 :**

Visualization: The self -learning material help students to visual the process and learn on their own (Visualization Object). This would help them to represent other fractional and decimal numbers on Number line (Introspective visualization). Finally the questioning based on the SLM would help students to mentally visualize the positions of different Rational numbers with respect to consecutive Integers (Interpretative visualization).

Generalization: 'Discover learning' approach was used by students through the SLM where they got engaged in inductive reasoning- to work with given set of data, analyze the processes used, identify the patterns or the relationship that exists and synthesize to infer the method to be used to place Rational numbers on Number line.

Estimation: Students were mentally engaged in finding approximate positions of fractional numbers and decimal numbers with respect to consecutive Integers on Number line.

Mathematical connections: the SLM also allowed the students to see mathematical connections among content areas, among the processes and within their own thinking.

Higher order questioning: The teacher questioning done after the students read and understand the SLM 2….helping students to strengthen their understanding about the process to be used, use their generalizing, visualizing and estimating skills.

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4.12 Lesson Plan 11

Unit : Real Numbers **Grade :** IX

Topic : The concept of UNIT **Duration :** 40 min

- **General Objective:**

1. Students will be able to understand the concept of Unit measurement.

- **Specific Objectives:**

1. Students will be able to deduce the conceptual meaning of Basic Unit in measurement.
2. Students will be able to use appropriate units to draw line segments of given lengths.

- **Teaching Aids:**

Calibrated Glass, Jar, Water, Teachers' Scale

- **Teacher's Activity and Student's Activity:**

T E : We are in process of representing Real numbers on Number line. In the previous class we have learnt how to represent Rational numbers on Number line, but before we learn how to place or find Irrational numbers on Number line, we need to understand the concept of UNIT. Like you must have come across measures like 3 cm, 5 m, 10 mm, 8 kg, 500g, 30 km, 50 km/hr, 5 litre, 300ml etc.

T Q : In each case, what is cm, m, km, kg, ………?

S A : Units

T Q : And what is the meaning of this term 'Unit'?

S A : It is the basic measure.

T Q : If I want to draw a line segment of 5 cm length. How will I draw it?

S A : We take a scale and then we start drawing a line from 0 and end at 5.

T Q : And what is the unit or the basic measure?

S A : 1 cm.

T E : That means I have drawn 5 continuous line segments of 1 cm each.

T Q : Now if want to draw a line of length 2 m, then what is the unit and how can draw such a line?

S A : Unit is *1 m* and you have to draw two lines one after the another of *1m* each.

T E : Good. Similarly for 5 kg weight, the basic unit is *1 kg* and for 500 g it is *1g*.

T Q : From all the examples we discussed above, should we understand that the basic unit is always 'one' i.e. *1mm* or *1cm* or *1m* or *1g* or *1kg* or *1litre* etc.

S A : Yes.

T E : So, let us see an experiment and check whether our understanding is correct or not.

Demonstration: Teacher demonstrates this concept with the help of a transparent glass, that is calibrated as full, half, three-fourth with a permanent marker.

1. Teacher takes full glass of water and pours in a jar three times.

T Q : What should be the basic unit in this case?

S A : 1 full glass.

2. Now, teacher fills the glass three-fourth each time and pours it in the jar again for three times.

T Q : What is the basic unit this time?

S A : Three-fourth glass.

Teacher fills the glass till half and pours the water three times.

T Q : What is the basic unit now?

S A : Half glass.

T Q : Very good, What do you understand from this experiment? Is it compulsory to have the basic unit as 1 always?

S A : No it may be half or three-fourth or anything else also.

T Q : In each of the three cases, was the total amount of water I was pouring in the jar same ?

S A : No, they were different, first case it was maximum, lesser in the second case and least in the last case.

T C : So in the first case, the basic unit = 'a full glass', in the second case basic unit = 'three-fourth glass' and in the third case it was 'half glass'.

Applying the same logic, suppose I want to draw a line segment of 5 units…

T Q : If I consider 1 unit as 1 cm, then 5 units = 5 cm and if I draw this line on this large Blackboard, it will be very small. So, instead I consider 1 unit as 2cm, the actual length of that line will be 10 cm but it can be written as 5 units.

[Teacher draws on the BB : 2cm lines one after the other five times]

A 5 units B Length of AB = 5 units (1 unit = 2 cm)

Actual length of AB = 10 cm

Similarly, if you are told to draw a line segment of 20 units in your book, if you take 1 unit as 1 cm, the line will not fit in your book properly, so you can consider 1 unit as 0.5 cm or 5mm and then draw the line. The actual length will be 10 cm, but it will be represented as 20 units (1 unit = 0.5 cm)

Since now we are dealing with Real numbers which can be very large and very small, in order to study them we have to use represented measures instead of actual measures by changing the basic unit measures instead of taking the usual standard 1 as the basic unit, like 1cm, 1m, 1mm, 1km etc.

This concept of ‘Unit’ is very important to represent Irrational numbers on the Number line.

T Q: Use your imagination and decide on the correct unit to be taken for drawing line segments of length

(1) 30 cm (2) 2cm (3) 100cm (4) 10m (5) 24m

Teacher instructs students to bring their Geometry boxes for the next few classes.

- **Teaching Strategies used in Lesson Plan 11 :**

Visualization: Students are engaged in visualizing the actual lengths from the given numerical values to derive methods to draw them on notebooks.

Estimation: Students are lead to mentally estimate the lengths of given numeric measures to find appropriate units to concretize the same.

Mathematical connections: Mathematical connections established between the concept of ‘unit measurement’ with measurements in real life scenarios through the demonstration and the different examples used.

Higher order questioning: Posing questions and tasks that elicit, engage, and challenge each student’s thinking throughout the lesson to establish the concept of Unit. The final question in which students need to decide appropriate units to draw line segments of given lengths need higher mental reasoning.

Cognitivist Strategies: Use of real world examples and providing visuals.

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4.13 Lesson Plan 12

Unit : Real Numbers **Grade :** IX

Topic : Irrational Nos. on Number Line **Duration :** 40 min

- **General Objective:**

1. Students will be able to understand how Irrational Numbers are placed on the Number line.

- **Specific Objectives:**

1. Students will be able to estimate the place of the Irrational numbers on the Number line with respect to the positions of Integers.
2. Students will be able to explain the use of Compass to construct line segments of given lengths on a Number line.
3. Students will be able to construct line segments of given lengths.
4. Students will be able to use the length of a constructed line segment to represent a number on the Number line.

- **Teaching Aids:**

PPT Slide 19 & 21: Line segments of equal lengths can be drawn using a compass

Worksheet 7: Construction of line segments of given lengths using a Compass

- **Teacher's Activity and Student's Activity:**

T E : Referring to the content-chart, we realize that we have clearly understood what are Rational numbers and what are Irrational numbers. We have seen how Rational numbers can be represented on the Number-line and today we will understand how Irrational numbers can be represented on a Number line.

Irrational numbers are Non-Recurring Decimals like $\sqrt{2}$ = 1.41421356237… or $\sqrt{3}$ =1.73205081… etc. Since we do not know the exact value it is tough to represent them on a Number line by the way we adopted to represent Rational numbers. Before we learn the method used to represent Irrational numbers on Number line, let us estimate their approximate places on the Number line.

[Teacher shows PPT slide 19 and explains] :

T E : *Let us draw a Number line XY with Integers 0 to 9 as shown in the slide.*

T Q : *Refer to Slide 18 (Square root table) and tell me the given Integers on the Number line are equal to which square root values …like 1 (No. line) would correspond to* $\sqrt{1}$*, similarly 2 would correspond to* $\sqrt{4}$

S A : *3 would correspond to* $\sqrt{9}$*, 4 would correspond to* $\sqrt{16}$*….*

***T E :** Very good, let us place these perfect squares below the corresponding Integers.*

***T Q :** Refer to the Square root table and tell where will be $\sqrt{2}$ and $\sqrt{3}$ placed on the Number line.*

***S A :** Their values are 1.4…. and 1.7….., so should be between 1 and 2*

Similarly teacher probes and helps students to estimate the positions of different Irrational numbers on the number line with respect to the position of Integers.]

T E : The slide 19 helped us to estimate the positions of Irrational numbers on Number line with respect to the positions of the Integers. But we still don't know how to point the exact place of an Irrational number on the Number line. Now we will focus on understanding the method to do so.

So we will use the Geometric Method to represent Irrational numbers on Number line.

But for that we first need to recapitulate a few concepts from your previous class:

1) Constructions of Line segments of given measures
2) Pythagoras Theorem

I. Constructions of Line segments of given measures on Number line

[Teacher draws a line-segment AB on the board.]

T Q : Suppose I want to construct another line-segment PQ of the same length as AB. How will I do it?

S A : Taking a scale, measuring AB and then drawing PQ of the same length.

T Q : And if I don't want to use the scale, is there any other way to construct PQ?

S A : May be by using the Compass as we did in Class VIII.

T E : Correct. But why do we use a Compass? What is the logic behind it? Let us try to understand from the PPT.

[Teacher shows PPT slide 20 and explains..]

***T E :** This slide shows a circle with center O and points P, Q, R & S on the circle. If we take out OP, OQ, OR, & OS, then we can see that their lengths are equal. This is because they are the radii of the same circle. Similarly in slide 21, instead of the whole circle if we take a part of it i.e. Arc, then as shown in the slide OA, OB & OC have equal lengths because they are the radii of the same circle/arc. Thus, what can we conclude?*

S A : From the center to the arc, all the line segments drawn will be of the same length

T Q : Can we use this fact to draw line segments of equal lengths without a scale?

S G : 'If we want to draw several line segments of the same lengths, then instead of using the scale again and again , we can simply use the compass by fixing its radius with the wanted length'

T E : This concept can be used to draw Line segments of required lengths, where more than exact measures, comparative measures are needed. [Solve Q1 of WS 7 and explain the procedure you used.]

S Act : Students do the construction using geometrical instruments and explain the process

1. Use a Compass and take A as center and radius as AB.
2. Draw a line XY and place the compass with radius AB at any point on the line XY, call that point P.
3. Draw an arc on the line XY. Mark the point Q, at which the arc cuts the line XY.
4. Line segment PQ represents the line of length equal to AB.

T I : Now Solve Q2 of WS 7

S A : 1. Take point O on Line XY as the and Radius as 5 cm (use a scale) and draw an Arc intersecting the Line XY at a point T.

1. Line segment OT is the required line of length 5 cm.

T E : Basically the Compass is used to draw circle of given radius. But the same concept can be used to construct line segments of given lengths. Read the instructions in Q3 and Q4 of WS 7 carefully and work them out.

Teacher discusses the answers with students.

T C : So we can use a Compass to construct line segments of required length on a given Line. By considering O that is the center as zero (0) and length of the given line segment as the radius ('h' cm or 'b' cm in Q4) on a simple Line, it can be converted into a Number Line. This method can be called as the Geometric method of representing numbers on a Number line which we are going to use further to represent Irrational numbers on a Number line.

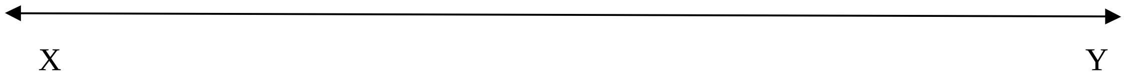
But such an elaborate method is not needed to represent Rational numbers on the Number line, which can be done by much easier methods as discussed in the earlier classes. This Geometric Method is applicable for Irrational no. We will learn that in detail in the next class

WORKSHEET 7

Q1. Line segment XY is given below. Without measuring it with a scale, construct another line segment AB of length same as XY using a Compass.

A B

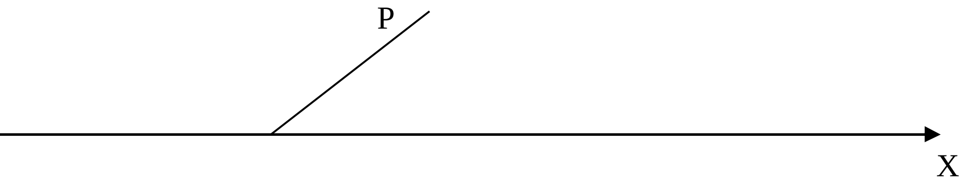
Q2. Construct a Line segment AB = 5 cm using a Compass on the given line XY.



Q3. Use the given figures and do as directed.

(a)Line OX in Fig.1 is a Line. Place a point A on ray OX in such a manner that OA = OP. You cannot use a Scale. Explain what will you do. (What would be the Center and the Radius?)

Fig. 1

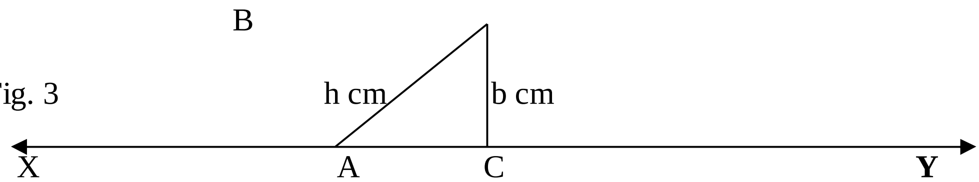


(b) Line XY is shown in Fig. 2. Line segment OP is perpendicular to Line XY. Place a point A on $\overrightarrow{OY}$, such that OP = OA. You cannot use a Scale. Explain what you will do.

Fig. 2

Q4. Attempt the following:

Fig. 3



$\triangle$ABC is a right-angled triangle on Line XY. Length of the hypotenuse is ‘h cm’ and length of side BC is ‘b cm’.

(a) Use a Compass to represent the measure ‘h cm’ on $\overrightarrow{AY}$ given in Fig. 3

Which point should be the center? : _____

What should be the radius? : _________

Draw an Arc using the above center and radius.

Let the Arc intersect $\overrightarrow{AY}$ at point M.

Thus, which line segment on the Line XY, represents h cm = ________

(b) Use a Compass to represent the length of BC i.e. ‘b cm’ on $\overrightarrow{CY}$ given in Fig. 3

Which point should be the center? _____

What should be the radius? ______

Draw an Arc using the above criteria, intersecting Line CY at P.

Thus, which line segment on the Line XY represents b cm = _______

- **Teaching Strategies used in Lesson Plan 12 :**

Visualization: Students are shown the relevance of compass to construct line segments of equal lengths through visual aids and hand-on tasks

Generalization: Students observe the constructions with the compass and generalize on its use to construct line segments with equal lengths.

Estimation: Students are helped to estimate the places of some Irrational numbers on Number line with reference to the positions of Integers

Mathematical connections: Mathematical connection between the concepts of geometry and arithmetic with lengths of line segments used to represent numbers of the numbering systems.

Higher order questioning: Questioning that lead students to estimate the approximate positions of Irrational numbers with respect to consecutive Integers. Questions 3 and 4 of WS 7 lead students to devise their own method to construct line segments of given lengths, and use the numeric values of these lengths to be represented on a Number line; thus can be referred to as higher order questioning.

Cognitivist Strategies: Comparing new to the already known concept i.e. numeric representations with constructions of line segments with given lengths.

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4.12 Lesson Plan 13

Unit : Real Numbers **Grade :** IX

Topic : Irrational Nos. on Number Line **Duration :** 40 min

- **General Objective:**

1. Students will be able to understand the Geometric Method.

- **Specific Objectives:**

1. Students will be able to use the Pythagoras theorem to construct a right-angled-triangle with hypotenuse of $\sqrt{2}$ units.
2. Students will be able to apply the Hypotenuse Geometric method to represent Irrational numbers on the Number line.

