APPENDICES

APPENDIX A

TOOLS USED FOR PRESENT STUDY

(1) POSTTEST (FIRST DRAFT)

I. KNOWLEDGE & UNDERSTANDING LEVEL :

1. State giving reasons whether 2.3×10^3 belongs to or does not belong to the following Numbering systems:

(i) Natural Numbers (ii) Integers (iii) Rational numbers (iv) Real numbers

2. How many Real numbers are there between 1 and 2? Write any three Irrational numbers between 1 and 2.

3. Write five Rational numbers between $\frac{2}{3}$ and $\frac{3}{2}$.

4. Show that $0.\overline{345}$ is a Rational number.

II. APPLICATION LEVEL :

1. 'The length of the Hypotenuse of a Right-angled triangle is 1.7 units.' Use this information to **find the specific Irrational number** that can be represented on a Number line. Also mention **which method** will be used to represent the number on the Number line.

2. Find which of the variables x, y, z and u represent Rational numbers and which Irrational numbers : **Show your working.**

(i) $x^2 = 5$ (ii) $y^2 = 9$ (iii) $z^2 = 0.04$ (iv) $u^2 = \frac{17}{4}$

3. Do as directed :

(a) Solve $10 \div 5 \times 3 - 2$

(b) Find the Square Root of the answer.

(c) Now Solve : $\sqrt{10} \div \sqrt{5} \times \sqrt{3} - \sqrt{2}$.

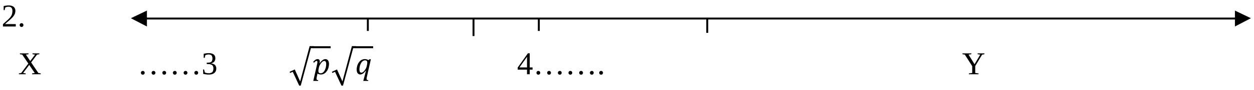
(d) Compare the answers (b) and (c) and make a General Mathematical Rule.

4. Find the value of : $2 + \sqrt{2} + \frac{1}{2+\sqrt{2}} + \frac{1}{\sqrt{2}-2}$

III. ANALYSIS LEVEL :

1. An investment policy had the following terms in very small font size .. "*The total amount invested by the policy owner will be returned after ten years with an additional amount of* - *the double of the square root of the amount invested.*" Would you invest in such a policy? Why?

2.



The figure above shows a Number Line XY, with Irrational numbers $\sqrt{p}$ and $\sqrt{q}$ lying between Real numbers 3 and 4. What should be the values of p and q? Choose the answer from the options given below. Give reasons for your answer. Also state why the other options are incorrect.

(a) p = 3.3, q = 3.4

(b) p = $3\frac{7}{9}$, q = $3\frac{8}{9}$

(c) p = $\sqrt{11}$, q = $\sqrt{12}$

(d) p = 11, q = 12

3. Predict whether the solution of ($x + \sqrt{x}$) where x is a terminating decimal number, will be a/an : (State reasons for your answer)

(a) Rational number

(b) Irrational number

(c) Integer

(d) Can be both Rational and Irrational

4. Which number is (approximately) greater among the two :

(a) $\sqrt{17} - \sqrt{10}$ or (b) $\sqrt{10} - \sqrt{5}$? Show your mental working in the space given below.

IV. SYNTHESIS LEVEL :

1. Determine the Rational numbers ‘a’ and ‘b’ if $\frac{\sqrt{3}-1}{\sqrt{3}+1} - \frac{\sqrt{3}+1}{\sqrt{3}-1} = a + 3\sqrt{3}\,b$.

2. If $x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$. Find the value of $x^2 + \frac{1}{x^2}$.

3. Represent ($\sqrt{2.3} + \sqrt{2}$) on a Number line. Explain the steps in brief in your own words.

4. A student was given a task to construct sums of the following type as per the given conditions. Mention in each case what kind of number should he take as ‘x’ and ‘y’

(i) x + y ; such that the sum is surely an Irrational number.

(ii) x - y ; such that the difference is surely a Rational number

(iii) x × y; such that the product may be Rational or an Irrational number

(iv) x ÷ y; such that the quotient is surely an Irrational number

V. EVALUATION LEVEL :

1. $\sqrt{2}$ = _____	$\sqrt{3}$ = _____
$\sqrt{2} \times \sqrt{2}$ = _____	$\sqrt{3} \times \sqrt{3}$ = ______
$\sqrt{2} \times \sqrt{2} \times \sqrt{2}$ = _____	$\sqrt{3} \times \sqrt{3} \times \sqrt{3}$ = ______

$\sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} =$ _____ $\sqrt{3} \times \sqrt{3} \times \sqrt{3} \times \sqrt{3} =$ ______

$\sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} =$ _____ $\sqrt{3} \times \sqrt{3} \times \sqrt{3} \times \sqrt{3} \times \sqrt{3} =$ ______

$\sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} =$ ___ $\sqrt{3} \times \sqrt{3} \times \sqrt{3} \times \sqrt{3} \times \sqrt{3} \times \sqrt{3} =$ ____

Observe the pattern and make as many generalizations as possible.

2. Prove that $\sqrt{7 + \sqrt{40}} = \sqrt{2} + \sqrt{5}$
3. Construct a Sum using Irrational numbers $\sqrt{2}$, $\sqrt{32}$ and $\sqrt{8}$; in order to prove that 'Distribution of multiplication of Irrational numbers over addition is possible.'
4. Why do we need Rational and Irrational numbers? Write as many reasons as you can think of.

(2) POST TEST (FINAL TOOL)

Total Marks : 60 **Time : 2 ½ hours**

I. COMPREHENSION LEVEL

1. Show that $7.\overline{34}5$ is a Rational number.
2. Simplify : $(\sqrt{m^2\ n^2} \times \sqrt[6]{m^2 n^2} \times \sqrt[3]{m^2 n^2})\ +\ (\sqrt{m^4 n^6}\ \div n^3)$
3. Write maximum two points of difference between the two Numbers in each of the following sets.

(i) 2^2 and $2^{1/2}$ (ii) $\sqrt{5}$ and $5.5\overline{}$ (iii) $\sqrt[6]{\sqrt[3]{64}}$ and $\sqrt[3]{2}$

II. APPLICATION LEVEL

4. If $x = \frac{2-\sqrt{5}}{2+\sqrt{5}}$ and $y = \frac{2+\sqrt{5}}{2-\sqrt{5}}$, find the value of $x^2 - y^2$.

5.Find whether the variables x, y, z and u represent a Rational number, Irrational number or both :(Show your working and give reasons for each of your answer)

(i) $x^2 = 9$ (ii) $y^2 = 0.04$ (iii) $z^2 = \frac{17}{4}$ (iv) $\sqrt{u} = \sqrt{81}$

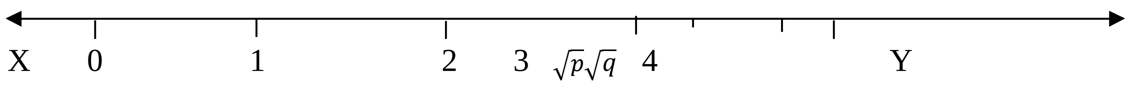
6. Determine the Rational numbers ‘a’ and ‘b’ if $\frac{\sqrt{3}-1}{\sqrt{3}+1} - \frac{\sqrt{3}+1}{\sqrt{3}-1} = a + 3\sqrt{3}\,b$

III. ANALYSIS LEVEL

7. How many Integers are there between 350×10^{-2} and 750×10^{-2}. Write four Rational numbers between $(-3\frac{1}{2})$ and $(-3\frac{3}{4})$ and four Irrational numbers between 3.5 and 3.5 .
8. ‘y’ is the reciprocal of Irrational number ‘$\sqrt{x}$’ and ‘x’ is the reciprocal of ‘$\sqrt{z}$’. If ‘z’ is the largest one-digit perfect square, then what is the value of ‘y’? If this value of y is added to its reciprocal, then the answer obtained will be Rational or Irrational?
9. Represent $(\sqrt{5} + \sqrt{2})$ on a Number line. Explain the steps in brief in your own words. Label clearly the line segment that represents $(\sqrt{5} + \sqrt{2})$ on the Number line?