- **Teaching Aids:**

PPT Slide 22: Showing the Construction steps

- **Teacher's Activity and Student's Activity:**

T E : In the previous class we have seen how lengths of different line-segments can be used to represent given numbers on a Number Line. In similar manner we will learn today to represent Irrational numbers on a Number Line. Let us call that as the 'Geometric Method to represent Irrational numbers (specially the square roots) on the Number line'.

We will be using two concepts that we learnt in the earlier classes to go ahead (1) the concept of 'Unit' and (2) the use of compass to represent lengths of line-segments on a Line.

Problem 1 : Representation of the Irrational number $\sqrt{2}$ on the Number line.

T Q : We will learn a method to locate the exact position of $\sqrt{2}$ on the Number line, but approximately, it should lie between which two Integers?

S A :$\sqrt{2}$ = 1.41….., so it should lie between the Integers 1 and 2.

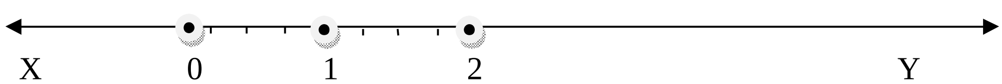
T E : Yes, correct. So then we start by drawing a Number line, and since we know that $\sqrt{2}$ will lie between 1 and 2, minimum numbers needed to be shown are 0, 1 and 2. What unit should be taken here?

S A : 1 cm

T P : We will get a very small version in that case, making the diagram looking messy. Let us take 1 unit = 4 cm

Step 1 : Teacher draws Fig. 1 on the blackboard a Number line XY with Integers 0, 1, and 2 by taking 1 unit = 4 cm and showing demarcations as follows :

Fig. 1



Step 2 : Since $\sqrt{2}$ = 1.4142…..is a non-recurring decimal number, we cannot find its exact place on the Number line, so we have to use the concept of representing length of a line-segment on a Number line.

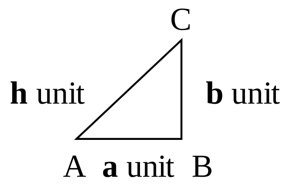
T Q : Think and tell me how can we get a line-segment of length $\sqrt{2}$ unit?

T P : You have studied Pythagoras Theorem in your earlier class, can you use that?

S A : The Pythagoras theorem states that 'For a right-angled-triangle, the square of the length of its hypotenuse is equal to the sum of the squares of its sides'.

[Teacher draws Fig.2 on blackboard, with **h** as the length of hypotenuse, **a** as length of the base and **b** as the length of the perpendicular side.]

Fig. 2



Pythagoras Theorem : $h^2 = a^2 + b^2$

or $h = \sqrt{a^2 + b^2}$

T Q : For what values of **a** and **b** will **h** become $\sqrt{2}$?

Students are given some time to work out and come to the solution : a = 1 unit and b = 1 unit, as $h = \sqrt{1^2 + 1^2}$; thus $h = \sqrt{2}$ unit

T C : So this helps us to conclude that if we construct a right-angled triangle ABC as shown in Fig.2, with a = 1 unit and b = 1 unit, then we can get a line-segment of length $\sqrt{2}$ unit as its hypotenuse.

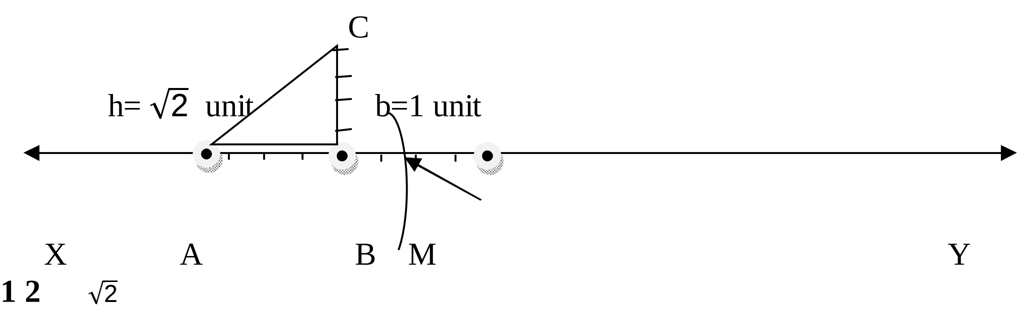
T Q : Now will we use this to represent $\sqrt{2}$ on the Number line we have drawn in Fig.1?

T P : Refer to WS 7 Q4 (a), and find out a way for it.

Students are given some time to work on it.

T C : Teacher shows the construction with help of the Fig.3 and shows the steps of construction by showing PPT Slide 22.

Fig. 3



Steps of Construction :

1. *Draw Line XY and place numbers 0, 1, and 2 using 1 unit = 4cm. Mark the points at 0 as A and 1 as B. Thus, length AB = a = 1 unit.*
2. *Use a set-square to draw a perpendicular from point B. Let the length of this perpendicular BC = b = 1 unit (4 cm).*
3. *Join points A and C. The length of AC = h =* $\sqrt{2}$ *units.*
4. *Use a compass and taking A as the center and radius = h =* $\sqrt{2}$ *units, draw an arc cutting Line XY and name that point as M.*
5. *Point M represents* $\sqrt{2}$ *and length of AM is* $\sqrt{2}$ *units.*

T Q : Verify whether $\sqrt{2}$ = 1.41…., lie between 1 and 2 from the construction.

Home Task : Problem 2: Represent $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$ on the Number line by the Hypotenuse Geometric Method.

- **Teaching Strategies used in Lesson Plan 13 :**

Visualization: Visualization Object with visual aids and Introspection visualization used in visualizing the value of Irrational numbers which do not have a real value and are abstract in nature.

Generalization: ' Discovery learning' and 'direct instruction' was used appropriately to promote generalization of the procedure used to represent $\sqrt{2}$ on the Number line.

Estimation: Probing students to estimate the position of the Irrational numbers (based on their decimal value) with respect to the Integers on the Number line, before using the procedure to represent the same.

Mathematical connections: Students are shown the mathematical connection between the new concept of 'representing the numerical value of $\sqrt{2}$ as length of a line segment' with the previously concept of Pythagoras theorem. Students make connections of 'the geometrical constructions of given lengths from WS 7' to understand 'the representation of $\sqrt{2}$ on the Number line.

Higher order questioning: Posing questions and tasks that elicit, engage, and challenge each student's thinking throughout the lesson to achieve the specific objectives.

Cognitivist Strategies: Providing visuals as diagrams on BB and through PPT slide.

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4.13 Lesson Plan 14

Unit : Real Numbers **Grade :** IX

Topic : Hypotenuse Geometric method for consecutive Irrational nos. **Duration :** 40 min

- **General Objective:**

1. Students will be able to understand the Geometric method of placing Irrational numbers on the Number line.

- **Specific Objectives:**

1. Students will be able to represent given Irrational number on the Number line by previous consecutive number representations.

- **Teaching Aids:**

Black Board Work: The Hypotenuse Geometric Method shown for consecutive initial Irrational numbers

- **Teacher's Activity and Student's Activity:**

Home Task : Problem 2: Represent consecutive Irrational numbers $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$ on the Number line by the Hypotenuse Geometric Method.

Try to find a general method to represent all Irrational numbers of the square root form on the same Number line. Students may come out with the following process as shown below. Teacher discusses the same ahead.

T E : Let us represent all the above on the same Number line by the Hypotenuse Geometric Method. *(Teacher explains and writes the same on the blackboard. Students note the same.)*

(1) $\sqrt{3}$Value of **a** and **b** for **h** = $\sqrt{3}$ unit **:**

h $= \sqrt{a^2 + b^2}$ …………(1)

$\sqrt{3} = \sqrt{a^2 + b^2}$

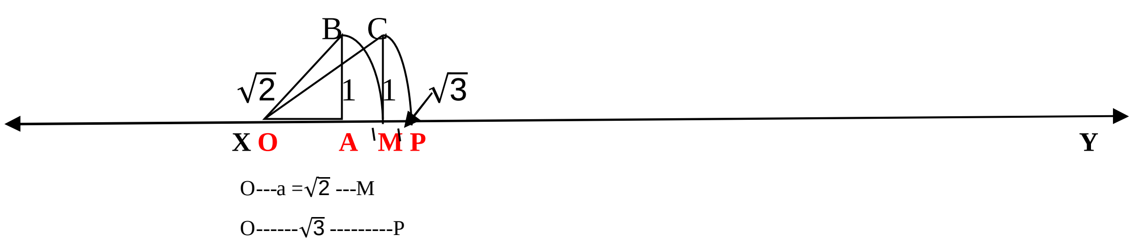
$\sqrt{3} = \sqrt{2 + 1}$…………(2)

From (1) and (2) values of a and b for which h = $\sqrt{3}$ are :

$a^2 = 2 \implies a = \sqrt{2}$ and $b^2 = 1 \implies b = 1$

Thus, to represent $\sqrt{3}$ on the No. line, we need to construct a right-angled triangle OMC on the No. line with one of its side **a** of length $\sqrt{2}$unit and another side **b** with length 1 unit.

Fig. 1



Construction Steps :

1. Represent $\sqrt{2}$ = OM on the Number line XY
2. Construct a right-angled triangle OMC, with side a = $\sqrt{2}$ units (OM shown in Fig. 1) and the perpendicular side MC = 1 unit. Thus OC = h = $\sqrt{3}$ units
3. Taking O as the center and OC as the radius, draw an arc intersecting line XY at P
4. Point P on the Number line represents $\sqrt{3}$

T Q : What is the decimal value of $\sqrt{3}$ **?**

S A : 1.7320……

T Q : Does the representation on the Number line verify that.

T E : As we represented $\sqrt{3}$ on the No. line with the help of $\sqrt{2}$, can we go ahead similarly for $\sqrt{4}$ (it is a Rational no. but helps us with the chain), $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$ and so on.

Just like $\sqrt{3} = \sqrt{2+1} \Longrightarrow$ a = $\sqrt{2}$ and b = 1 (We have to represent $\sqrt{2}$ first)

Similarly $\sqrt{4} = \sqrt{3+1} \Longrightarrow$ a = $\sqrt{3}$ and b = 1 (We have to represent $\sqrt{2}$ first, then $\sqrt{3}$)

$\sqrt{5} = \sqrt{4+1} \Longrightarrow$ a = $\sqrt{4}$ and b = 1 (We have to represent $\sqrt{2}$ first, then $\sqrt{3}$ and $\sqrt{4}$)

$\sqrt{6} = \sqrt{5+1} \Longrightarrow$ a = $\sqrt{5}$ and b = 1 (We have to represent $\sqrt{2}$ first, then $\sqrt{3}$, then $\sqrt{4}$ then $\sqrt{5}$

T Q : So, can you see the pattern? In the above cases for the values of the hypotenuse $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$, etc. what are the respective values of ‘a’ and ‘b’ i.e. the sides of the right-angled triangle? Say the hypotenuse is $\sqrt{n}$, then what will be the respective value of ‘a’& ‘b’?

S A : $\sqrt{n} = \sqrt{(n-1)+1}$ **a** = $\sqrt{n-1}$ unitsand **b** = 1unit

T Q : So what does the rule you made just now help you do?

S A : If we want to represent consecutive Irrational numbers as above, they can be represented on the same Number line by the process as shown by you earlier.

Home Task : Represent consecutive numbers $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, $\sqrt{8}$ on the same Number line by the Hypotenuse Geometric Method. Mention in case the value of **a** and **b** taken and the Construction steps.

- <u>**Teaching Strategies used in Lesson Plan 14 :**</u>

Visualization: Object visualization through diagrams on black board and then Introspective visualization where students were made to imagine the process they need to carry out to represent consecutive Irrational numbers on Number line.

Generalization: Empirical Generalization has been used. It is a gradual process of analyzing a series of concrete examples in which the non-essential attributes are systematically changed.

The same was done with the values of ‘h’, ‘a’ and ‘b’ which helped students’ generalize the relationship and internalize the geometric process of representing consecutive Irrational numbers on Number line.

Estimation: Students are probed to verify the positions of the Irrational numbers on the Number line by using estimation techniques.

Mathematical connections: Students were shown mathematical connections among the contents - ‘Pythagoras theorem’, ‘Construction of line segments’ and ‘Representation of Irrational numbers on Number line’; among mathematical processes - ‘finding the values of ‘a’ and ‘b’ from the value of ‘h’’ with that of the steps used to represent Irrational numbers on Number line; finally among their thinking - by giving a chance to work out the process on their own with the home task which made them reflect on their own methods (errors) in thinking as well as connect the same with the process later shown by the teacher.

Higher order questioning: Posing questions and tasks that elicit, engage, and challenge each student’s thinking throughout the lesson.

Cognitivist Strategies: Use of well-connected steps with visuals.

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4.14 Lesson Plan 15

Unit : Real Numbers **Grade :** IX

Topic : Hypotenuse Geometric method for non-consecutive Irrational no. **Duration:**40 min

- **General Objective:**

1. Students will be able to understand how Irrational numbers are placed on the Number line.

- **Specific Objectives:**

1. Student will be able to represent the given Irrational number on the Number line without representing previous Irrational numbers.

- **Teaching Aids:**

PPT Slide 23, 24 & 25: Representation of $\sqrt{15}$ on Number line and Steps of Construction to represent $\sqrt{15}$

Worksheet 8: Practice for representing Irrational numbers on Number line using the Hypotenuse Geometric method

- **Teacher's Activity and Student's Activity:**

T E : Students, in the previous class we have learnt how we can represent a given Irrational number on the Number line by representing all the consecutive previous numbers of the form $\sqrt{x}$, where $x \in$ N; $x \geq 2$.

But the process becomes a really long one, is there any other way in which we can directly find, place or represent the given Irrational number on the Number line without representing all the previous numbers first and is applicable to a large Irrational number as well.

Let us try with $\sqrt{15}$ and come down to a general method.

Problem : Represent $\sqrt{15}$ on the Number line using the Hypotenuse Geometric method (without representing the previous consecutive numbers)

T Q : Use the Pythagoras theorem, to find the values of the sides of a right-angled triangle **a** and **b**, where **h** = $\sqrt{15}$ i.e $\sqrt{15} = \sqrt{a^2 + b^2}$ ……………..(1)

Students are given time to work on it.

T P : Remember, since the formula above has squares (a^2 and b^2) try to find perfect squares that fits into the equation (1). So which perfect square do you suggest for the value of a^2 that would take is nearest to 15?

S A : 9

T Q : Very good, so we take a^2 as 9, then what is the value of **a** and that of **b**?

S A :a would be 3.

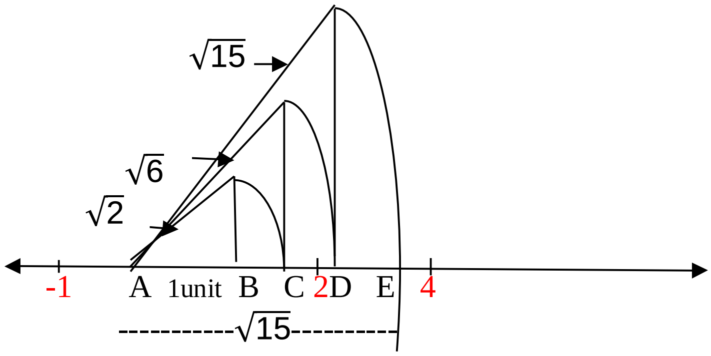
T E : You know the values of **a** and **h**, put them in equation (1) and get the value of **b.**

S W : $\sqrt{15} = \sqrt{9 + b^2}$

$\sqrt{15} = \sqrt{9 + 6}$

We get **a** = 3 and **b** = $\sqrt{6}$

T E : In order to construct hypotenuse of $\sqrt{15}$ unit length, the values of a = 3 unit and b = $\sqrt{6}$ unit. Again, since **b** is Irrational we consider it as the hypotenuse of another right-angled triangle and find its respective sides, a_1 = 2 unit and b_1 = $\sqrt{2}$ unit, again to represent $\sqrt{2}$, a third hypotenuse is to be constructed with a_2 = 1unit and b_2 = 1 unit.



[Teacher shows the PPT slides 23, 24 and 25 and explains in an organized way the steps that are followed to represent $\sqrt{15}$ on number line. Also questions on the estimated positions of $\sqrt{2}$, $\sqrt{6}$, and $\sqrt{15}$ are put forward at appropriate junctures.]

Home Task : Complete Worksheet 8

WORKSHEET 8

Use the Hypotenuse Geometric method to represent the following Irrational numbers on the Number line. 1) $\sqrt{6}$ 2) $\sqrt{10}$ 3) $\sqrt{11}$ 4) $\sqrt{13}$ 5) $\sqrt{18}$ 6) $\sqrt{15}$

- **Teaching Strategies used in Lesson Plan 15 :**

Visualization: The PPT with custom animation to show the construction used to represent $\sqrt{15}$ on Number line.

Estimation: Scope given to estimate the positions of different Irrational numbers on Number line, line square root of 2, 6 and 15.

Mathematical connections: Connections between two processes that can be used to represent Irrational numbers on Number line is shown.

Higher order questioning: Questioning done to find the values of ‘a’ and ‘b’ for ‘h’ as 15.

Cognitivist Strategies: Establishing the relevance of the present mathematical content with reference to the previously learnt similar content (representing Irrational numbers on Number line by representing all the previous consecutive Irrational numbers).

4.15 Lesson Plan 16

Unit : Real Numbers **Grade :** IX

Topic : Perpendicular Geometric method to represent Irrational nos. **Duration :** 40 min

- **General Objectives:**

1. Students will be able to understand how Irrational numbers are placed on the Number line.

- **Specific Objectives:**

1. Students will be able to deduce that the Hypotenuse Geometric method is not applicable to all Irrational numbers like for square roots of decimal numbers.
2. Students will be able to find the lengths of the hypotenuse and the base when the length of the perpendicular side of a right-angled triangle is of the form $\sqrt{x}$ unit.

- **Teaching Aids:**

Worksheet 9: To prove that for a Right angled triangle, if $\sqrt{x}$ is the measure of the perpendicular side, then lengthof the hypotenuse would be (x + 1)/2 units and that of the base would be (x – 1)/2 units and verifying the formula by the Pythagoras theorem

- **Teacher's Activity and Student's Activity:**

T E : When we use the Hypotenuse Geometric method to represent $\sqrt{x}$, if x is a large number, the representation becomes quite complex. Moreover, can we use this Hypotenuse Geometric method to represent $\sqrt{x}$, where x is a decimal number? Let us find out…..

T Q : Using the same Hypotenuse Geometric method, represent $\sqrt{3.2}$ on the Number line.

[Students take h = $\sqrt{3.2}$ and try to find the value of **a** and **b**, but fail.]

T E : Is it possible to get the values of **a** and **b** by the trial-n-error method that we have been using for square roots of Whole numbers till now? **S A :** No

T E : So in order to represent such decimal Irrational numbers on Number line, we might need to use Case 2, i.e. the *Perpendicular Geometric method* where the length of the perpendicular of a right-angled triangle is the given Irrational number $\sqrt{x}$.

The Perpendicular Geometric method can be used to represent $\sqrt{x}$ on the Number line, where *x* can be a Whole number as well as a decimal number.

To understand this method properly, let us play a game.

T I : The instructions are given in WS 9.

1. Let **x** be your Roll No., **x =** _____
2. Calculate $\frac{x+1}{2}$ and $\frac{x-1}{2}$
3. Now find $\left(\frac{x+1}{2}\right)^2$ and $\left(\frac{x-1}{2}\right)^2$

4. Now subtract : $\left(\frac{x+1}{2}\right)^2 - \left(\frac{x-1}{2}\right)^2$

[Teacher discusses the answers and writes around five to six values on BB.]

BB : For x = 7, $\left(\frac{x+1}{2}\right)^2 - \left(\frac{x-1}{2}\right)^2 = 7$

T Q : Does the answer help you to generalize something?

S G : $\left(\frac{x+1}{2}\right)^2 - \left(\frac{x-1}{2}\right)^2 = x$

T E : Let us write the above equation as :

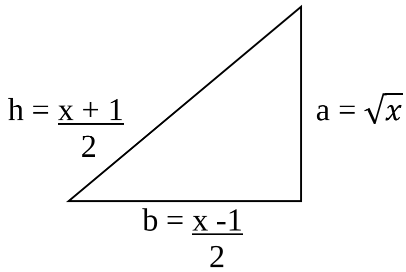
$\left(\frac{x+1}{2}\right)^2 = \left(\sqrt{x}\right)^2 + \left(\frac{x-1}{2}\right)^2$ -----------------(1)

Let us compare equation (1) with the Pythagoras theorem: $h^2 = a^2 + b^2$ -----------------(2)

Thus If $a^2 = \left(\sqrt{x^2}\right) \Longrightarrow a = \sqrt{x}$

Then $h^2 = \left(\frac{x+1}{2}\right)^2 \Longrightarrow h = \frac{x+1}{2}$

And $b^2 = \left(\frac{x-1}{2}\right)^2 \Longrightarrow b = \frac{x-1}{2}$



Thus, for any Right-angled triangle, if the length of one of its perpendicular side is $\sqrt{x}$, then the lengths of its hypotenuse will be $\frac{x+1}{2}$ and the length of the other adjacent side will be $\frac{x+1}{2}$ ------------------------- (3)

Example :

For a right-angled triangle, if its perpendicular side a = $\sqrt{7}$ units,

then hypotenuse h = $\frac{7+1}{2}$ = 4 units and the other side b = $\frac{7-1}{2}$ = 3 units.

Verifying the results with the Pythagoras theorem:

$h^2 = a^2 + b^2 \Longrightarrow 4^2 = \left(\sqrt{7}\right)^2 + 3^2$

LHS = $4^2 = 16$

RHS = $\left(\sqrt{7}\right)^2 + 3^2 = 7 + 9 = 16$

Thus, LHS = RHS

Home Task : Similarly solve Q3 of WS 10, and check on your own that statement (1) is correct or no for both x as Whole numbers and decimal numbers.

WORKSHEET 9

Q1. Do as directed

1. Write your roll number, say it is x . i.e. x = _____ (your Roll No.)

2. Calculate $\frac{x+1}{2}$ = ________ and $\frac{x-1}{2}$ = ___________

3. Now find $\left[\frac{x+1}{2}\right]^2$ = _________ and $\left[\frac{x-1}{2}\right]^2$ = ________

4. Now subtract : $\left[\frac{x+1}{2}\right]^2$ - $\left[\frac{x-1}{2}\right]^2$ = __________

Q2. Repeat the same steps as in Q1 by taking the value of x = 2.6

1. Calculate $\frac{x+1}{2}$ = ________ and $\frac{x-1}{2}$ = ___________

2. Now find $\left[\frac{x+1}{2}\right]^2$ = _________ and $\left[\frac{x-1}{2}\right]^2$ = ________

3. Now subtract : $\left[\frac{x+1}{2}\right]^2$ - $\left[\frac{x-1}{2}\right]^2$ = __________

Thus, what can you conclude from the answers got in Q1 and Q2?

$$\left[\frac{x+1}{2}\right]^2 - \left[\frac{x-1}{2}\right]^2 = \text{\_\_\_\_\_\_\_\_\_\_}$$

Q3. Given are the lengths of the perpendicular side of Right-angled triangles, find the lengths of the hypotenuse and base in each case and also verify your results with the Pythagoras theorem: a) $\sqrt{15}$ units b) $\sqrt{39}$ units c) $\sqrt{100}$ units d) $\sqrt{3.7}$ units e) $\sqrt{6.8}$ units f) $\sqrt{125.7}$ units

- **Teaching Strategies used in Lesson Plan 16 :**

Visualization: Students are made to go through the process and visualize the evolving of the relationship between the perpendicular side and the base and hypotenuse lengths of a right angled triangle.

Generalization: Students are guided through inductive reasoning to conclude the formula that can be used to find the lengths of hypotenuse and base of a right-angled triangle when the perpendicular side is of the form $\sqrt{x}$.

Mathematical connections: Students are guided with logically connected steps to conclude the formula. The connection between the Pythagoras theorem in different forms with Irrational numbers is reinforced through this lesson.

Higher order questioning: Students are probed to discover that the Hypotenuse geometric method cannot be used to represent Irrational numbers of the decimal forms and the proving the relevance of the Perpendicular geometric method.

4.16 Lesson Plan 17

Unit : Real Numbers **Grade :** IX

Topic : Perpendicular Geometric Method to represent Irrational nos. on Number line **Duration :** 40 min

- **General Objective:**

1. Students will be able to understand the Perpendicular Geometric method.

- **Specific Objectives:**

1. Students will be able to represent Irrational numbers (square roots of x, x ∈ Q) on the Number line using the Perpendicular Geometric method.

- **Teaching Aids:**

PPT Slide 26 & 27: Representation of Irrational numbers on Number line by the Perpendicular Geometric method

Worksheet 10: Perpendicular Geometric method (Practice work)

SLM 3: Representation of Irrational numbers on Number line by the Perpendicular Geometric method

- **Teacher's Activity and Student's Activity:**

T E : After solving WS 9, it must be clear that if the length of a perpendicular side of a right-angled triangle is known then the length of the hypotenuse and the other side can be found.

We now use this concept to represent Irrational numbers like $\sqrt{x}$, where x ∈ Q on the Number line.

T I : The process of representing Irrational numbers on Number line using the Perpendicular Geometric method is explained in detail in SLM 3.

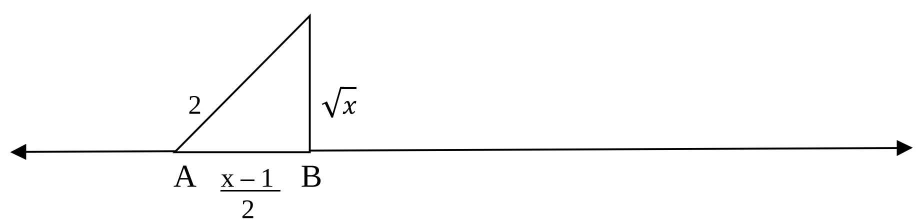
Teacher explains the process in detail using the PPT slide 26 and 27 and also provides the SLM 3 to students to go through it and understand the same in individual pace.

[Teacher uses the PPT to explain the steps in the following manner.]

Representation of Irrational numbers by the Perpendicular Geometric method

In this method, we need to construct a right-angled triangle on the Number line as shown on the black board.

x + 1



I) Constructing line segment of length $\frac{x+1}{2}$ on the Number line:

- Draw a Number line XY, construct a line segment OB of length x units on it.
- Construct the length x + 1 units, by placing the point P on the Number line such that BP = 1 unit. Thus OP = x + 1 units
- To construct the length $\frac{x+1}{2}$ units, we need to divide OP into half, or find the mid-point of OP. We use the construction of perpendicular bisector to find point A. Thus OA = AP = $\frac{x+1}{2}$ units.

II) Constructing line segment of length $\frac{x-1}{2}$ on the Number line:

- Check out the length of AB…
- AB = AP – BP

$= \frac{x+1}{2} - 1$

$= \frac{x+1-2}{2}$

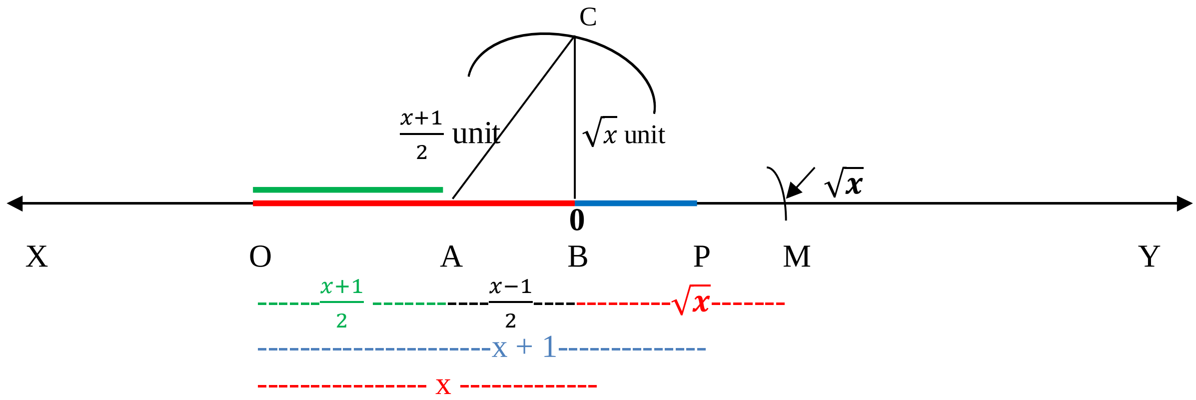
AB = $\frac{x-1}{2}$ units

III) Constructing the right-angled triangle ABC, where AB = $\frac{x-1}{2}$ units, AC is the hypotenuse with length $\frac{x+1}{2}$ units and BC is the perpendicular side with the Irrational number $\sqrt{x}$ units:

- Draw a perpendicular from B, in the space above line XY.
- In order to construct AC (hypotenuse) of △ABC, use a compass and draw an arc, with A as the center and OA as the radius i.e. $\frac{x+1}{2}$ unit, intersecting the perpendicular from B at C.
- Since the thus constructed right-angled triangle, has the length of its hypotenuse as $\frac{x+1}{2}$ unit, its base as $\frac{x-1}{2}$ unit, automatically the length of its perpendicular side will be $\sqrt{x}$ unit.

V) Representation of $\sqrt{x}$ on the Number line:

- Since the length of BC is $\sqrt{x}$ units, taking BC as the radius and B as the , draw an arc on line XY with a compass at point M.



T E : Teacher helps students to represent $\sqrt{3.5}$ on the Number line using the Perpendicular Geometric method. Steps of construction to represent $\sqrt{x}$ = $\sqrt{3.5}$to be done by the teacher through questioning.

T Q : Firstly estimate the tentative position of $\sqrt{3.5}$ on the Number line with respect to two consecutive Integers.

S A : $\sqrt{4}$ is 2, so $\sqrt{3.5}$ should be between 1 and 2 and will be near to 2.

T Q : If x = 3.5, then what are the values of $\frac{x+1}{2}$ and $\frac{x-1}{2}$

S A : $\frac{x+1}{2}$ = 2.25 and $\frac{x-1}{2}$ = 1.25

T Q : So now how should we use this to represents $\sqrt{\mathbf{3.5}}$on the Number line?

S A : We can construct a right angled triangle whose base length is 1.25 units and hypotenuse length is 2.25 units, the length of the perpendicular side will be $\sqrt{\mathbf{3.5}}$**.**

T Q : And how will you construct the length of 1.25 and 2.25 units?

S A : With a scale.

T Q : Are you sure you will be able to get the exact measure of 1.25 and 2.25 with a scale?

S A : Almost.