IV. SYNTHESIS LEVEL

10.



The figure above shows a Number Line XY, with Irrational numbers $\sqrt{p}$ and $\sqrt{q}$ lying on it. What should be the values of p and q? Choose the answer from the options given below. Give reasons for your answer. Also state why the other options are incorrect.

(a) p = 3.3, q = 3.4

(b) p = $3\frac{7}{9}$, q = $3\frac{8}{9}$

(c) p = $\sqrt{16}$, q = $\sqrt{25}$

(d) p = 11, q = 12

11. 'The length of the Hypotenuse of a Right-angled triangle is 3 units'. Use this information to *find the specific Irrational number ($\sqrt{x}$)* that can be represented on a Number line. Use appropriate method to represent that Irrational number on the Number line. Label the diagram properly.

12. Do as Directed :

(a)(i) Solve : 100 + 25 - 16 - 9 and find the square root of the solution

(ii) Solve : $\sqrt{100} + \sqrt{25} - \sqrt{16} - \sqrt{9}$

(b) (i) Solve : 100 ÷ 25 × 16 × 9 and find the square root of the solution

(ii) Solve : $\sqrt{100} \div \sqrt{25} \times \sqrt{16} \times \sqrt{9}$

(c) Compare the solutions of (a) and (b) and frame two General Rules.

V. EVALUATION LEVEL

13. A student was given a task to construct problems of the following type as per the given conditions. Mention in each case what kind of number should he take as 'x' and 'y'. Give one example in each case to substantiate your answer.

(i) x + y ; such that the sum is surely an Irrational number (one example)

(ii) x - y ; such that the difference is surely a Rational number (one example)

(iii) x × y; such that the product may be Rational or an Irrational number (two examples)

(iv) x ÷ y; such that the quotient is surely an Irrational number (one example)

14. An investment policy offered four options to its investors to choose from. If an investor wants to invest Rs.10,000 for ten years, which of the following would be the best option for him.

"*At the end of the term the investor would get back*

(i) Approximately $\sqrt{30}$ *times the original amount*

(ii) Double the square root of the original amount + the original amount

(iii)($\sqrt{2^5} \div 2^{\frac{3}{2}}$) *times the original amount*

(iv) 2 *times the original amount*"

15. Construct a problem using Irrational numbers $\sqrt{2}$, $\sqrt{32}$ and $\sqrt{8}$; in order to prove that 'Distribution of Multiplication of Irrational numbers over subtraction is possible.'

APPENDIX A

(3) BLUE PRINT BASED ON WHICH POSTTEST WAS CONSTRUCTED

	TOPIC	**Understanding**			**Application**			**Analysis**			**Synthesis**			**Evaluation**			**Total**
		1(4)	**2(4)**	**3(4)**	**4(4)**	**5(4)**	**6(4)**	**7(4)**	**8(4)**	**9(4)**	**10(4)**	**11(4)**	**12(4)**	**13(4)**	**14(4)**	**15(4)**	
1.	Real Numbers & Numbers within	1(4)						1(4)		½(2)							**10**
2.	Relation between the different Numbering systems included in Real Number			1(4)		½(2)					½(2)						**8**
3.	Representation of Real Numbers on Number Line				1(4)					½(2)		1(4)					**10**
4.	Properties of Real Numbers													1(4)		1(4)	**8**
5.	Mathematical Operations on Real Numbers						1(4)		½(2)		½(2)		½(2)		1(4)		**14**
6.	Algebraic Operations on Real Numbers		1(4)			½(2)			½(2)				½(2)				**10**
	No. of Questions(Marks)	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**15(60)**

APPENDIX A

(4) ANSWER-KEY AND SCORING RUBRIC FOR POSTTEST

Comprehension level question:
Q1. Show that 7.345 is a Rational number.

Sample Response:
Let x = 7.345345….
Multiplying by 1000 on both sides,
1000x = 7345.345345…..
Subtracting x from both sides,
1000x – x = 7345.345345… - 7.345…..
999x = 7338
x = $\frac{7338}{999}$
Thus, 7.345345…. = $\frac{7338}{999}$
Since $\frac{7338}{999}$ is in the fractional form (p/q), it is a Rational number because it aligns with the definition of Rational number …(A number that is of the p/q form, $p \in Z$, $q \in Z$ and $q \neq 0$)
Here $7338 \in Z$, $999 \in Z$ and $q = 999 \neq 0$

Scoring for Basic level competencies		Scoring for Higher level competencies	
2 point	Correct and complete computations	2 point	Correct – Grasping of the holistic meaning and justifying it with complete description of the definition of Rational No.
1 point	Partially correct/Incomplete computations	1 point	Partial correct - Grasping of the holistic meaning and justifying it with complete description of the definition of Rational No.
0 point	Incorrect/no computations or algorithmic procedures	0 point	Incorrect or no (above) skills displayed

Comprehension level question:
Q2. Simplify :$(\sqrt{m^2\ n^2} \times \sqrt[6]{m^2\ n^2} \times \sqrt[3]{m^2\ n^2}) + (\sqrt{m^4 n^6} \div n^3)$

Sample Response:
$(m^2n^2)^{1/2} \times (m^2n^2)^{1/6} \times (m^2n^2)^{1/3} + [(m^4n^6)^{1/2} \div n^3]$
$= m^1n^1 \times m^{1/3}n^{1/3} \times m^{2/3}n^{2/3} + \frac{m^2n^3}{n^3}$
$= m^{1+\frac{1}{3}+\frac{2}{3}}n^{1+\frac{1}{3}+\frac{2}{3}} + m^2$
$= m^2n^2 + m^2$
$= m^2\,(n^2 + 1)$

Scoring for Basic level competencies		Scoring for Higher level competencies	
2 point	Correct and complete computations with respect to basic exponential rules	2 point	Correct ordering, grouping of exponential functions and differentiating between mathematical and exponential operation

1 point	Partially correct /incomplete computations	1 point	Partial ordering, grouping and differentiating of the same
0 point	Incorrect/no computation	0 point	Incorrect or no (above) skills displayed

Comprehension level question:

Q3. Write maximum two points of difference between the two Numbers in each of the following sets. (i) 2^2 and $2^{1/2}$ (ii) $\sqrt{5}$ and 5.5 (iii) $\sqrt[6]{\sqrt[3]{64}}$ and $\sqrt[3]{2}$

Sample Response:

(i) 2^2 is a Rational number and $2^{1/2}$ is an Irrational number

Because $2^2 = 4$ and $2^{1/2} = 1.414$……. which is non-recurring decimal number

(ii) $\sqrt{5}$ is an Irrational number and $5.\overline{5}$ is an Rational number