T E : Here we are aspiring to ideally find the exact position of $\sqrt{\mathbf{3.5}}$on the Number line, so we have avoid even the minor errors. So we have to construct using compass and not the scale. So refer to the steps in the earlier explanation and use the same here. Students see the SLM 2 and speak out the steps as follows:

- Draw the Number line XY
- Construct OB of length x units i.e. 3.5 units. (1 unit = 1 cm)
- Construct BP of length 1 unit.
- Construct perpendicular bisector to find the mid-point of OP, thus OA = 2.25 units …$\frac{x+1}{2}$
- Construct a perpendicular from B.
- Taking A as the center and OA as the radius, draw an arc intersecting the perpendicular from B at point C. Thus BC = $\sqrt{3.5}$ units.
- Represent $\sqrt{3.5}$ on the Number line by taking B as the center and radius BC, at point M on line XY.
- Thus point M represents $\sqrt{3.5}$ with respect to B (zero point).

T E : Now measure BM with a scale and check are we getting what we estimated earlier, i.e. $\sqrt{3.5}$ should be lying between 1 and 2, very near to 2?

[Students check and verify the result]

Home Task : WS 10

WORKSHEET 10

Q1. Use the Perpendicular Geometric Method to represent the following Irrational numbers on the Number line. a)$\sqrt{3.7}$ units b) $\sqrt{2.5}$ units c) $\sqrt{9}$ units d)$\sqrt{5}$ units

- **Teaching Strategies used in Lesson Plan 17 :**

Visualization: Students are made to visualize the steps of construction through visual aid (PPT) with the explanation of the concept behind each step in order to help students with the Interpretative visualization that helped students apply the same for other Irrational numbers.

Estimation: The position of $\sqrt{3.5}$ on the Number line is estimated prior to using the steps of construction and the same is verified later.

Mathematical connections: Students are shown mathematical connections among mathematical processes (behind every construction step).

Higher order questioning: Asking Focus-questions that lead student through the steps of thinking while representing $\sqrt{3.5}$ on the Number line.

Cognitivist strategies: Algorithmic procedure used for the Perpendicular geometric method shown with logical reasoning and appropriate explanation, naming all the critical and additional features of the concept.

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4.17 Lesson Plan 18

Unit : Real Numbers **Grade :** IX

Topic : Commutative & Associative Properties for different Operations on Real Number **Duration :** 40 min

- **General Objective:**

1. Students will be able to understand the relevance of the Commutative and Associative property for Real Number.

- **Specific Objectives:**

1. Students will be able to prove that the Commutative property holds true for addition & multiplication but not for subtraction & division of Real numbers.
2. Students will be able to prove that the Associative property holds true for addition & multiplication but not for subtraction and division of Real nos.

- **Teaching Aids:**

Worksheet 11 : Commutative and Associative Property of Real Numbers

- **Teacher's Activity and Student's Activity:**

T E : Referring to the content-chart, we realize that we have clearly understood (1) what are Real Numbers- Set of Rational and Irrational numbers and (2) how to represent them on Number line. Now we proceed forward to understand how to add, subtract, multiply and divide these numbers. i.e. 'Mathematical Operations on Real Numbers'.

In earlier classes where we studied the Mathematical Operations on the set of Whole nos., Integers, Rational nos., we have also seen that there are certain limitations or rules while adding, subtracting, multiplying and dividing two or more numbers of a given Numbering system. For example we can add two or more Integers/Rational numbers in any order but we cannot change the order of the numbers while subtracting them.

Ex. $5 + 3 = 3 + 5$; but $5 - 3 \neq 3 - 5$

So, today we will try to find whether those rules/properties/laws guiding the mathematical operations on Rational numbers are applicable or hold true on Irrational numbers as well. So today's topic is 'Commutative Property of Addition /Subtraction /Multiplication /Division on Real Numbers.

I. Commutative Property of Addition on Real Numbers

T I : Solve Q1 of WS 11

T P : After solving the first two problems can you estimate the answers of the (b)s after solving the (a)s?

S A : The answers of the (b)s will be same as that of the (a)s.

T Q : What kind of numbers are adding in Q1?

S A : Fractional and Decimal numbers.

T Q : Which Numbering system do they belong to?

S A : Rational Numbers.

T Q : Compare the (a)s and the (b)s in each case and say what is happening?

S A : The orders are changed but answer remain the same. So, Commutative property holds true for Rational numbers.

T I : Very good, now solve Q2 of WS 11. Use the decimal values as shown in the PPT slide 18 to solve the question.

T Q : So what can be said for 'Addition of Irrational number'?

S A : The Commutative property holds true for addition of Irrational Numbers.

T C : Thus the Commutative property of addition holds true for Real Numbers.

H T : Solve Q2, Q3 and Q4 of WS 11 and check whether the Commutative property holds true for subtraction, multiplication and division of Real Numbers.

Home Task : Complete WS 11

WORKSHEET 11

Q1. Solve the following :

(1) a) $\frac{2}{13} + \frac{3}{13} + \frac{-1}{13}$ = _____ b) $\frac{3}{13} + \frac{2}{13} + \frac{-1}{13}$ = ______

(2) a) 0.8 + 1.8 = ______ b) 1.8 + 0.8 = ______

(3) a) $\sqrt{2} + \sqrt{3}$ = ______ b) $\sqrt{3} + \sqrt{2}$ = _____

(4) a) $\sqrt{2} + \sqrt{3} + \sqrt{5}$ = ___ b) $\sqrt{3} + \sqrt{5} + \sqrt{2}$ = _____

√2 ~	**1.414**
√3 ~	**1.732**
√5 ~	**2.236**
√6 ~	**2.449**

(i) Observe each of the numbers on both the sides of the *equal to* sign and State the Numbering System they belong to. _______________

(ii) Compare the results of a) and b) and write down the General Rule that is guiding the Operation of Addition in this Numbering System.

(iii) What is this *Property* called?

-Finally, can we declare that the Commutative Property holds true for Addition of Real Numbers?

Commutative Property for Subtraction, Multiplication and Division

Q2. Solve the following:

(1) a) $7 - 3 =$ ______ b) $3 - 7 =$ _______

(2) a) $\frac{2}{3} - \frac{5}{3} =$ ______ b) $\frac{5}{3} - \frac{2}{3} =$ _______

(3) a) $5.7 - 3 =$ ______ b) $3 - 5.7 =$ _______

(4) a) $\sqrt{2} - \sqrt{3} =$ ___________ b) $\sqrt{3} - \sqrt{2} =$ _____________

(i) Compare the answers of (a)s and (b)s and State whether Commutative property holds true for *Subtraction of Real numbers.*

Q3. Solve the following (take only two decimal places in case of Irrational numbers):

(1) a) $7 \times 8 =$ ______ b) $8 \times 7 =$ _______

(2) a) $\frac{-2}{3} \times \frac{4}{6} =$ ______ b) $\frac{4}{6} \times \frac{-2}{3} =$ _______

(3) a) $1.2 \times (-5.3) =$ ______ b) $(-5.3) \times 1.2 =$ _______

(4) a) $(-\sqrt{5} \times \sqrt{7}) =$ ______ b) $(\sqrt{7} \times -\sqrt{5}) =$ _______

Thus, what can you say about the Commutative property for Multiplication of Real numbers?

Q4. Solve the following (take only one decimal place for Irrational numbers) :

(1) a) $14 \div 2 =$ ________ b) $2 \div 14 =$ _________

(2) a) $\frac{-2}{3} \div \frac{3}{4} =$ ________ b) $\frac{3}{4} \div \frac{-2}{3} =$ _________

(3) a) $\sqrt{2} \div \sqrt{3} =$ _________ b) $\sqrt{3} \div \sqrt{2} =$ __________

(4) a) $\sqrt{5} \div \frac{1}{\sqrt{5}} =$ _________ b) $\frac{1}{\sqrt{5}} \div \sqrt{5} =$ ___________

Does the Commutative property hold for Division of Real Numbers?

Thus, the Commutative Property holds true for _________________________________ mathematical Operations for Real Numbers and does not hold true for __________operations for Real Numbers.

Q5. Prove that Associative property holds true for addition and multiplication of Real numbers and not for subtraction and division.

[Associative property means that, when three numbers or more are to be added, subtracted, multiplied or divided; any of the pairs of given set of numbers may be operated first]

Ex. $2 + 3 + 4 + 5 = (2 + 3) + 4 + 5 = 2 + (3 + 4) + 5 = 2 + 3 + (4 + 5)$

$= 5 + 4 + 5 = 2 + 7 + 5 = 2 + 3 + 9 = 14]$

Q6. Show with one example for Irrational numbers that, *'Associative Property holds true for the Addition of Real Numbers'.*

Q7. Show with one example for Irrational numbers that, *'Associative Property does not hold true for the Subtraction of Real Numbers'.*

- **Teaching Strategies used in Lesson Plan 18 :**

Visualization: Students visualize through concrete examples the processes involved in proving the applicability of the Commutative property, which is further used by deductive reasoning or interpretative visualization to prove the same for Associative property.

Generalization: Empirical Generalization has been used in Worksheet 11, in which students are made to go through a gradual process of analyzing a series of concrete examples in which the non-essential attributes are systematically changed. Students were engaged with inductive reasoning i.e. to observe or work with given set of data, analyze it in the process and identify the pattern or the relationship that exists within the components and synthesize them to infer the mathematical property (Commutative property).

Estimation: Students were probed to estimate the answers in most of the problems of the worksheet.

Mathematical connections: Connections between the mathematical operations and the mathematical properties (rules that guide the application of different operations on different Numbering systems) was made apparent through the reasoning that was used by students while solving the worksheet.

Higher order questioning: Posing questions and tasks that elicit, engage, and challenge each student's thinking in the worksheet and also including open-ended questions (Associative property) encouraging students to justify their reasoning in writing.

Cognitivist Strategies: Best examples and non-examples are given to help students deduce the applicability of Commutative and Associative properties on Real numbers.

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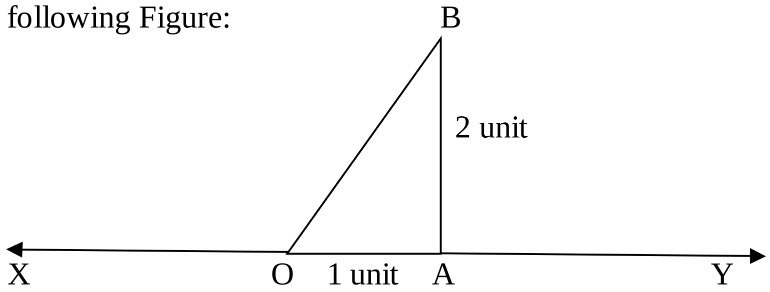
4.18 Formative Assessment - Evaluation 2

Total Marks : 60 **Time : 2 ½ hours**

1. 3.455 lie between which two Integers. In order to represent 3.455 on a Number Line, the space between those two Integers is to be divided into how many equal parts? Following that, the given number will be in between which two parts?
2. Represent (-1.35) on the Number Line.
3. Write four points of difference between Rational and Irrational Numbers.
4. Observe the following Figure:



Line XY is the Number Line and △OAB is a Right-angled-triangle. OA = 1 unit and AB = 2 unit. Use this information to find the Irrational Number that can be represented on Number Line XY. Write the Steps that should be used to represent that Number on the Number Line.

5. Write six Rational Numbers (three fractional and three decimal form) and six Irrational numbers (three square root and three decimal form) that lie between 5 and 6.
6. Represent $\sqrt{12}$ on a Number line. Label the diagram very clearly. Which point is taken as the center and what is the radius taken to represent the given Irrational number on the Number line?
7. Use the Commutative Property and the Associative Property to solve the following sums. Indicate the steps where the above properties are used in both the cases.

   (1) 93.6 + 7.8 + 3.5 + 6.4 + 6.5 + 42.2

   (2) 0.25 × 0.5 × 1.25 × 40 × 0.2 × 0.8
8. Note the conversation between a Teacher and a Student.

   *T : Speak out ten **Irrational numbers** between 9 and 10.*

   S: 9.5, 9.7532…, $\sqrt{81}$, $\sqrt{9.5}$, $\sqrt{90}$, $\sqrt{85.5}$, $\frac{19}{2}$, $\sqrt{89}$, $\frac{99}{9}$, *9.5055005550……*

   Do you think the answers given by the student are correct? Find the errors that the student has made. Write all the correct answers and wrong answers with reasons for each.

9. The length of the Hypotenuse of a Right-angled triangle can be used to represent

Irrational numbers on the Number line. How will you use this concept to represent $\sqrt{68}$ on the Number Line? Show the same on the given Number line.

10. An architect needs to make a plan for a triangular plot. He starts from point O and moving towards the east, stops at point B which is 5 units away from point O. Now he moves towards the north and reaches point A. From A, he takes a slanting path downwards and reaches back to O. Find the lengths of BA and AO.

11. Estimate, the following numbers lie between which two consecutive Integers. Also write the reasoning for your answer.

    1) $\sqrt{17}$: 2) $\sqrt{12.5}$: 3) 7^2 : 4) $\frac{7}{3}$:

12. Construct a Sum using Irrational numbers $\sqrt{5}$, $\sqrt{3}$ and $\sqrt{2}$; and prove that 'The Associative Property does not hold true for Subtraction of Irrational numbers'. *[$\sqrt{2}$ = 1.414…, $\sqrt{3}$ = 1.732…, $\sqrt{5}$ = 2.236….]*

13. In a Right-angled triangle, if the length of the perpendicular side is $\sqrt{17}$, then what is the value of the other two sides. Verify your answer using the Pythagoras theorem.

14. If the variables '***p***', '***q***' and '***r***' are such that,

    '***p***' is a positive Proper Fraction,

    '***q***' is an Irrational number more than $\sqrt{5}$ and less than $\sqrt{6}$ and

    '***r***' is a Rational number between the first two Natural numbers

    Then arrange '***p***', '***q***' and '***r***' in ascending order as per their values. Answer with reasons.

15. Teacher brings a mathematical game in the class. She has five cards of different colours and different Numbers are written on each card. She calls three students A, B and C one by one and asks them to pick up randomly the five cards. She tells them to add the Numbers on the cards in the same order as they pick up each card. The Colours and respective Numbers of the cards are :

    Red : 5, Blue : 2.5, Green : (-3), Yellow : $\frac{1}{2}$, White : $\sqrt{2}$*[$\sqrt{2}$~ 1.414]*

Student A picks up the cards in the order:	Red	Yellow	White	Blue	Green
Student B picks up the cards in the order:	White	Blue	Green	Red	Yellow
Student C picks up the cards in the order:	Yellow	Red	White	Green	Blue

(1) What answers will Student A and Student B get? Predict the answer of Student C.

(2) Generalize the mathematical Property that the Teacher wants to teach from this game.

(3) Will the same Property work for Subtraction, Multiplication and Division of the respective Numbering system? Verify with one example each.

4.19 Lesson Plan 19

Unit : Real Numbers **Grade :** IX

Topic : Distributive Property on Real Numbers **Duration :** 40 min

- **General Objective:**

1. Students will be able to understand the relevance of the Distributive property of multiplication over addition and subtraction in case of Real Numbers.

- **Specific Objectives:**

1. Students will be able to prove the relevance of the use of Distributive property over the BODMAS rule for Algebraic expressions and Irrational numbers.
2. Students will be able to verify that the Distributive property of multiplication and division over addition and subtraction holds true for Real numbers.
3. Students will be able to prove that the Distributive property of addition/subtraction over multiplication/division does not hold true for Real numbers.

- **Teaching Aids:**

Worksheet 12 : Understanding Distributive property holistically

- **Teacher's Activity and Student's Activity:**

T E : We are trying to understand the different laws and properties that are to be used when we add, subtract, multiply and divide Real numbers. We already learnt two properties in the previous classes i.e. Commutative and Associative Properties which basically dealt with the order that we need to follow while we add, subtract, multiply or divide Real numbers.

Today we will take up a very important property called *Distributive Property of Multiplication over Addition and Subtraction.*

As the name suggests it is distribution of multiplication over addition/subtraction. You have already studied this in your previous classes for Rational numbers.

Example : (a) $2 \times (3 + 5) = \mathbf{2} \times 3 + \mathbf{2} \times 5$

(b) $\frac{2}{3} \times \left(\frac{1}{3} + \frac{5}{3} - \frac{7}{3}\right) = \frac{\mathbf{2}}{\mathbf{3}} \times \frac{1}{3} + \frac{\mathbf{2}}{\mathbf{3}} \times \frac{5}{3} - \frac{\mathbf{2}}{\mathbf{3}} \times \frac{7}{3}$ …..(1)

T Q : Consider the first example2 × (3 + 5) and this example : 2 × 3 + 5. Are they both same? If no then what is the difference.

S A : In the first case 2 is multiplied to both 3 and 5 and in the second case 2 is multiplied only to 3 and not 5.

T Q : Yes Good ! If you solve them what are the answers?

S A : (1) 16 and (2) 11

T Q : Again consider the first sum 2 × (3 + 5). This sum can be solved by the BODMAS rule (we start with the operation inside the *Bracket*, then proceed for *Of*, next *Division*, then *Addition* and last *Subtraction*) easily…. Like this 2 × (3 + 5) = 2 × 8 = 16; then why do we use the Distributive property as shown in (1)?

T E : It is true that in order to solve sums of the types shown in (1) where we are dealing with Rational numbers, use of the BODMAS rule is easy and appropriate. But in case of Algebraic expressions and for Irrational numbers for similar sums as in (1), the use of Distributive property is widely used when we are left with unlike terms inside the bracket. Let us understand with help of some examples.

In Algebra, the Distributive property is used to Expand Algebraic expressions.

Example of Expansion : (a) $2 \times (x + y) = 2x + 2y$ (b) $7x (y + 2xz) = 7xy + 14x^2z$

In the above examples where there are unlike terms inside the bracket, the use of Distributive property is effectively seen.

T E : Similarly, the same is true for Irrational numbers also. Let us understand this with help of an example.

Example : Solve $\sqrt{2}\,(\sqrt{2} + \sqrt{8})$ by the using the Distributive property and the BODMAS rule and check out which is easier.

(1) Solving by using the Distributive property:

$$\sqrt{2}\,(\sqrt{2} + \sqrt{8}) = \sqrt{2} \times \sqrt{2} + \sqrt{2} \times \sqrt{8}$$

$$= \sqrt{4} + \sqrt{16} = 2 + 4 = 6$$

(2) Solving by using BODMAS rule :

$$\sqrt{2}\,(\sqrt{2} + \sqrt{8}) \approx 1.414\,(1.414 + 2.828) \qquad (\sqrt{2} = 1.414\ldots.. ,\ \sqrt{8} = 2.828\ldots.)$$

$$\approx 1.414\ (4.242) \approx 5.998 \approx 6$$

T E : Which of the method is easy, (1) or (2)?

S A : (1)

T E : Thus, we have seen that though we can solve similar sums as we solved today by the BODMAS rule, the use of Distributive property makes our work easy and less time consuming.

T E : We have already proved that the Distributive property of multiplication holds true over addition and subtraction for Rational numbers, we now have to prove that it holds true for the Real numbers as well. So for that we have to prove that 'The Distributive property of multiplication over addition and subtraction holds true for Irrational numbers'

Let us use an example to prove the same.

Prove that $\sqrt{2}\ (\sqrt{3} + \sqrt{5} - \sqrt{7}) = \sqrt{2} \times \sqrt{3} + \sqrt{2} \times \sqrt{5} - \sqrt{2} \times \sqrt{7}$

Proof : LHS $= \sqrt{2}\ (\sqrt{3} + \sqrt{5} - \sqrt{7})$

$\approx$ 1.41 (1.73 + 2.25 - 2.65)

$\approx$ 1.41 (1.33) $\approx$ 1.9 (By BODMAS rule)

RHS $= \sqrt{2} \times \sqrt{3} + \sqrt{2} \times \sqrt{5} - \sqrt{2} \times \sqrt{7}$

$\approx 1.41 \times 1.73 + 1.41 \times 2.25 - 1.41 \times 2.65$

$\approx$ 2.43 + 3.17 - 3.74 $\approx$ 1.9

Thus, LHS = RHS i.e. $\sqrt{2}\ (\sqrt{3} + \sqrt{5} - \sqrt{7}) = \sqrt{2} \times \sqrt{3} + \sqrt{2} \times \sqrt{5} - \sqrt{2} \times \sqrt{7}$

The same can be proved for all such examples.

Thus, The Distributive property of multiplication over addition and subtraction holds true for Irrational numbers and thus for Real Numbers as well.

Home Task : Worksheet 12

WORKSHEET 12

Q1. Check whether the *Distributive property of Addition over Multiplication* holds true for

(i) Rational numbers (ii) Irrational numbers… using proper examples.

Q2. Check whether the *Distributive property of Division over Addition* holds true for

(i) Rational numbers (ii) Irrational numbers…. using proper examples.

- **Teaching Strategies used in Lesson Plan 19 :**

Visualization: Examples are shown to students to help them visualize the further application of Distributive property on Irrational numbers.

Mathematical connections: Connecting the previously learnt concepts of Distributive property on - Rational numbers, algebraic expressions, unlike Irrational numbers; and BODMAS rule on Rational numbers and decimal numbers to understand the relevance of the use of Distributive property over the BODMAS rule.

Higher order questioning: Open ended questions in worksheet 12 challenge students to think, reason and come out with appropriate examples that can be used to prove the respective mathematical property.

Cognitivist Strategies: Comparing the new (solving problems using Distributive property for Irrational numbers) to the already known concept (solving problems using the BODMAS rule) and use of appropriate examples for each.

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4.20 Lesson Plan 20

Unit : Real Numbers **Grade :** IX

Topic : Closure Property on Real Numbers **Duration :** 40 min

- **General Objective:**

1. Students will be able to understand the Closure Property with respect to different mathematical operations for Real numbers.

- **Specific Objectives:**

1. Students will be able to prove that the Closure property holds true for addition, subtraction, multiplication and division (divisor $\neq 0$) for Rational numbers.
2. Students will be able to prove that the Closure property does not hold true for Irrational numbers alone, but holds true for Real numbers.

- **Teaching Aids:**

Worksheet 13 : Closure Property for the different Operations on Real numbers

- **Teacher's Activity and Student's Activity:**

T E : In the earlier class, we were discussing about the applicability of Distributive property on Real numbers. We had proved that this property holds true for Multiplication over Addition and Subtraction only. In Worksheet 12, what answer did you get for Q1 and Q2?

S A : Distributive property for Real numbers does not hold true for Addition over Multiplication and it holds true for Division over Addition.

T E : Good! So till now we have seen that the Commutative Property and Associative property holds true for Addition and Multiplication of Real numbers and the Distributive property holds true for Multiplication/Division over Addition and Subtraction of Real numbers. Today we will study one more very important property called the Closure property as to how it affects the mathematical operations (addition, subtraction, multiplication and division) of Real numbers.

T E : What is Closure Property?

S A : 'If we add, subtract, multiply and divide two or more numbers of a particular Numbering system, and the answer we get, also belongs to the same Numbering system then it is said that the Closure property holds true for that respective Operation for that Numbering system'.

T Q : Let us consider the Numbering system to be 'Real Numbers' and check whether the Closure property holds true for addition, subtraction, multiplication and division. Since Real numbers = Rational numbers + Irrational numbers, thus we check first for Rational numbers then we will go for Irrational numbers.

1. Checking whether Closure property holds true for addition, subtraction, multiplication & division of Rational numbers :

Example :

(a) $\frac{2}{3} + \frac{1}{3} + 3 + 0 + 1.5 = \frac{3}{3} + 4.5 = 1 + 4.5 = 5.5$ ($5.5 \in Q$)

(b) $7 - 2.5 = 4.5$ ($4.5 \in Q$)

(c) $\frac{2}{7} - \frac{5}{7} = \frac{-3}{7}$ ($\frac{-3}{7} \in Q$)

(d) $1.5 \times \frac{10}{3} \times (-8) = \frac{15}{10} \times \frac{10}{3} \times (-8) = (-40)$ ($-40 \in Q$)

(e) $3.5 \div 0.5 = \frac{35}{10} \div \frac{5}{10} = \frac{35}{10} \times \frac{10}{5} = 7$ ($7 \in Q$)

(f) $45 \div 0 =$ undefined

In all the above cases we can see that, when we add, subtract, multiply two or more Rational numbers, the answer is always a Rational number. The same is true for Division also, except for one case i.e. shown in (f) when we divide a Rational number by 0. So, leaving this exception the Closure property holds true for all operations for Rational numbers**.**

2. Checking whether Closure property holds true for addition, subtraction, multiplication & division of Irrational numbers:

T E : Worksheet 13 will help us find this answer.

H T : Worksheet 13

Checking whether Closure property holds true for addition, subtraction, multiplication & division of Real numbers:

Check what happens when we add, subtract, multiply or divide two Irrational numbers.

WORKSHEET 13

Q1. Solve each of the following sums below using the Square-root-table/Calculator. Also write in the bracket whether the numbers are Rational or Irrational. (mark R for Rational and I for Irrational)

(a) $\sqrt{2}$ ( ) + $\sqrt{3}$ ( ) = __________ ( )

(b) $\sqrt{2}$ ( ) + ($-\sqrt{2}$) ( ) = __________ ( )

(c) π ( ) + $(1 - \pi)$ ( ) = __________ ( )

(d) $\sqrt{8}$ ( ) - $\sqrt{7}$ ( ) = __________ ( )

(e) $\sqrt{5}$ ( ) - $\sqrt{5}$ ( ) = __________ ( )

(f) $\sqrt{2}$ ( ) × $\sqrt{3}$ ( ) = __________ ( )

(g) $\sqrt{3}$ ( ) × $\sqrt{3}$ ( ) = _________ ( )

(h) $\sqrt{7}$ ( ) ÷ $\sqrt{5}$ ( ) = _________ ( )

(i) $\sqrt{13}$ ( ) ÷ (- $\sqrt{13}$ ( ) = _________ ( )

Observe the above sums and answer the following questions in terms of Rational/Irrational.

1) When two or more Irrational numbers are added, the answer :
2) When two Irrational numbers are subtracted, the answer :
3) When two or more Irrational numbers are multiplied, the answer :
4) When two Irrational numbers are divided, the answer :

Thus, does the Closure property for addition, subtraction, multiplication and division holds true for Irrational numbers? _____________

Q2. Does the Closure property for all four operations holds true for Real numbers? Why?

Q3. Check what happens when we add, subtract, multiply or divide two Rational numbers. Use your own examples. Find the answers belong to which Numbering system?

(1) ________ + ________ = __________ (   )

(2) ________ - ________ = __________ (   )

(3) ________ × ________ = ___________ (   )

(4) ________ ÷ ________ = ___________ (   )

Q4. Check what happens when we add, subtract, multiply or divide one Rational and one Irrational number. Use your own examples. Find the answers belong to which Numbering system? 1.________ + ________ = __________ (   )

2. ________ - ________ = __________ (   )

3. ________ × ________ = ___________ (   )

4. ________ ÷ ________ = ___________ (   )

FINAL CONCLUSIONS

Rational (+ - × ÷) Rational = _____________
Irrational (+ - × ÷) Irrational = ____________________
Rational (+ - × ÷) Irrational = _______________

- **Teaching Strategies used in Lesson Plan 20 :**

Visualization: Questions 1 and 2 of worksheet 13 worked as Object visualization, which was further used as Interpretative visualization to design own examples in questions 3 & 4. Concluding with the final rule can be categorized as Introspective visualization.

Generalization: Empirical Generalization has been used in worksheet 13. Students went through the gradual process of analyzing a series of concrete examples in which the essential attributes are systematically arranged. They were engaged in inductive reasoning i.e. to observe or work with given set of data, analyze it in the process and identify the pattern or the relationship that exists within the components and synthesize them to infer the general operational rule on Rational and Irrational numbers.

Estimation: Problem posed with known backgrounds and students were encouraged to guess the answer (Worksheet 13, Q 3 & 4). Further probing was used to modify the guess to a reasoned guess. This was done by having students estimate an unknown quantity by either comparing it to a known quantity or partitioning it into known quantities or by using mental computation (choosing the examples in Q3 & 4).

Mathematical connections: Students were allowed to explore the mathematical connections among content areas (numbers and operations), among mathematical processes and within their own thinking while choosing appropriate examples to be operated upon and also analyzing the solutions to conclude mathematical properties (in Worksheet 13).

Higher order questioning: Questions included in worksheet 13 engaged and challenged students to carry out cognitive acts of analysis, synthesis and evaluation.

Cognitivist Strategies: Giving best examples and non-examples while explaining the concept for Rational numbers and Irrational numbers (Worksheet 13).

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4.21 Lesson Plan 21

Unit : Real Numbers **Grade :** IX

T**opic :** Operations on Irrational numbers **Duration :** 40 min

- **General Objective:**

1. Students will be able to understand the application of mathematical operations on Irrational numbers.

- **Specific Objectives:**

1. Students will be able to apply the operational rules used in algebra on Irrational numbers with Like terms (like square roots).

- **Teaching Aids:**

Worksheet 14 : Operations on Like Irrational terms (Square roots)

- **Teacher's Activity and Student's Activity:**

T E : From the previous class as per the results of WS 14, we can conclude that :

Though the Closure property does not hold true for Irrational numbers, it is true for Real numbers because application of all four mathematical operations on Real numbers (Q or/and I) always results into a Real number.

- If two or more *Rational numbers* are added, subtracted, multiplied and divided (except for divisor $\neq 0$), the answer is always a *Rational number.*
- If two or more *Irrational numbers* are added, subtracted, multiplied and divided, the answer is either a *Rational number or an Irrational number.*
- If a *Rational and an Irrational number* is added, subtracted, multiplied and divided, the answer is always an *Irrational number.*

T E : Let us recapitulate that we studied till now. We have understood that the collection of all Rational numbers and all Irrational numbers are called Real numbers. Rational numbers means the set of all Fractions (terminating and non-termination-recurring decimals) and Irrational numbers means the set of all non-terminating-non-recurring decimals, which are generally found in the form of square roots of non-perfect squares. Further, we represented Rational numbers (specially decimals with two or more decimal places) on the Number line, which gave us an idea that there are uncountable Rational numbers between any two Rational numbers. We then represented Irrational numbers on Number line by two methods (1) Perpendicular Geometric method and (2) Hypotenuse Geometric method. We also deduced that the Perpendicular Geometric method is a more general way to represent Irrational numbers on the Number line because it can be used for both whole as well as decimal square root Irrational numbers.

Before proceeding to the most important part i.e. how to add, subtract, multiply and divide Real numbers specially Irrational numbers, we learnt the properties that guide these operations. The Commutative and Associative properties holds true for addition and multiplication of Real numbers and so Real numbers can be added or multiplied in any order or in any sequence. The Distributive property of multiplication over addition and subtraction is valid for Real numbers and we also saw that how this property makes the calculations easier in case of Algebra and for Irrational numbers over the BODMAS rule.