Because $\sqrt{5} = 2.236$….. and $5.5 = 5.55555$……

(iii) $\sqrt[6]{\sqrt[3]{64}} = \left(\left(64^{\frac{1}{3}}\right)\right)^{1/6} = (4)^{1/6} = 2^{1/3}$ and $\sqrt[3]{2} = 2^{1/3}$

Thus, $\sqrt[6]{\sqrt[3]{64}} = \sqrt[3]{2}$ and both are Irrational number

Scoring for Basic level competencies		**Scoring for Higher level competencies**	
2 point	Correct and complete computations and identification of Rational and Irrational number	2 point	Correct understanding of - given information, interpretation of facts after comparing & contrasting and justifying the same with mathematical reasoning
1 point	Partially correct/incomplete computation and identification of Rational and Irrational number	1 point	Partially correct understanding of - given information, interpretation of facts after comparing & contrasting and justifying the same with mathematical reasoning
0 point	Incorrect/no computations, identification	0 point	Incorrect or no (above) skill displayed

Application level question:

Q4. If $x = \frac{2-\sqrt{5}}{2+\sqrt{5}}$ and $y = \frac{2+\sqrt{5}}{2-\sqrt{5}}$, find the value of $x^2 - y^2$.

Sample Response:

$x = \frac{(2-\sqrt{5})^2}{4-5}$ and $y = \frac{(2+\sqrt{5})^2}{4-5}$ (Rationalizing the denominator)

$x = -(4 - 4\sqrt{5} + 5)$ $y = -(4 + 4\sqrt{5} + 5)$

$x = -9 + 4\sqrt{5}$ $y = -9 - 4\sqrt{5}$

$\mathbf{x^2 - y^2 = (x + y)(x - y)}$ **………………(1)**

$(x + y)(x - y) = (-9 + 4\sqrt{5} - 9 - 4\sqrt{5})(-9 + 4\sqrt{5} + 9\ 4\sqrt{5})$

$x^2 - y^2 = -144\sqrt{5}$

Scoring for Basic level competencies		Scoring for Higher level competencies	
2 point	Correct and complete computations (operations on Real nos.): Values of x and y.	2 point	Correct use the information (Operations on Real numbers) in a different context (Algebra): use of algebraic identity to solve the problem.
1 point	Partially correct/incomplete computations	1 point	Partially correct use of the given information to solve the problem
0 point	Incorrect/ no computations	0 point	Incorrect/ no use of information to solve the problem

Application level question:

Q5. Find whether the variables x, y, z and u represent a Rational number, Irrational number or both :

(i) $x^2 = 9$ (ii) $y^2 = 0.04$ (iii) $z^2 = \frac{17}{4}$ (iv) $\sqrt{u} = \sqrt{81}$

Show your working and give reasons for each of your answer.

Sample Response:

(i) $x^2 = 9 \rightarrow x = 3$, since 3 can be written as 3/1 (p/q form) or since 3 is an Integer, x is a Rational number.

(ii) $y^2 = 0.04 \rightarrow y = 0.2$, since 0.2 can be written as 2/10 (p/q form) or since 0.2 is a recurring decimal, y is Rational.

(iii) $z^2 = \frac{17}{4} \rightarrow z = \sqrt{17}/2$, since $\sqrt{17}$ is Irrational no. and Irrational ÷ Rational is Irrational, z is Irrational no.

(v) $\sqrt{u} = \sqrt{81} \rightarrow u = 81$, since …(same as (i))

Scoring for Basic level competencies		Scoring for Higher level competencies	
2 point	Correct and complete computation of values of x, y, z and u	2 point	Correct use of - the concept of square root and property of Rational and Irrational numbers to make inference.
1 point	Partially correct /incomplete computation of values of x, y, z and u	1 point	Partially correct use of - concept of square root and property of Rational and Irrational numbers to make inference.
0 point	Incorrect/no computations	0 point	Incorrect or no use of the concepts to make inference.

Application level question:

Q6. Determine the Rational numbers ‘a’ and ‘b’ if $\frac{\sqrt{3}-1}{\sqrt{3}+1} - \frac{\sqrt{3}+1}{\sqrt{3}-1} = a + 3\sqrt{3}\,b$

Sample Response:

LHS = $\frac{(\sqrt{3}-1)^2}{3-1} - \frac{(\sqrt{3}+1)^2}{3-1} = ½\{(3 - 2\sqrt{3} + 1) - (3 + 2\sqrt{3} + 1)\} = ½(-4\sqrt{3}) = -2\sqrt{3}$

Comparing LHS with RHS, $-2\sqrt{3} = a + 3\sqrt{3}\,b$,

We get, a = 0 and b = -2/3

Scoring for Basic level competencies		Scoring for Higher level competencies	
2 point	Correct and complete computation of LHS	2 point	Correct use of the concept of the algebraic property of equating like terms on both sides of an equation to find values of ‘a’ and ‘b’, (application of Real numbers in different context)
1 point	Partially correct/incomplete computations	1 point	Partially correct use of the information to solve new problem
0 point	Incorrect/no computations	0 point	Incorrect or no use of information to solve problem in different context

Analysis level question:

Q7.How many Integers are there between 350×10^{-2} and 750×10^{-2}. Write four Rational numbers between ($-3\frac{1}{2}$) and ($-3\frac{3}{4}$) and four Irrational numbers between 3.5 and $\overline{3}.5$.

Sample Response:

(i) $350 \times 10^{-2} = 3.5$ and $750 \times 10^{-2} = 7.5$: There are 3 Integers (4,5, 6) between 3.5 and 7.5

(ii)Four Rational numbers between -3 ½ and -3 ¾ are :-3.51, -3.52, -3.53, -3.6 -3.7 (Higher level)

-3 ½ = -7/2 and -3 ¾ = -15/4 → -14/4 and -15/4 → -141/40, -142/40, -143/40, -144/40 (Basic level)

(iii)Four Irrational numbers between 3.5 and $\overline{3}.5$ are:

3.5010011000…., 3.5101100111…., 3.5232233222….., 3.5404400444…..

Scoring for Basic level competencies		Scoring for Higher level competencies	
2 point	Correct computations (decimal values of (i)) and algorithmic procedure followed in (ii) and (iii)	2 point	Recognition of the components [(i) values 3.5 and 7.5, (ii) -3.5 and -3.75 or equivalent fractions, (iii) 3.5000… and 3.5555…] Organizing them to use the hidden meaning [(i) sequencing nos. between 3.5 and 7.5 and picking out the Integers to be counted, (ii) identifying the hidden meaning and sequencing in the form of decimal nos. or equivalent fractions]
1 point	Partially correct/incomplete computation	1 point	Partial recognition, organization of components, and identification of hidden meanings to solve.
0 point	Incorrect or no computations and algorithmic procedure	0 point	Incorrect or no recognition, organization of components, and identification of hidden meanings to solve.

Analysis level question:

Q 8. ‘y’ is the reciprocal of Irrational number ‘$\sqrt{x}$’ and ‘x’ is the reciprocal of ‘$\sqrt{z}$’. If ‘z’ is the largest one-digit perfect square, then what is the value of ‘y’? If this value of y is added to its reciprocal, then the answer obtained will be Rational or Irrational?

Sample Response:

$y = \frac{1}{\sqrt{x}}$ and $x = \frac{1}{\sqrt{z}}$ $z = 9$

So, $x = \frac{1}{\sqrt{9}} = \frac{1}{3}$ and $y = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$

$y + \frac{1}{y} = \sqrt{3} + \frac{1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$

Since Rational no. ÷ Irrational no. always is an Irrational no., $y + \frac{1}{y}$ is an Irrational no.

Scoring for Basic level competencies		Scoring for Higher level competencies	
2 points	Correct use of previous knowledge in computing reciprocals and mathematical operations	2 points	Correct identification of components x, y, z and organizing them appropriately to infer value of y and y + 1/y
1 point	Partially correct use of computing reciprocals and mathematical operations	1 point	Partially correct identification of components x, y, z and organizing them appropriately to infer value of y and y + 1/y
0 point	Incorrect or no use of computing reciprocals and mathematical operations	0 point	Incorrect or no identification of components x, y, z and organizing them appropriately to infer value of y and y + 1/y

Analysis level question:

Q 9. Represent ($\sqrt{5} + \sqrt{2}$) on a Number line. Explain the steps in brief in your own words. Label clearly the line segment that represents ($\sqrt{5} + \sqrt{2}$) on the Number line?

Sample Response:

To represent $\sqrt{5}$, hypotenuse = $\sqrt{5}$ and sides a and b:

$h = \sqrt{a^2 + b^2}$

$\sqrt{5} = \sqrt{4 + 1}\sqrt{2} = \sqrt{1 + 1}$

$a^2 = 4 \rightarrow a = 2$

$b^2 = 1 \rightarrow b = 1$

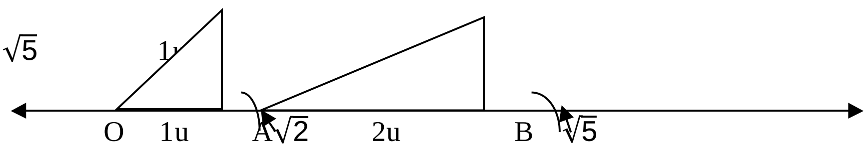
To represent $\sqrt{2}$, $h_1 = \sqrt{2}$ and sides a_1 and b_1 :

$h_1 = \sqrt{a1^2 + b1^2}$

$a1^2 = 1 \rightarrow a1 = 1$

$b1^2 = 1 \rightarrow b1 = 1$

$\sqrt{2}$ 1u $\sqrt{5}$ 1u



………..……$\sqrt{5} + \sqrt{2}$ ………..............

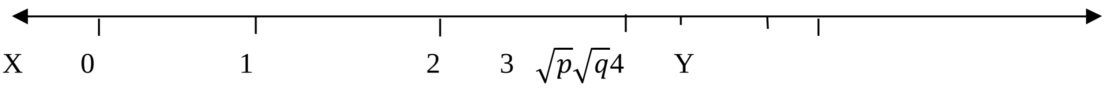
OB = $\sqrt{5} + \sqrt{2}$

Scoring for Basic level competencies		Scoring for Higher level competencies	
2 point	Correct and complete computation and use of procedure to find a, b, a1, and b1	2 point	Correct recognition of hidden meanings: using the values of a, b, a1 and b1 to represent $\sqrt{2}$ and $\sqrt{5}$ separately on a Number Line and organizing them correctly to represent $\sqrt{5} + \sqrt{2}$ on the Number Line

1 point	Partially correct/incomplete computation and procedure to find a, b, a, a1 and b1	1 point	Partially correct recognition of hidden meanings and use of given information
0 point	Incorrect or no computations/process	0 point	Incorrect or no recognition of hidden meaning and use of given information

Synthesis level question:

Q 10.



The figure above shows a Number Line XY, with Irrational numbers $\sqrt{p}$ and $\sqrt{q}$ lying on it. What should be the values of p and q? Choose the answer from the options given below. Give reasons for your answer. Also state why the other options are incorrect.

(e) p = 3.3, q = 3.4 (b) p = $3\frac{7}{9}$, q = $3\frac{8}{9}$ (c) p =$\sqrt{16}$,q =$\sqrt{25}$ (d) p = 11, q = 12

Sample Response:

(a) p = 3.3, q = 3.4 → $\sqrt{p}$ and $\sqrt{q}$ is less than 2, as $\sqrt{3}$ = 1.732… and should $\sqrt{p}$ and $\sqrt{q}$ be between 1 and 2. So, (a) is incorrect.

(b) p =$3\frac{7}{9}$, q = $3\frac{8}{9}$ → $\sqrt{p}$ and $\sqrt{q}$ is $\sqrt{3.77..}$ and $\sqrt{3.888..}$ respectively, $\sqrt{p}$ and $\sqrt{q}$ is between 1& 2. So, (b) is incorrect.

(c) p =$\sqrt{16}$, q =$\sqrt{25}$ → $\sqrt{p}$ = $\sqrt{4}$ = 2 and $\sqrt{q}$ = $\sqrt{5}$, since $\sqrt{p}$ is 2 unlike the figure where $\sqrt{p}$ is between 3 & 4. So, (c) is incorrect.

(d) p = 11, q = 12→ $\sqrt{p}$ = $\sqrt{11}$ and $\sqrt{q}$ = $\sqrt{12}$, both lie between $\sqrt{9}$ = 3and $\sqrt{16}$ = 4. So, (d) is correct.

Scoring for Basic level competencies		**Scoring for Higher level competencies**	
2 point	Correct and complete computation of the approximate values of $\sqrt{p}$ and $\sqrt{q}$	2 point	Correctly using known concepts (approximate values of square roots) to create new ones (determining the positions of Irrational numbers with respect to Integers) and exact use of estimation skills
1 point	Partially correct/ incomplete computation	1 point	Partially correct in using of known concepts to create new ones or error in estimation skills
0 point	Incorrect or no computation	0 point	Incorrect or no use of known concepts to create new one and use of estimation skill

Synthesis level question:

Q 11.'The length of the Hypotenuse of a Right-angled triangle is 3 units'. Use this information *to find the specific Irrational number* ($\sqrt{x}$)that can be represented on a Number line. Use appropriate method to represent that Irrational number on the Number line. Label the diagram properly.

Sample Response: h = 3 units, using Pythagoras theorem, OR $h = \frac{x+1}{2} = 3 \rightarrow x + 1 = 6 \rightarrow x = 5$ $h^2 = a^2 + b^2$ *So,* $a = \sqrt{x} \rightarrow a = \sqrt{5}$ $\sqrt{h^2} = \sqrt{a^2 + b^2}$ $b = \frac{x-1}{2} \rightarrow b = \frac{5-1}{2} = 2$ $\sqrt{9} = \sqrt{4+5}$ *Construction by Perpendicular Geometric method* So, $a^2 = 4 \rightarrow a = 2$ *by using the values of h, a and b* $b^2 = 5 \rightarrow b = \sqrt{5}$ Construction by Hypotenuse Geometrical method

Scoring for Basic level competencies		**Scoring for Higher level competencies**	
2 points	Correct and complete use of theory (Pythagoras theorem/Formula where perpendicular side of right angle triangle is $\sqrt{x}$) and computations to find the required values (sides a and b)	2 points	Correctly relate knowledge from several areas (Pythagoras theorem, square roots, construction of right angles, representing Irrational numbers on Number line) to form conclusions (identify correct procedure)
1 point	Partially correct use of the formula and computations	1 point	Partially correct in relating knowledge from several areas to form conclusions
0 point	Incorrect or no use of formula and computations	0 point	Incorrect or no skill of relating knowledge from several areas to form conclusions