T E : In order to add, subtract, multiply and divide Like Irrational numbers (square roots), we can use the mathematical operations done in Algebra.

1. Operations on Like Terms :

   In Algebra :

- $x + x = 2x$ or $x + x = x(1 + 1) = 2x$ (Distributive property)
- $2y + 3y - y = 4y$ or $2y + 3y - y = y(2 + 3 - 1) = 4y$
- $x \times x \times x = x^3$ whereas $x + x + x = 3x$
- $3x \times 4x = 12x^2$
- $4y \div 2y = 2$
- $15x^2 \div 5x = 3x$

For Like Irrational numbers same rules can be applied:

- $\sqrt{2} + \sqrt{2} + \sqrt{2} = \sqrt{2}\,(1 + 1 + 1) = \sqrt{2} \times 3$ or $\mathbf{3\sqrt{2}}$
- $2\sqrt{5} + 3\sqrt{5} - \sqrt{5} = \mathbf{4\sqrt{5}}$
- $\sqrt{2} \times \sqrt{2} \times \sqrt{2} = (\sqrt{2})^3 = (\sqrt{2})^2 \times \sqrt{2} = 2 \times \sqrt{2} = \mathbf{2\sqrt{2}}$
- $3\sqrt{7} \times 4\sqrt{7} = 12(\sqrt{7})^2 = 12 \times 7 = \mathbf{84}$
- $4\sqrt{2} \div 2\sqrt{2} = 2$
- $15(\sqrt{2})^3 \div 5\sqrt{2} = 5(\sqrt{2})^2 = 5 \times 2 = 10$
- $\sqrt{3} \times \sqrt{3} \times \sqrt{3} \times \sqrt{3} \times \sqrt{3} = (\sqrt{3})^2 \times (\sqrt{3})^2 \times \sqrt{3} = 3 \times 3 \times \sqrt{3} = \mathbf{9\sqrt{3}}$

Home Task : Q 1 of WS 14

WORKSHEET 14

Q1. Solve the following for *Like Irrational numbers.* (Use the mathematical operations used in Algebra for Like terms):

1. $\sqrt{5} + \sqrt{5} + \sqrt{5} + \sqrt{5} =$ _______

2. $\sqrt{5} \times \sqrt{5} \times \sqrt{5} \times \sqrt{5} =$ _______ = ________

3. $\sqrt{7} + \sqrt{7} - \sqrt{7} + 3\sqrt{7} - 2\sqrt{7} =$ _______

4. $3\sqrt{13} \times 2\sqrt{13} =$ _______ = ________

5. $3\sqrt{13} + 2\sqrt{13} =$ _______

6. $\frac{1}{\sqrt{5}} + \frac{3}{\sqrt{5}} + \frac{4}{\sqrt{5}} =$ _______

7. $\frac{1}{\sqrt{5}} \times \frac{3}{\sqrt{5}} \times \frac{5}{\sqrt{5}} =$ _______

8. $\frac{5\sqrt{2}}{\sqrt{3}} - \frac{3\sqrt{2}}{\sqrt{3}} =$ _______

9. $\frac{5\sqrt{2}}{\sqrt{3}} \times \frac{3\sqrt{2}}{\sqrt{3}} =$ _______ = _______

10. $\frac{\sqrt{5}}{\sqrt{7}} \div \frac{\sqrt{7}}{\sqrt{5}} =$ _______

11. $3\sqrt{7} \div 5\sqrt{7} =$ _______

12. $3\sqrt{7} - 5\sqrt{7} =$ _______

13. $3\sqrt{7} \times 5\sqrt{7} =$ _______ = ________

14. $3\sqrt{7} + 5\sqrt{7} =$ _______

15. $\sqrt{19} + \sqrt{19} \times \sqrt{19} \div \sqrt{19} - \sqrt{19} =$ _______

16. $3\left(\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right) + \sqrt{3}\left(\frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}}\right) + \sqrt{3}\left(\frac{1}{\sqrt{3}} \div \frac{1}{\sqrt{3}}\right) =$ _______

17. $\frac{11}{\sqrt{11}} + \sqrt{11} + \frac{\sqrt{11}}{11} + \frac{1}{\sqrt{11}} =$ _______ (Hint : $11 = \sqrt{11} \times \sqrt{11}$)

18. $\sqrt{5}\,(2\sqrt{5} - \sqrt{5}) \div \sqrt{5} + (5 \div \frac{1}{\sqrt{5}}) =$ _______ (Use BODMAS rule)

19. $\sqrt{3}\{3(\sqrt{3} + 2\sqrt{3}) \div \sqrt{3}(5\sqrt{3} - 4\sqrt{3})\} =$ _______

20. $3\sqrt{5} + \frac{3}{\sqrt{5}} - \frac{\sqrt{5}}{3} \times 3\sqrt{5} \div \frac{3}{\sqrt{5}} =$ _______ (Use BODMAS rule)

- **<u>Teaching Strategies used in Lesson Plan 21 :</u>**

Visualization: Encouraging visualization of algebraic terms and processes to simplify processes for Irrational numbers.

Mathematical connections: Inter-connections of all the previous topics of the Chapter learnt so far explained very clearly along with the connection of the current topic. Also, connections between the Mathematical processes involved in ‘the application of different operations on Like algebraic terms’ with that of those ‘used for Like Irrational Nos.(square roots)’, was shown with examples to promote better understanding.

Cognitivist Strategies: Use of appropriate examples to explain the concept of ‘addition, subtraction, multiplication and division of Like Irrational numbers (square roots).

4.22 Lesson Plan 22

Unit : Real Numbers **Grade :** IX

Topic : Operations on Irrational numbers **Duration :** 40 min

- **General Objective:**

1. Students will be able to understand the application of mathematical operations on Irrational numbers.

- **Specific Objectives:**

1. Students will be able deduce the rule for addition, subtraction, multiplication & division of Unlike Irrational numbers (square roots).
2. Students will be able to apply the operational rules used in algebraic identities on Unlike Irrational numbers.

- **Teaching Aids:**

Worksheet 14: Applying different operations on unlike Irrational numbers

- **Teacher's Activity and Student's Activity:**

T E : In the previous class, we have seen that how we can add, subtract, multiply and divide Like Irrational numbers using the rules of Algebra. Similarly, we can use Algebraic rules to carry out the same operations on Unlike Irrational numbers as well. First we begin with simple rules. You will deduce the rules yourself by solving Q2 of Worksheet 14.

Operations on Unlike Terms :

T E : After solving Q2, what can we conclude?

S G : $\sqrt{a} + \sqrt{b} \neq \sqrt{a+b}$

$\sqrt{a} - \sqrt{b} \neq \sqrt{a-b}$

$c\sqrt{a} \times d\sqrt{b} = cd\sqrt{a \times b}$

$\sqrt{a} \div \sqrt{b} = \sqrt{a \div b}$

$(\sqrt{a} \times \sqrt{b}) \div \sqrt{c} = \sqrt{\frac{a \times b}{c}}$

T Q : Keeping the above concept in mind, state whether the following statements are true or false. Justify your answers.

(1) $\sqrt{6} + \sqrt{3} = \sqrt{9}$ [False, because $\sqrt{6} + \sqrt{3} \neq \sqrt{6+3}$]

T P : Justify using decimals…. S A: $\sqrt{6} \approx 2.44$ & $\sqrt{3} \approx 1.67$; and 2.44 + 1.67 ≈4 and not 3]

(2) $\sqrt{6} - \sqrt{3} \neq \sqrt{3}$ [True, because $\sqrt{6} - \sqrt{3} \neq \sqrt{6-3}$]

(3) $\sqrt{6} \times \sqrt{3} = \sqrt{18}$ [True, because $\sqrt{6} \times \sqrt{3} = \sqrt{6 \times 3}$]

(4) $\frac{\sqrt{6}}{\sqrt{3}} \neq \sqrt{2}$ [False, because $\sqrt{6} \div \sqrt{3} = \sqrt{6 \div 3}$]

(5) $\frac{\sqrt{3} \times \sqrt{7}}{\sqrt{21}} = 1$ [True, because $(\sqrt{3} \times \sqrt{7}) \div \sqrt{21}$ $\sqrt{\frac{3\times 7}{21}}$

(6) $\frac{\sqrt{3} + \sqrt{7}}{\sqrt{10}} \neq 1$ [True, because $(\sqrt{3} + \sqrt{7}) \div \sqrt{10} \neq \sqrt{\frac{3+7}{10}}$]

T E : So , with this we understood how we can use algebraic rules to add, subtract, multiply and divide Like and Unlike Irrational numbers. Similarly for solving more complex sums involving Irrational numbers, let us understand how Algebraic Identities can be used.

T Q : Which are the Algebraic Identities that you have learnt in your previous classes?

S A : 1. $(a \pm b)^2 = a^2 \pm 2ab + b^2$

2. $(a + b)(a - b) = a^2 - b^2$

3. $(a + b)(c + d) = ac + ad + bc + bd$

T Q : Since we are going to use these identities for Irrational numbers, consider square roots of a, b, c & din the above identities and find the values of

$(\sqrt{a} \pm \sqrt{b})^2$, $(\sqrt{a} + \sqrt{b}).(\sqrt{a} - \sqrt{b})$, $(\sqrt{a} + \sqrt{b}).(\sqrt{c} + \sqrt{d})$

T I : Solve the same in Q3 of Worksheet 14**.**

[Teacher discusses the answers].

Home Task : Try to solve Q4 of Worksheet 14. Apply the operations on Irrational numbers just as you do for Algebraic expressions.

WORKSHEET 14

Q2. Some Important points to note while dealing with Square Roots made up of *Unlike Terms*. Solve the sums to discover the points :

(1) a) $\sqrt{16} + \sqrt{9}$ = ______________ b) $\sqrt{16 + 9}$ = ________________

(2) a) $\sqrt{16} - \sqrt{9}$ = ______________ b) $\sqrt{16 - 9}$ = ________________

(3) a) $\sqrt{16} \times \sqrt{9}$ = _______________ b) $\sqrt{16 \times 9}$ = _______________

(4) a) $2\sqrt{25} \times 3\sqrt{4}$ = ____________ b) $2 \times 3\sqrt{25 \times 4}$ = ____________

(5) a) $\sqrt{16} \div \sqrt{4}$ = ______________ b) $\sqrt{16 \div 4}$ = _________________

(6) a) $\sqrt{4} \times \sqrt{9} \div \sqrt{16}$ = __________ b) $\sqrt{\frac{4 \times 9}{16}}$ = _________________

Observe the solutions and represent the concept mathematically for a, b, c $\in R^+$. (Fill the boxes with '=' or '$\neq$') :

- Sums (1) implies $\sqrt{a} + \sqrt{b}$ ☐ $\sqrt{a+b}$

- Sums (2) implies $\sqrt{a} - \sqrt{b}$ ☐ $\sqrt{a-b}$
- Sums (3) and (4) implies $c\sqrt{a} \times d\sqrt{b}$ ☐ $cd\sqrt{a \times b}$
- Sums (5) implies $\sqrt{a} \div \sqrt{b}$ ☐ $\sqrt{a \div b}$
- Sum (6) implies that $\sqrt{a} \times \sqrt{b} \div \sqrt{c}$ ☐ $\sqrt{\frac{a \times b}{c}}$

These Rules apply for Adding, Subtracting, Multiplying and Dividing Unlike Irrational Numbers

Q3. Use the Algebraic Identities and find solutions for Irrational numbers.

(Remember $(\sqrt{a})^2 = a$)

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

$(a + b)(a - b) = a^2 - b^2$

$(a + b)(c + d) = ac + ad + bc + bd$

1. $(\sqrt{a} + \sqrt{b})^2 =$
2. $(\sqrt{a} - \sqrt{b})^2 =$
3. $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) =$
4. $(a + \sqrt{b})(a - \sqrt{b}) =$
5. $(\sqrt{a} + b)(\sqrt{a} - b) =$
6. $(\sqrt{a} + \sqrt{b})(\sqrt{a} + \sqrt{c}) =$
7. $(\sqrt{a} + \sqrt{b})(\sqrt{c} + \sqrt{d}) =$
8. $(\sqrt{a} - \sqrt{b})(\sqrt{c} - \sqrt{d}) =$

Q4. Use Algebraic Identities to solve/simplify the following expressions:

1. $(\sqrt{2} + \sqrt{3})^2 =$
2. $(\sqrt{11} - \sqrt{8})^2 =$
3. $(\sqrt{7} + \sqrt{13})(\sqrt{7} - \sqrt{13}) =$
4. $(5 + \sqrt{5})(5 - \sqrt{5}) =$
5. $(\sqrt{5} + 2)(\sqrt{5} - 2) =$
6. $(\sqrt{15} + \sqrt{3})(\sqrt{15} + \sqrt{5}) =$
7. $(\sqrt{2} + \sqrt{3})(5 + \sqrt{7}) =$
8. $(\sqrt{8} - \sqrt{3})(\sqrt{7} - \sqrt{2}) =$
9. $(\sqrt{2} + 3)(5 - \sqrt{7}) =$
10. $(\sqrt{8} - \sqrt{3})(\sqrt{7} + \sqrt{2}) =$

[Answers Q4 : (1) $5 + 2\sqrt{6}$ **(2)** $19 - 4\sqrt{11}$ **(3)** (-6) **(4)** 20 **(5)** 1
(6) $15 + 5\sqrt{3} + 3\sqrt{5} + \sqrt{15}$ **(7)** $5\sqrt{2} + 5\sqrt{3} + \sqrt{14} + \sqrt{21}$ **(8)** $2\sqrt{10} - \sqrt{21} + \sqrt{6} - 4$
(9) $5\sqrt{2} - \sqrt{14} - 3\sqrt{7} + 15$ **(10)** $4 + 2\sqrt{14} - \sqrt{21} - \sqrt{6}$

- **Teaching Strategies used in Lesson Plan 22 :**

Visualization: Object visualization while solving Q2 (WS 14), introspective visualization while establishing the algebraic rules and interpretative visualization while solving the true/false statements.

Generalization: Students are engaged into inductive reasoning while solving Q2 (WS 14) where they work with given set of problems, analyze them in the process and identify the relationship that exists while adding/subtracting/multiplying/dividing unlike square roots to infer the generalized mathematical rule that needs to be followed while applying mathematical operations on unlike square roots (Irrational numbers).

Estimation: Students are probed to use decimal numbers to prove the truth of the statement, thus encouraging them to use estimation skills.

Mathematical connections: Mathematical connections established between operations on unlike square roots with operations on algebraic terms; and that between algebraic identities and the procedure used to solve unlike square roots (Irrationals) with similar structures.

Higher order questioning: Questions present in the worksheet target higher order thinking with use of skills of analysis, synthesis and evaluation by the students.

Cognitivist Strategies: Use of appropriate examples and non-examples to deduce the required procedure aimed for in this lesson.

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4.23 Lesson Plan 23

Unit : Real Numbers **Grade :** IX

Topic : Operations on Irrational numbers **Duration :** 40 min

- **General Objective:**

1. Students will be able to understand the application of mathematical operations on Irrational numbers.

- **Specific Objectives:**

1. Students will be able rationalize Monomial Irrational expressions.
2. Students will be able to rationalize Binomial Irrational expressions.

- **Teaching Aids:**

Worksheet 15: Rationalization of Monomial, Binomial, Fractional Irrational terms

- **Teacher's Activity and Student's Activity:**

T E : In the previous class, we have seen that how we can add, subtract, multiply and divide Like and Unlike Irrational numbers using the rules of Algebra. Today we will study how to solve or simplify Monomial and Binomial Irrational numbers which are in Fractional forms, like…. $\frac{1}{\sqrt{2}+\sqrt{3}}$ or $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{5}+\sqrt{2}}$ or $\frac{3\sqrt{2}-\sqrt{5}}{\sqrt{3}+2\sqrt{2}}$ etc.