Synthesis level question:

Q 12.Do as Directed :

(a) (i) Solve : 100 + 25 - 16 - 9 and find the square root of the solution

(ii) Solve : $\sqrt{100} + \sqrt{25} - \sqrt{16} - \sqrt{9}$

(b) (i) Solve : 100 ÷ 25 × 16 × 9 and find the square root of the solution

(ii) Solve : $\sqrt{100} \div \sqrt{25} \times \sqrt{16} \times \sqrt{9}$

(c) Compare the solutions of (a) and (b) and frame two **General Rules.**

Sample Response:

(a) (i) 100 + 25 – 16 – 9 = 100 and $\sqrt{100} = 10$

(ii) $\sqrt{100} + \sqrt{25} - \sqrt{16} - \sqrt{9} = 10 + 5 - 4 - 3 = 8$

(b) (i) 100 ÷ 25 × 16 × 9 = 576 and $\sqrt{576} = 24$

(ii) $\sqrt{100} \div \sqrt{25} \times \sqrt{16} \times \sqrt{9} = 10 \div 5 \times 4 \times 3 = 24$

(c) (a) (i) ≠ (ii)

$\sqrt{100 + 25 - 16 - 9} \neq \sqrt{100} + \sqrt{25} - \sqrt{16} - \sqrt{9}$ $\rightarrow \sqrt{a \pm b} \neq \sqrt{a} \pm \sqrt{b}$

(b) (i) = (ii) or

$\sqrt{100 \div 25 \times 16 \times 9} = \sqrt{100} \div \sqrt{25} \times \sqrt{16} \times \sqrt{9}$ $\rightarrow \sqrt{a \times/\div b} = \sqrt{a} \times/\div \sqrt{b}$

Scoring for Basic level competencies		Scoring for Higher level competencies	
2 point	Correct and complete computations (a) and (b)	2 point	Correct generalizing from (a) and (b) as shown in (c) or concluding the general rule
1 point	Partially correct/incomplete computations (a) & (b)	1 point	Partially correct in generalizing from (a) and (b) as in (c) or concluding the general rule
0 point	Incorrect or no computations	0 point	Incorrect or no generalization

Evaluation level question:

Q 13. A student was given a task to construct problems of the following type as per the given conditions. Mention in each case what kind of number should he take as 'x' and 'y'. Give one example in each case to substantiate your answer.

(i) $x + y$; such that the sum is surely an Irrational number (one example)

(ii) $x - y$; such that the difference is surely a Rational number (one example)

(iii) $x \times y$; such that the product may be Rational or an Irrational number (two examples)

(iv) $x \div y$; such that the quotient is surely an Irrational number (one example)

Sample Response:

1. If two or more Rational numbers are added, subtracted, multiplied and divided (except for divisor $\neq 0$), the answer is always a Rational number.
2. If two or more Irrational numbers are added, subtracted, multiplied and divided, the answer is either a Rational number or an Irrational number.
3. If a Rational and an Irrational number is added, subtracted, multiplied and divided, the answer is always an Irrational number.

(i) If x is a Rational number than y is an Irrational number or vice versa (Property 3).

Example: $2 + \sqrt{2} = 2 + 1.414\ldots. = 2.414\ldots..$ (Irrational no.)

(ii) Both x and y should be a Rational number (Property 1).

Example: $7 - 8 = -1$ (Rational number)

(iii) Both x and y are Irrational numbers (Property 2).

Example: $\sqrt{2} \times \sqrt{2} = 2$ (Rational no.) and $\sqrt{2} \times \sqrt{3} = \sqrt{6}$ (Irrational no.)

(iv) If x is a Irrational number than y is an Rational number or vice versa (Property 3).

Example: $\sqrt{2} \div 2 = 1.414\ldots \div 2 = 0.707106\ldots\ldots$ (Irrational no.)

Scoring for Basic level competencies		Scoring for Higher level competencies	
2 point	Correct examples given in all the cases and correct computations done	2 point	Different properties of operations on Rational and Irrational numbers are compared and contrasted to infer the values of x and y: correctly for all the cases

1 point	Correct examples given in half of the cases and partially correct computations done	1 point	Different properties of operations on Rational and Irrational numbers are compared and contrasted to infer the values of x and y: correctly for half of the cases
0 point	No examples given and no computations done	0 point	Compare and discriminate between ideas to infer values : incorrect or not done

Evaluation level question:

Q 14. An investment policy offered four options to its investors to choose from. If an investor wants to invest Rs.10,000 for ten years, which of the following would be the best option for him.

"*At the end of the term the investor would get back*

(vi) Approximately $\sqrt{30}$ *times the original amount*

(vii) Double the square root of the original amount + the original amount

(viii) $(\sqrt{2^5} \div 2^{\frac{3}{2}})$ *times the original amount*

(ix) 2 *times the original amount"*

Sample Response:

(i) $\sqrt{30}$ times the original amount~ 5 times 10,000 ~ 50,000 (Investor would get back approximately something more than Rs. 50,000 after 10 years)

(ii) Double the square root of the original amount + the original amount = $2 \times \sqrt{10000}$ + 10,000 = 200 + 10000 = 10,200 (Investor would get back Rs. 10,200 after 10 years)

(iii) $(\sqrt{2^5} \div 2^{\frac{3}{2}})$ times the original amount = $(2\sqrt{2} \div 2\sqrt{2}) \times 10{,}000 = 1 \times 10{,}000 = 10{,}000$ (Investor would get back Rs. 10,000)

(iv) 2 times the original amount = 2 × 10,000 = 20,000 (Investor gets back Rs. 20,000)

Option (i) would be the best deal for the Investor.

Scoring for Basic level competencies		Scoring for Higher level competencies	
2 point	Correct and complete computations for cases (ii), (iii) and (iv)	2 point	Correct approximation in case (i) and Making choices based on reasoned arguments: done correctly in all four cases
1 point	Partially correct/incomplete computations in cases (ii), (iii) and (iv)	1 point	Partially correct approximation in case (i) and Making choices based on reasoned arguments: correct in half cases
0 point	Incorrect or no computations	0 point	Incorrect approximation in case (i) and Making choices based on reasoned arguments: incorrect/ not done in all

Evaluation level question:

Q 15. Construct a problem using Irrational numbers $\sqrt{2}$, $\sqrt{32}$ and $\sqrt{8}$; in order to prove that 'Distribution of Multiplication of Irrational numbers over subtraction is possible.'

Sample Response:

$\sqrt{32} = 2\sqrt{8}$

Constructing the problem (distribution of multiplication of Irrational nos. over subtraction):

$\sqrt{2}(\sqrt{32}-\sqrt{8})$ or $\sqrt{2}(2\sqrt{8}-\sqrt{8}) = \sqrt{2} \times 2\sqrt{8} - \sqrt{2} \times \sqrt{8}$

LHS = $\sqrt{2}(2\sqrt{8}-\sqrt{8}) = \sqrt{2}(\sqrt{8}) = \sqrt{32}$

RHS = $\sqrt{2} \times 2\sqrt{8} - \sqrt{2} \times \sqrt{8} = 2\sqrt{2}\sqrt{8} - \sqrt{2}\sqrt{8} = \sqrt{2}\sqrt{8} = \sqrt{32}$

Thus, LHS = RHS

So, the Distributive property of multiplication over subtraction is valid for Irrational numbers.