To simplify the above kinds of Fractional sums, first we need to convert the Denominators into Rational numbers. This process of converting an Irrational number to a Rational number is called Rationalization.

1. Rationalization of Monomial Irrational terms :

T I : Solve Q1 of Worksheet 15 and find out on your own the process.

T Q : What can you generalize after solving Q1?

S G : An Irrational Monomial term can be rationalized by multiplying it by itself.

2. Rationalization of Binomial Irrational terms:

T E : Binomial means an Irrational Expression having two terms like $\sqrt{5} + \sqrt{2}$ or $\sqrt{3} - \sqrt{5}$ or $\sqrt{a} \pm \sqrt{b}$…..

Try to work out the process to rationalize such binomials on your own for the terms given in Q2 of WS 15. Hint : You may use one of the Identities we learnt in the previous class.

T Q : What can you generalize after solving Q2?

S G :$(\sqrt{a} + \sqrt{b})$ can be rationalized by multiplying it by $(\sqrt{a} - \sqrt{b})$.

Because $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$ = a – b; which is a Rational number and similarly $\sqrt{a} - \sqrt{b}$ can be rationalized by multiplying it by $\sqrt{a} + \sqrt{b}$.

T E : Thus, Monomial Irrational terms can be rationalized by taking the product of the term with itself and Binomial Irrational terms can be rationalized by taking the product of $(\sqrt{a} \pm \sqrt{b})$ with $(\sqrt{a} \mp \sqrt{b})$.

We will now use these concepts to solve/simplify Fractional Irrational terms.

3. Rationalization of Fractional Irrational terms :

T E : In order to solve or simplify an expression like $\frac{1}{(\sqrt{7}-\sqrt{6})}$, we need to rationalize the denominator. $\frac{1}{(\sqrt{7}-\sqrt{6})} = \frac{1}{(\sqrt{7}-\sqrt{6})\ (\sqrt{7}+\sqrt{6})}$ ……………………….. (1)

There is an error in the above equation. Identify the error.

T P : Since (1) is an equation, the values on both sides of the equality sign should be same. Are they same?

T P : You may also use the concept of Equivalent fractions. Are the fractions on both sides of the equality sign Equivalent?

T P : How can we make them equivalent?

S A : Since we multiplied $(\sqrt{7} + \sqrt{6})$ in the denominator, the same operation has to be done in the numerator as well, only then the value remains same on both sides of the equation.

Thus, to simplify $\frac{1}{(\sqrt{7}-\sqrt{6})}$, we need to follow the steps:

$$\frac{1}{(\sqrt{7}-\sqrt{6})} = \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7}-\sqrt{6})\ (\sqrt{7}+\sqrt{6})} = \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7} + \sqrt{6}$$

T I : Based on this understanding solve Q3 of WS 15.

T E : Thus, $\frac{1}{\sqrt{a}+\sqrt{b}}$ when simplified, gives $\frac{\sqrt{a}-\sqrt{b}}{a-b}$ and

$\frac{1}{\sqrt{a}-\sqrt{b}}$ when simplified, gives $\frac{\sqrt{a}+\sqrt{b}}{a-b}$

T E : We have studied how Irrational numbers can be added, subtracted, multiplied, divided using Algebraic laws and properties. We have also seen how to simplify Fractional Irrational numbers. But did you note one thing? In all the above operations we have only considered positive Irrational numbers…Why?

T E : That is for Irrational numbers like $\sqrt{a}$, $\sqrt{b}$, or $\sqrt{c}$; a, b, c ∈ R^+. What does this statement mean?

S A : a, b, and c are Positive Real numbers.

T P : Correct. Try to understand the fact by taking a = 4 and b = (-4). Find $\sqrt{a}$ and $\sqrt{b}$.

S A : $\sqrt{a} = \sqrt{4} = 2$ and $\sqrt{b} = \sqrt{(-4)} = (-2)$.

T P : Are you sure $\sqrt{(-4)} = (-2)$ is correct?

T P : What is the value of 2^2 and $(-2)^2$?

S A : For both answer is +4. ………………….. (1)

T P : That means $\sqrt{a} = \sqrt{4} = \sqrt{(2)^2} = 2$. This is fine.

But can we say that $\sqrt{b} = \sqrt{(-4)} = \sqrt{(-2)^2} = (-2)$? ………………(2)

S A : Yes.

T P : But in statement (1), you said $(-2)^2 = +4$; then in statement (2), $\sqrt{(-2)^2} = \sqrt{4}$ and the value of $\sqrt{4}$ is 2. This implies what? What is the value of $\sqrt{(-2)^2}$?

S A : $\sqrt{(-2)^2} = 2$ and not (-2)

T P : And so what should be the value of $\sqrt{-4}$?

S A : $\sqrt{-4} = +2$ and not (-2).

T E : Thus, $(\pm 2)^2 = 4$ So, irrespective of 2 being + or - , its square is always positive. Thus, Square root of its square is also taken as only positive… $\sqrt{(\pm 2)^2} = 2$

So, when we apply mathematical operations on Irrational numbers, we will consider square roots of positive numbers only.

T E : Simplify : $\frac{2\sqrt{2} - \sqrt{3}}{\sqrt{3} + 4} = \frac{(2\sqrt{2} - \sqrt{3})\ (\sqrt{3} - 4)}{(\sqrt{3} + 4)\ (\sqrt{3} - 4)} = \frac{2\sqrt{2} \times \sqrt{3} - 2\sqrt{2} \times 4 - \sqrt{3} \times \sqrt{3} + \sqrt{3} \times 4}{(\sqrt{3})^2 - 4^2}$

$$= \frac{2\sqrt{6} - 8\sqrt{2} - 3 + 4\sqrt{3}}{3 - 16}$$

$$= \frac{2\sqrt{6} - 8\sqrt{2} - 3 + 4\sqrt{3}}{-13}$$

Home Task : Practice Sums

WORKSHEET 15

Q1. Solve the following to understand the process of Rationalization of Monomial Irrational terms: 1) $\sqrt{5} \times \sqrt{5}$ 2) $\sqrt{39} \times \sqrt{39}$ 3) $\sqrt{x} \times \sqrt{x}$ 4) $\frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{3}}{\sqrt{2}}$

5) $\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$ 6) $\frac{2}{\sqrt{5}} \times \frac{2}{\sqrt{5}}$ 7) $\frac{\sqrt{3}}{7} \times \frac{\sqrt{3}}{7}$ 8) $\frac{\sqrt{x}}{\sqrt{y}} \times \frac{\sqrt{x}}{\sqrt{y}}$

The answers obtained in each of the above case are Rational or Irrational numbers? ______

Thus, explain how will you Rationalize the following Irrational Monomial terms :

(1) $\sqrt{a}$ (2) $\frac{1}{\sqrt{b}}$ (3) $\frac{\sqrt{c}}{\sqrt{d}}$

In general, State how are Monomial Irrational numbers Rationalized?

Q2. Rationalize the following Binomial Irrational numbers :

1. $\sqrt{3} + \sqrt{2}$ 2. $\sqrt{5} - \sqrt{3}$ 3. $\sqrt{7} + \sqrt{2}$ 4. $\sqrt{11} - \sqrt{7}$

Thus, What can you conclude? How can we rationalize Binomial Irrational numbers like:

$(\sqrt{a} + \sqrt{b}) : (\sqrt{a} - \sqrt{b})$:

Q3. Rationalize the denominators of the following:

1. $\frac{1}{\sqrt{5}+\sqrt{2}}$ (2) $\frac{1}{\sqrt{3}-\sqrt{2}}$ (3) $\frac{1}{\sqrt{13}-\sqrt{7}}$ (4) $\frac{1}{\sqrt{7}+\sqrt{3}}$ (5) $\frac{1}{\sqrt{23}-\sqrt{19}}$

(6) $\frac{1}{\sqrt{11}+\sqrt{8}}$ (7) $\frac{1}{3-\sqrt{2}}$ (8) $\frac{1}{\sqrt{5}+2\sqrt{3}}$ (9) $\frac{1}{2\sqrt{5}+3\sqrt{2}}$

Solutions of the above sums show a specific pattern. Identify the pattern and generalize:

- $\frac{1}{\sqrt{a}+\sqrt{b}}$ = ____________________
- $\frac{1}{\sqrt{a}-\sqrt{b}}$ = ____________________

PRACTICE SUMS

Q1. Simplify the following by rationalizing the denominator:

1. $\frac{1+\sqrt{2}}{2-\sqrt{2}}$ **2.** $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ **3.** $\frac{7+3\sqrt{5}}{7-3\sqrt{5}}$ **4.** $\frac{3\sqrt{2}-\sqrt{5}}{3\sqrt{5}+2\sqrt{2}}$ **5.** $\frac{\sqrt{2}+3\sqrt{5}}{3\sqrt{2}+5\sqrt{5}}$ **6.** $\frac{5\sqrt{7}-2\sqrt{3}}{3\sqrt{7}-4\sqrt{3}}$ **7.** $\frac{4+\sqrt{5}}{4-\sqrt{5}} + \frac{4-\sqrt{5}}{4+\sqrt{5}}$ **8.** $\frac{\sqrt{5}-2}{\sqrt{5}+2} - \frac{\sqrt{5}+2}{\sqrt{5}-2}$

Q2. Prove that :

1. $\frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{3}+2} = 1$
2. $\frac{1}{\sqrt{6}-\sqrt{5}} - \frac{3}{\sqrt{5}-\sqrt{2}} - \frac{4}{\sqrt{6}+\sqrt{2}} = 0$
3. $\frac{2\sqrt{2}}{\sqrt{5}+\sqrt{3}} - \frac{3\sqrt{3}}{\sqrt{5}+\sqrt{2}} + \frac{\sqrt{5}}{\sqrt{3}+\sqrt{2}} = 0$
4. $\frac{1}{\sqrt{9}-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-\sqrt{4}} - \frac{1}{\sqrt{4}-\sqrt{3}} + \frac{1}{\sqrt{3}-\sqrt{2}} - \frac{1}{\sqrt{2}-1} = 2$

- **Teaching Strategies used in Lesson Plan 23 :**

Visualization: Object visualization of the facts and processes throughout the lesson and introspective visualization used for solving the Practice sums.

Generalization: Students are guided to discover the general rules for rationalizing monomial and binomial Irrational numbers.

Mathematical connections: Students are encouraged to discover the mathematical connections among contents (square roots squares, Irrational numbers, algebraic identities), among mathematical processes (operations of squares and square roots, operations on Irrational numbers and algebraic terms), and within their own thinking.

Higher order questioning: Questioning and probing done to engage and challenge students' thinking through the worksheet questions and by probing questions for the topic 'application of operations only positive real numbers and not negative real nos.'.

Cognitivist Strategies: Classifying or chunking this concept into three categories – Rationalization of (1) Monomial Irrationals (2) Binomial Irrationals (3) Fractional forms of Irrationals.

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4.24 Lesson Plan 24

Unit : Real Numbers **Grade :** IX

Topic : Laws of Exponents on Real Numbers **Duration :** 40 min

- **General Objective:**

1. Students will be able to understand the application of laws of exponents on Real numbers.

- **Specific Objectives:**

1. Students will be able to deduce the rule : $\sqrt[n]{x} = x^{1/n}$
2. Students will be able to apply the exponential laws on Irrational numbers.
3. Students will be able to use the rule: $a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = (a^m)^{\frac{1}{n}}$ decisively.

- **Teaching Aids:**

Black board only

- **Teacher's Activity and Student's Activity:**

T E : In the last class we have seen that in order to work with Irrational numbers of the square root form, we used the Rationalization process, which helped us to simplify the computation. But Irrational numbers are not limited to only square roots. As indicated in the definition, even cube roots of non-perfect cubes, fourth roots of non-perfect fours etc. are Irrational numbers. Example $\sqrt[3]{5}$, $\sqrt[4]{7}$, $\sqrt[7]{13}$,……and an uncountable set of such numbers are also Irrational nos.

T E : Also, in previous classes, while solving equations in Algebra, you have been using the strategy of adding, subtracting, multiplying, dividing the same number on both the sides of the equation in order to maintain the balance while finding the solution for the variable. For example, to solve the equation 2x + 8 = 14,

first 8 is subtracted on both sides (2x + 8 – 8 = 14 – 8) to obtain 2x = 6.

Then 2 is divided on both sides (2x/2 = 6/2), to get the solution x = 3 -------------------- [a]

Similarly, since we are dealing with Irrational numbers like square roots, cube roots, fourth roots, fifth roots etc., we need to device a method to remove square roots, cube roots, fifth, sixth…. roots like $\sqrt[2]{x}$, $\sqrt[3]{x}$, $\sqrt[4]{x}$ etc. to get the value of x.

Consider the following equations:

(1) $x^2 = 5$ (2) $y^3 = 13$ (3) $z^{13} = 100$

T Q : Keep in mind the strategy used in [a] and find out how the exponents of x, y and z in the above equations can be removed without disturbing the balance on both sides of the equation.

S A : Maybe by doing $x^{2-2} = 5^{-2}$

T P : But what is the value of x^{2-2}?

S A : x^0, and that is 1

T P : So the equation becomes $1 = 5^{-2}$ (both sides of equal to sign are not equal so this method cannot be right)

S A : We can instead do $x^{2/2} = 5^{1/2}$, then we get $x = 5^{1/2}$. --------------------[b]

T E : Very good, similarly $y = 13^{1/3}$ and $z = 100^{1/13}$.

T E : Our aim was to get the value of x when $\sqrt[2]{x}$, $\sqrt[3]{x}$, $\sqrt[4]{x}$ etc. are given. How do we do that?

T E : We all know that $2^2 = 4$, using the above strategy [b], the equation can be written as

$$2 = 4^{1/2} \quad \text{--------------(1)}$$

But we also know that $2 = \sqrt{4}$ --------------(2)

From (1) and (2) we can say that $4^{1/2} = \sqrt{4}$

Similarly for any variable x, $\sqrt[2]{x} = x^{1/2}$, $\sqrt[3]{x} = x^{1/3}$, $\sqrt[4]{x} = x^{1/4}$

In general, $\sqrt[n]{x} = x^{1/n}$----------------------[A]

T Q : Using the above rule, write the following in the exponential form :

(1) $\sqrt{64}$ (2) $\sqrt[5]{32}$ (3) $\sqrt[3]{125}$

S A : (1) $64^{\frac{1}{2}}$ (2) $32^{\frac{1}{5}}$ (3) $125^{\frac{1}{3}}$

T E : This means that the different roots can be written in the exponential form, i.e. Irrational numbers can be written in the exponential form. So the rules or laws of exponents that you have studied in your previous classes can be applied to Irrational numbers.

Laws of Exponents are :

(1) $x^p \cdot x^q = x^{p+q}$

(2) $\frac{x^p}{x^q} = x^{p-q}$ (here x, y> 0 and are real numbers and p and q are Rational numbers)

(3) $(x^p)^q = x^{pq}$

(4) $x^p y^p = (xy)^p$ ----------------------------------[B]

T I : If we consider x^p and x^q as Irrational numbers, then p and q are in fractional form.

So see for your own selves how Exponential laws are applicable to Irrational numbers by taking $p = ½$ and $q = 1/3$ in each of the above cases in [B]. Find RHS in each of the above case.