Scoring for Basic level competencies		**Scoring for Higher level competencies**	
2 point	All computations involved throughout the solution, correctly done	2 point	Construction of problem done correctly and theory (property) correctly verified
1 point	Partially correct computations done	1 point	Construction of problem correctly done but verification of LHS and RHS incorrect or vice versa
0 point	Incorrect or no computations	0 point	Incorrect or no construction and verification

APPENDIX A

(5) PRETEST (FOR COMPARISON OF GROUPS)

Total Marks : 60 **Time : 2 ½ hours**

I. COMPREHENSION LEVEL

1. Show that 3.5 is a Rational number.
2. If $\frac{x}{y} = \frac{3}{4}$, then find the value of $\left(\frac{6}{7} + \frac{y-x}{y+x}\right)$
3. The Fraction $\frac{3}{4}$ is a Proper fraction and $\frac{4}{3}$ is an Improper fraction. How are these two types of Fractions different from each other? (Write four points of difference)

II. APPLICATION LEVEL

4. If $x = \frac{\frac{1}{5}+\frac{1}{3}}{\frac{1}{5}-\frac{1}{3}}$. Find the value of '$y$', if $y^2 - x^2 = 0$.
5. Find which of the variables x, y and z *represent a Rational number (Q) but is not an Integer (Z) :*(Show your working and give reasons for each of your answer)

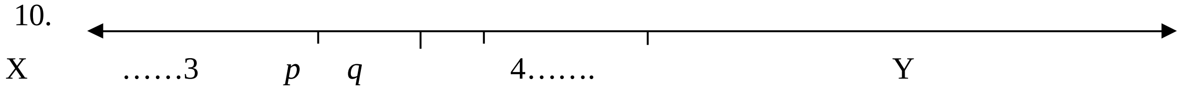
   (i) $x - \frac{1}{2} = (-9)$ (ii) $\frac{y}{100} = (-0.04)$ (iii) $z + \frac{17}{4} = \frac{17}{4}$
6. Determine the Rational number '**a**' if $\frac{\frac{1}{3}-1}{\frac{1}{3}+1} - \frac{\frac{1}{3}+1}{\frac{1}{3}-1} = \frac{3}{2}\mathbf{a}$

III. ANALYSIS LEVEL :

7. How many Integers are there between 7×10^2 and 8×10^2 and between 7×10^{-2} and 8×10^{-2}.Write five Integers between $(-4\frac{2}{3})$ and $4\frac{2}{3}$.
8. 'y' is the reciprocal of Rational number 'x' and 'x' is the reciprocal of Rational number 'z'. If 'z' is three times the value of the only even prime number, then what is the value of 'y'? If this value of 'y' is added to its reciprocal, then the answer obtained will be an Integer or a Fraction?
9. Find the Additive inverse of the Multiplicative inverse of $2\frac{2}{3}$. Subtract $\frac{1}{2}$ from that number and then represent that number on a Number line.

IV. SYNTHESIS LEVEL :

10.



The figure above shows a Number Line XY, with Rational numbers *p* and *q* lying on it. What should be the values of *p* and *q*? Choose the answer from the options given below. Give reasons for your answer. Also state why the other options are incorrect.

(a) $p = \frac{1}{2}$, $q = \frac{1}{3}$

(b) $p = 3\frac{1}{2}$, $q = 3\frac{3}{4}$

(c) $p = 0.3$, $q = 0.4$

(d) $p = \frac{33}{10}$, $q = \frac{34}{10}$

11. Represent $(\frac{2}{3} + 3 - \frac{1}{2})$ on Number line, without doing any kind of calculations. Draw a Number line with clearly marked points.

Mark $(\frac{2}{3})$ *as A,* $(\frac{2}{3} + 3)$ *as B, and* $(\frac{2}{3} + 3 - \frac{1}{2})$ *as C.*

12. Write all the possible similarities between 3^x and 7^x, where x = 1, 2, 3, 4, 5 and 6.

V. EVALUATION LEVEL :

13. A student was given a task to frame some sums using x and y of the form : **x + y** such that the answers obtained in each case is always an *Integer*. What kind of numbers can he take as x and y? He chooses to use all the three options given below. Will he be able to use all the three options, if yes…how? Show giving two examples of each.

(a) He can take both x and y as Integers

(b) He can take x as Integer and y as Rational number

(c) He can take x and y both as Fractional numbers

14. An investment policy in its policy terms gave four options to the investors to choose from. It stated that- "At the end of ten years, the investor will get back the original invested amount and in addition :

(a) 0.8 times the invested amount

(b) $\frac{8}{3}$ times the invested amount

(c) $1\frac{1}{8}$ times the invested amount

(d) $\frac{10}{8}$ times the invested amount"

As an investor which option would you choose and why?

15. Construct a problem using any three Rational numbers; in order to prove that 'Distribution of multiplication of Rational numbers over subtraction is possible.

APPENDIX A

(6) BLUE PRINT BASED ON WHICH PRETEST WAS CONDUCTED

	TOPIC	**Understanding**			**Application**			**Analysis**			**Synthesis**			**Evaluation**			**Total**
		1(4)	**2(4)**	**3(4)**	**4(4)**	**5(4)**	**6(4)**	**7(4)**	**8(4)**	**9(4)**	**10(4)**	**11(4)**	**12(4)**	**13(4)**	**14(4)**	**15(4)**	
1	Rational Numbers & Numbers within	1(4)						½(2)		½(2)			¾(3)				**11**
2	Relation between the different Numbering systems included in Rational Number			1(4)		½(2)		½(2)			½(2)						**10**
3	Representation of Rational Numbers on Number Line				½(2)					½(2)		1(4)					**8**
4	Properties of Rational Numbers													1(4)		1(4)	**8**
5	Mathematical Operations on Rational Numbers		½(2)		½(2)		1(4)		½(2)						1(4)		**14**
6	Algebraic Operations on Rational Numbers		½(2)			½(2)			½(2)		½(2)		¼(1)				**9**
	No. of Questions(Marks)	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**15(60)**

APPENDIX A

(7) REACTION SCALE

Instructions for Reaction Scale

Dear Students,

A heartfelt thanks to you for sincerely participating in the experiment conducted by me for my research project titled, “Developing, Implementing and Assessing an Instructional Package for Higher Order Thinking Skills in Mathematics”

Please give me your feedback regarding the Methods used for Teaching, the Worksheets and the Evaluations conducted during the experiment. Read the following Instructions carefully and give your feedback accordingly.

A set of 25 statements are given along with a rating scale for each. Encircle the most appropriate abbreviation for each statement, in the following manner.

SA	A	ND	DA	SD

The meaning of the abbreviations are :

SA : Strongly Agree **A :** Agree **ND :** Not Decided **DA :** Disagree **SD :** Strongly Disagree

1.	The previous knowledge discussed before the beginning of a new topic helped me to understand the topic better.	SA	A	ND	DA	SD
2.	The detailed in-depth explanation of each concept helped in better understanding.	SA	A	ND	DA	SD
3.	The examples, counter-examples, contrasts, similarities used to explain concepts made my understanding better.	SA	A	ND	DA	SD
4.	The sequencing of the sub-topics which was different than that given in textbook, helped in better understanding.	SA	A	ND	DA	SD
5.	The questions put forward by the teacher during her instructions forced me think further than the usual.	SA	A	ND	DA	SD
6.	The time given for each sub-topic was sufficient to bring about proper understanding regarding the concept.	SA	A	ND	DA	SD
7.	I have understood all the concepts related to the topic 'Real numbers' very clearly.	SA	A	ND	DA	SD
8.	I have understood the holistic meaning and structure of the Numbering system – Real numbers.	SA	A	ND	DA	SD
9.	I have understood the inter-connections between the sub-topics within Real numbers completely.	SA	A	ND	DA	SD
10.	I got better understanding about some basic mathematical facts which were not clear to me earlier.	SA	A	ND	DA	SD
11.	I have understood complex aspects of Mathematics like estimation, proofs, verification, and generalization with reference to Real numbers.	SA	A	ND	DA	SD
12.	I paid less attention to the concepts explained beyond the textbook.	SA	A	ND	DA	SD
13.	I am motivated and interested to learn more about Real numbers and other Numbering systems.	SA	A	ND	DA	SD
14.	I am more confident now to proceed further with the other mathematical topics in my syllabus.	SA	A	ND	DA	SD
15.	I love Mathematics more now.	SA	A	ND	DA	SD
16.	Mathematics seems to be more difficult and complex now.	SA	A	ND	DA	SD
17.	Solving the worksheets increased my understanding about that topic.	SA	A	ND	DA	SD
18.	The worksheets were appropriate and interesting.	SA	A	ND	DA	SD
19.	The worksheets gave me scope to observe patterns and generalize.	SA	A	ND	DA	SD
20	I could not understand the language used in the worksheets.	SA	A	ND	DA	SD
21	There should have been lesser number of worksheets.	SA	A	ND	DA	SD
22.	New, complex, unfamiliar problems posed in the Evaluations gave me scope to think at higher levels.	SA	A	ND	DA	SD
23.	The Evaluations motivated me to understand concepts of math rather than memorizing the procedure.	SA	A	ND	DA	SD
24.	The Evaluation questions were very tough and I have lost interest in Mathematics because of them.	SA	A	ND	DA	SD
25.	The teaching, worksheets, evaluations helped me to look at Mathematics in a different way, which is logical, inter-connected and interesting.	SA	A	ND	DA	SD

1. How was the teaching and learning experience for the present topic ‘Real Numbers’ different for you in comparison to the way Mathematics was taught in your earlier classes till Class VIII?

Navrachana University
School of Liberal Studies and Education
Vasna Bhayli Road,
Vadodara

(Sangeeta Pramanik)
Investigator

APPENDIX B

TEACHING-LEARNING MATERIALS USED IN THE INSTRUCTIONAL PACKAGE

(1) CONTENT CHART

1. MEANING OF REAL NUMBERS

Rational Numbers (Q)

Irrational Numbers (I)

2. RATIONAL NUMBERS

Meaning & Definition

Relation between fractions and decimals

Conversion of each type of decimals into fractions to prove that they belong to (Q)

Rational numbers between two Rational numers

3. IRRATIONAL NUMBERS

- Conversion of different types of Fractions into decimals to identify the types of Decimals

-Definition of Irrational Numbers (Decimals & Roots of Non-perfect squares, cubes etc.)

-Irrational numbers between two numbers

Place in the set of Real Numbers

4. REPRESENTATION OF REAL NUMBERS ON NUMBER LINE

Rational Numbers (Q)

- Fractions
- Decimals

Irrational Numbers (I)

- Hypotenuse Geometric Method
- Perpendicular Geometric Method

5. PROPERTIES OF DIFFERENT OPERATIONS ON REAL NUMBERS

Commutative Property

Associative Property

Distributive Property

Closure Property

6. OPERATIONS ON REAL NUMBERS

Addition

Subtraction

Multiplication

Division

of Irrational Numbers by using

-rules of Algebra

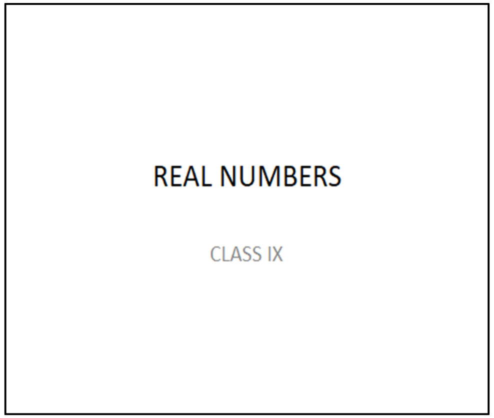
-algebraic identities,

- Irrational expressions in fractional form

-laws of exponents on Irrational numbers

APPENDIX B

(2) POWERPOINT PRESENTATION



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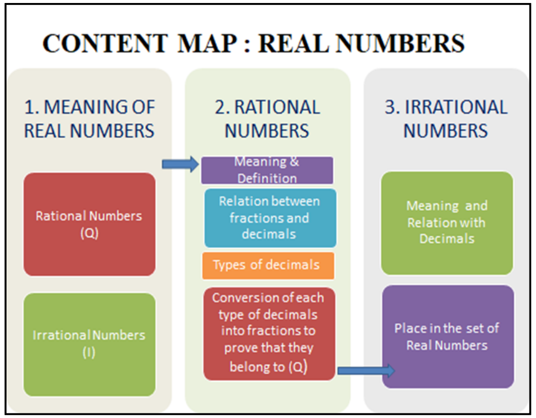

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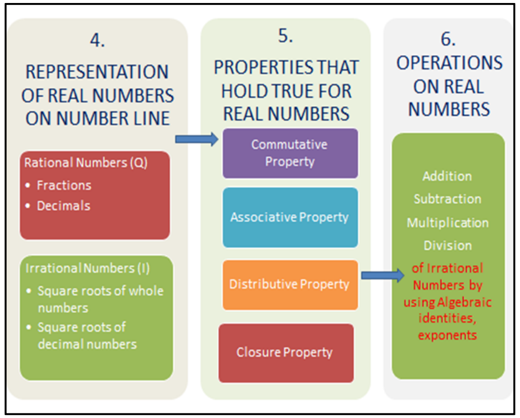

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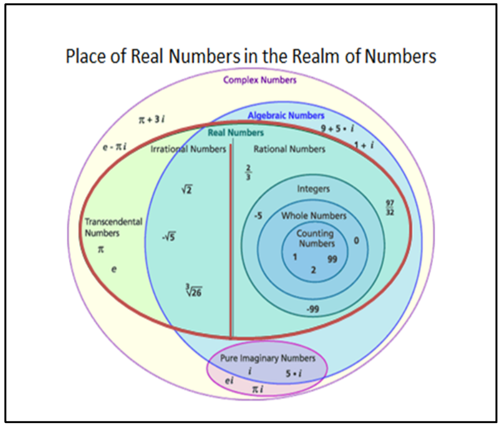

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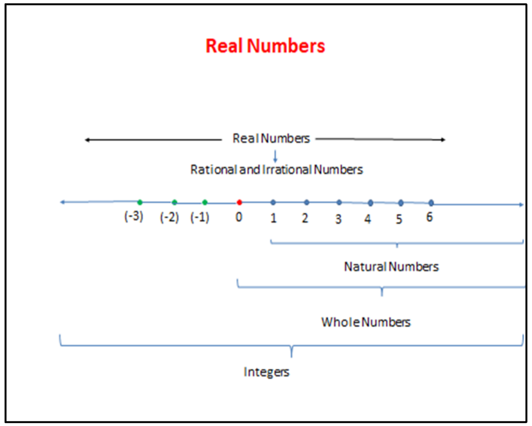