[Students put in the values in each of the above equations by the using the operations on fractions and understand the rules.]

T E : Now let us focus on the third rule : $(x^p)^q = x^{pq}$ and see its application on the problems:

(1) $\sqrt{64}$ (2) $\sqrt[5]{32}$ (3) $\sqrt[3]{125}$

(1) $\sqrt{64} = 64^{\frac{1}{2}}$ (in the exponential form)

Further again writing 64 in the exponential form we get $(8^2)^{\frac{1}{2}}$

Now using the third rule $(8^2)^{\frac{1}{2}} = 8^{2\times\frac{1}{2}} = 8$

Similarly solve (2) and (3) and see how finding the values of different roots can be simplified by converting them to exponents.

T Q : Consider the following definition :

"Let *a* > 0 be a real number. Let *m* and *n* be Integers such that *m* and *n* have no common factors other than 1, and *n*> 0. Then, $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$ -------------------------[C]

(By writing the roots in the exponential form we get)

$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = (a^m)^{\frac{1}{n}}$

Now taking the values of *a* = 4, *m* = 3 and *n* = 2, prove the above definition [C].

S A : $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$

$= 4^{\frac{3}{2}} = \left(\sqrt[2]{4}\right)^3 = 2^3 = 8$ --------------(1)

$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

$= 4^{\frac{3}{2}} = \sqrt[2]{4^3} = \sqrt[2]{64} = 8$ -------------(2)

Thus, $4^{\frac{3}{2}} = \left(\sqrt[2]{4}\right)^3 = \sqrt[2]{4^3}$

T C : Thus , if we are asked to find the value of an exponent of the value $4^{\frac{3}{2}}$, we can either solve it as in (1) or (2); whichever path is easier.

For example, if you want to find the value of $32^{\frac{2}{5}}$, which of (1) or (2) would be easier?

[Students try to solve the same in both ways and find that (1) $32^{\frac{2}{5}} = \left(32^{\frac{1}{5}}\right)^2 = 2^2 = 4$]

Whereas (2) $32^{\frac{2}{5}} = (32^2)^{\frac{1}{5}}$ would be difficult to solve, as first square of 32 had to be found and then its fifth root, making the problem tedious and time consuming.

T Q : Simplify $5^{\frac{4}{2}}$

S A : Some students use the method (2) and solve : $5^{\frac{4}{2}} = (5^4)^{\frac{1}{2}} = \sqrt{625} = 25$

Whereas some use their logic and solve : $5^{\frac{4}{2}} = 5^2 = 25$

Teacher points out the fact and encourages students to use logic and use methods that simplify their work.

- **Teaching Strategies used in Lesson Plan 24:**

Visualization: Object visualization with the different rules, procedures, facts and strategies discussed in the lesson with the exponential forms of Irrational numbers.

Generalization: Students are guided to deduce the rule : $\sqrt[n]{x} = x^{1/n}$ through well thought of examples.

Estimation: Students are given scope to explore and find different strategies to solve the same problem (use of exponential laws or square roots -cube roots operations or fractional rules).

Mathematical connections: Minute intrinsic mathematical connections displayed throughout the lesson while deducing the representation of Irrational numbers in the exponent form and applying the exponent rules on Irrational numbers.

Higher order questioning: Questions and tasks were posed throughout the lesson to elicit, engage, and challenge each student's thinking.

Cognitivist Strategies: Comparing the new (exponent rules on Irrational numbers) on already known concept (exponent rules).

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4.25 Lesson Plan 25

Unit : Real Numbers **Grade :** IX

Topic : Laws of Exponents on Real Numbers **Duration :** 40 min

- **General Objective:**

1. Students will be able to apply the laws of exponents on Real numbers.

- **Specific Objectives:**

1. Students will be able to compute problems involving Real numbers.

Teaching Aids:

Practice Sums

- **Teacher's Activity and Student's Activity:**

Teacher guides the students to solve the following problems.

Q1. Solve the following:

(1) $9^{\frac{3}{2}}$ (2) $16^{\frac{3}{4}}$ (3) $3^{\frac{4}{2}}$ (4) $27^{\frac{4}{3}}$ (5) $2^{\frac{6}{5}}$

(6) $125^{\frac{-1}{3}} = (5^3)^{\frac{-1}{3}} = 5^{-1} = \frac{1}{5}$

Q2. Apply the Exponential rules and simplify the following: (Remember the laws are applicable for like bases)

(1) $2^{\frac{2}{3}} . 2^{\frac{1}{5}}$

(2) $\left(\frac{1}{3^3}\right)^7$

(3) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

(4) $7^{\frac{1}{2}} . 8^{\frac{1}{2}}$ (unlike base)

(5) $13^{\frac{1}{5}} . 17^{\frac{1}{5}}$ (unlike bases)

(6) $3^{\frac{3}{2}} . 3^{\frac{4}{5}}$

(7) $16^{\frac{4}{3}} \times 4^{\frac{2}{3}}$ (Convert into like bases first)

(8) $3^{\frac{1}{2}} . 12^{\frac{1}{2}}$ (simplify the base 12 first)

(9) $\frac{5^{\frac{4}{5}}}{25^{\frac{3}{2}}}$ (Convert into like bases first)

(6) (10) $\frac{2^{\frac{1}{3}}}{5^{\frac{2}{3}}} \times \frac{15^{\frac{1}{3}}}{6^{\frac{4}{3}}} \div \frac{4^{\frac{1}{2}}}{3^{\frac{1}{3}}}$ (simplify to get as many like bases as possible)

Q3. Simplify :

(1) $\left(a^{\frac{1}{2}} . b^{\frac{1}{3}}\right)^{\frac{1}{4}} \left(a^{\frac{1}{3}} . b^{\frac{1}{4}}\right)^{\frac{1}{2}} \left(a^{\frac{1}{4}} . b^{\frac{1}{2}}\right)^{\frac{1}{3}}$

(2) $\frac{81^{\frac{1}{4}}}{625^{\frac{1}{4}}} + \frac{216^{\frac{1}{3}}}{8^{\frac{1}{3}}} - 729^{\frac{1}{6}}$

(3) $\left(\frac{81}{16}\right)^{\frac{-3}{4}} \times \left(\frac{25}{9}\right)^{\frac{-3}{2}}$

(4) $\left[5\left[8^{\frac{1}{3}} + 27^{\frac{1}{3}}\right]^{3}\right]^{\frac{1}{4}}$ [Remember : $(a + b)^2 \neq a^2 + b^2$]

(5) $(1^3 + 2^3 + 3^3)^{\frac{-3}{2}}$

[Answers : **Q1.** (1) 27 (2) 8 (3) 1/5 (4) 81 (5) $2\sqrt[5]{2}$ (6) 9 **Q2.** (1) $2^{\frac{13}{15}}$ (2) $\frac{1}{3^{21}}$ (3) $11^{\frac{3}{4}}$ (4) $56^{\frac{1}{2}}$

(5) $221^{\frac{1}{5}}$ (6) $3^{\frac{23}{10}}$ (7) $4^{\frac{10}{3}}$ (8) 6 (9) $5^{\frac{-6}{5}}$ (10) $\frac{1}{4\times 2^{\frac{2}{3}} \times 5^{\frac{1}{5}}}$

Q3. (1) $(ab)^{\frac{3}{4}}$ (2) $\frac{3}{5}$ (3) $\frac{8}{125}$ (4) 5 (5) $\frac{1}{216}$]

- **Teaching Strategies used in Lesson Plan 25:**

Visualization: Introspective visualization wherein the rules and the strategies understood in the previous class is being applied.

Estimation: Students are engaged in estimating the correct procedure to adopt in each problem that would simplify the working.

Mathematical connections: Connection between theory and practical experienced with the problems.

Higher order questioning: Question 3 of offers enough scope to students to engage with higher order thinking.

Cognitivist Strategies: Appropriate examples selected and organized so that students move on from easy to difficult in a sequential manner.

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4.26 Lesson Plan 26

Unit : Real Numbers **Grade :** IX

Topic : Summary of the entire Unit **Duration :** 40 min

- **General Objective:**

1. Students will be able to summarize the contents included in the Unit – Real Numbers.

- **Specific Objectives:**

1. Students will be able to connect the sub-topics in a logical order and describe the content holistically.

- **Teaching Aids:** *NA*
- **Teacher's Activity and Student's Activity:**

The teacher summarizes the entire unit :

The Numbering system Real Numbers comprises of the two disjoint sets of Rational numbers (Q) and Irrational numbers (I).

➢ *Rational Numbers :*

- The set Q includes all Integers and all Fractional numbers.
- Fractions are of two types (1) Ten-based which always gets converted to terminating decimal numbers and (2) Non-ten-based fractions which always gets converted to recurring decimal numbers.
- Rational numbers can be defined as 'Numbers of the form p/q, where p, q $\in$ Z and q $\neq$ 0.
- Use of the definition to prove that the terminating decimals and recurring decimals are Rational numbers.

➢ *Irrational Numbers :*

- The set I includes only non-terminating-non-recurring decimal numbers.
- When square roots of non-perfect squares or cube roots of non-perfect cubes etc. are found, they turn out to be non-terminating-non-recurring decimals. Thus such numbers are Irrational numbers.
- Examples of Irrational numbers : In decimal form – 0.121122111222…… and in root form - $\sqrt{2}$, $\sqrt{3}$, $\sqrt{73.7}$, $\sqrt[3]{7}$, $\sqrt[4]{111}$, $\sqrt[5]{99}$ etc.

➢ *Real Numbers :*

- It comprises of all the Rational numbers and all the Irrational numbers.
- Since there are uncountable Rational numbers and Irrational numbers. So, the set of Real numbers in infinite.

- Use of different methods to find Rational numbers and Irrational numbers between two given Rational numbers to prove that there are uncountable Real numbers between any two Real numbers.

➢ *Representation of Real numbers on Number line :*

- Representing Rational numbers (Fractions and Decimal numbers having three to four decimal places) on Number line.
- Representing Irrational numbers on Number line by the Hypotenuse Geometric method by using $\sqrt{n-1}$ method and by using the Pythagoras theorem (applicable to large Irrational numbers).
- Limitations of the Hypotenuse Geometric method to represent Irrational numbers (square roots of decimal numbers).
- Representing Irrational numbers on Number line by the Perpendicular Geometric method to overcome the above limitation.

➢ *Properties that guide the mathematical operations (+, -, ×, ÷) on Real numbers :*

- Commutative property
- Associative property
- Distributive property
- Closure property
- Use of Closure property to prove (1) Operations on Rational numbers always result to only Rational number (2) Operations on Irrational numbers may result into Rational or Irrational number (3) Operations on Rational and Irrational numbers always result into Irrational number.

➢ *Mathematical Operations on Real numbers (specifically Irrational numbers) :*

- Noting the similarity in the application of different operations on Irrational numbers with that of algebraic expressions.
- Application of algebraic identities to simplify Irrational expressions.
- Simplifying Irrational expressions in fractional forms by Rationalizing the denominator.
- Expressing Irrational numbers in the exponential form and applying the exponential laws on them.
- Application of Exponential laws to simplify Irrational expressions.

- **Teaching Strategies used in Lesson Plan 26 :**

Mathematical connections: Consolidation of the entire unit helps students to visualize the holistic value of the unit and connect the entire content in a meaningful way.

4.27 Implementation Schedule of Instructional Plan

The Lesson Plans, Worksheets, and Evaluations were implemented for a period of 25 days. Total 48 sessions were taken of 40 minutes each. The details regarding the date, respective contents taken up and number of session is included in the Table 12 below.

Table 12: Implementation Schedule of Instructional Plan

Date	Content (Session No.)	Content (Session No.)	Content (S. No.)
12/06/2017	Pretest	Pretest	Pretest
13/06/2017	(LP 1) - Numbers &Relationship between different Numbering systems: Concept **(1)**		
14/06/2017	(LP 2) - Fractions, Decimals, Whole numbers: Rational Numbers **(2)**	Worksheet 1**(3)**	
15/06/2017	(LP 3) –Density of Numbering system: Q**(4)**	(LP 4) – Numbers lying between two given Rational nos. **(5)**	Worksheet 2**(6)**
16/06/2017	(LP 5) – Conversion of Fractions to Decimals **(7)**	(LP 6) – Types of Decimal numbers (WS 3)**(8)**	
17/06/2017	(LP 7) – Converting Recurring Decimals into Fractions**(9)**	Worksheet 4**(10)**	
19/06/2017	Evaluation 1**(11)**	Evaluation 1**(12)**	Evaluation 1**(13)**
20/06/2017	(LP 8) – Irrational numbers: Non-recurring decimal numbers& Roots of non-perfect squares/cubes/ fourths…. **(14)**	Worksheet 5**(15)**	
21/06/2017	(LP 9) - Definition of Irrational numbers and Real numbers **(16)**	(LP 10) Representation of Fractions & Decimals on Number line **(17)**	Worksheet 6 **(18)**
22/06/2017	(LP 11) – Concept of 'Unit'**(19)**	(LP 12) – Estimation of Irrational numbers on Number line **(20)**	Worksheet 7 **(21)**
23/06/2017	(LP 12) – Basics of Geometrical Constructions (WS 7) **(22)**	(LP 13) – Hypotenuse Geometric method of representing Irrational numbers on No. line **(23)**	

Date	Content (Session No.)	Content (Session No.)	Content (S. No.)
24/06/2017	(LP 14) – Hypotenuse Geometric method to represent consecutive Irrational numbers on number line **(24)**	(LP 15) – Hypotenuse Geometric method to represent larger Irrational numbers directly on Number line **(25)**	Worksheet 8 **(26)**
26/06/2017	(LP 16) – Concept A: In a right-angled triangle if, perpendicular side length is $\sqrt{x}$, then hypotenuse is (x + 1)/2 and base is (x-1)/2 **(27)**	Worksheet 9 – Verification of the Concept A with Pythagoras theorem and its application**(28)**	
27/06/2017	(LP 17) – Perpendicular Geometric method to represent Irrational numbers on Number line **(29)**	Worksheet 10 – Practice **(30)**	
28/06/2017	Worksheet 11 **(31)**	(LP 18) – Commutative and Associative Property of Real numbers **(32)**	
29/06/2017	Evaluation 2 **(33)**	Evaluation 2 **(34)**	Evaluation 2 **(35)**
30/06/2017	(LP 19) – Distributive Property on Real numbers **(36)**	Worksheet 12 **(37)**	
1/07/2017	(LP 20) – Closure Property on Real numbers **(38)**	Worksheet 13 **(39)**	
3/07/2017	(LP 21) – Mathematical operations on like Irrational numbers (square roots) **(40)**	Worksheet 14 **(41)**	
4/07/2017	(LP 22) – Mathematical operations on unlike Irrational numbers (square roots) using algebraic identities **(42)**	Worksheet 14 **(43)**	
5/07/2017	(LP 23) – Mathematical operations on Irrational expressions with fractional forms: Rationalization **(44)**	Worksheet 15 **(45)**	
6/07/2017	(LP 24) – Mathematical operations on Irrational numbers (apart from square roots): $\sqrt[n]{x} = x^{\frac{1}{n}}$ **(46)**		

7/07/2017	(LP 25) – Application of Exponential laws on Real numbers **(47)**	Practice Work	
8/07/2017	(LP 26) – Summarizing the entire unit **(48)**		
10/07/2017	Posttest	Posttest	Posttest

The data thus available as a product of the implementation of the Instructional Package was captured in the responses of the Posttest, which is analysed quantitatively in the next Chapter. Data Analysis and its Interpretation are presented in the succeeding Chapter.