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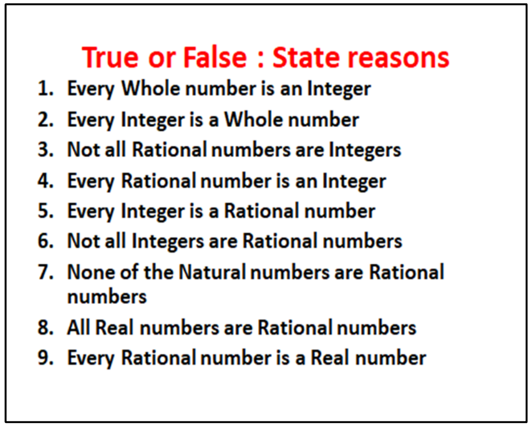

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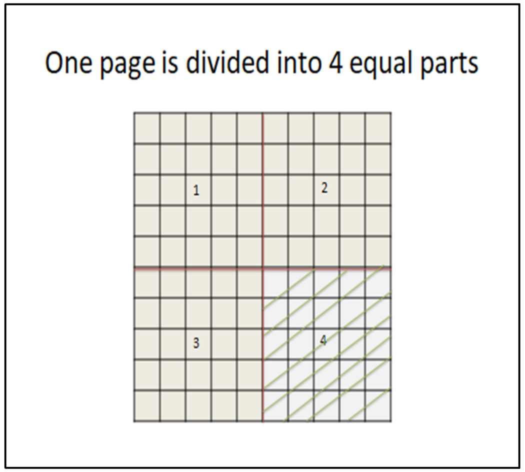

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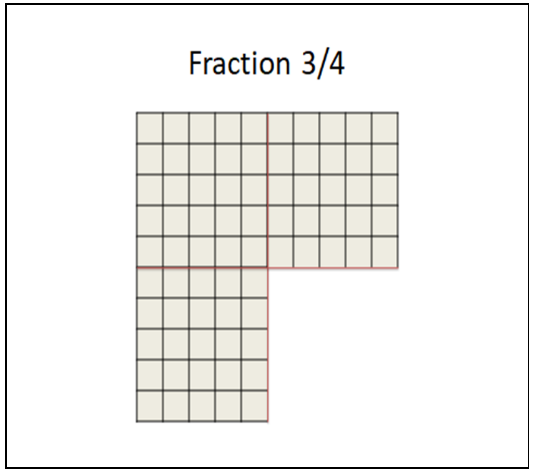

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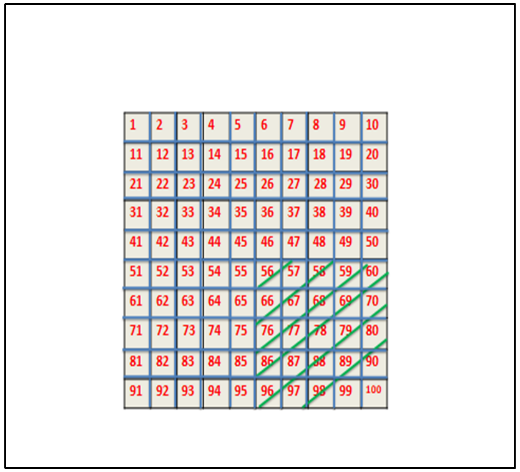

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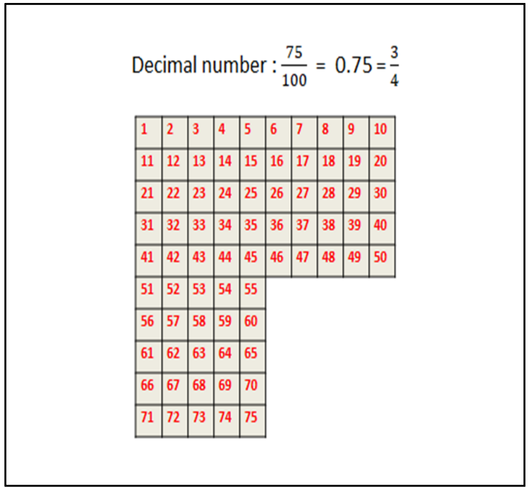

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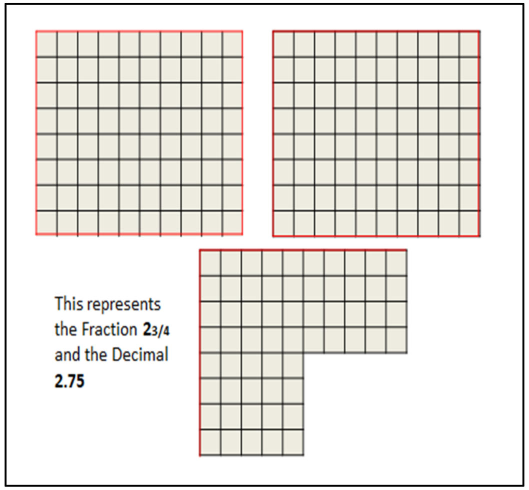

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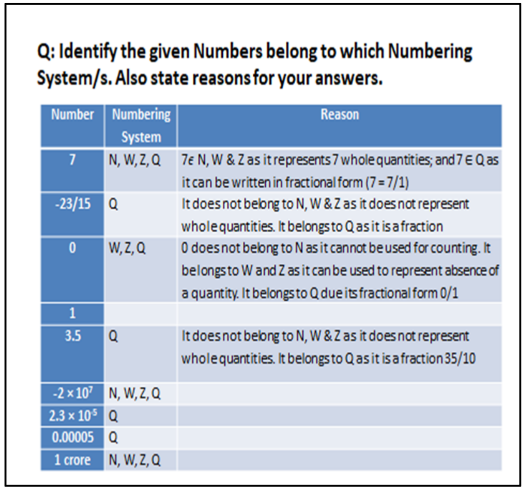
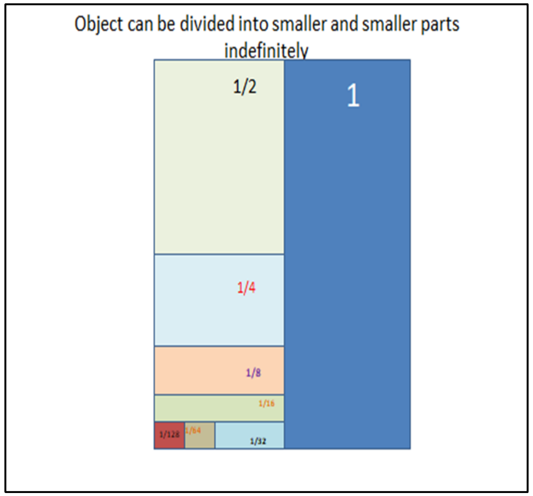


Number	Numbering System	Reason
7	N, W, Z, Q	7ϵ N, W & Z as it represents 7 whole quantities; and 7 ∈ Q as it can be written in fractional form (7 = 7/1)
-23/15	Q	It does not belong to N, W & Z as it does not represent whole quantities. It belongs to Q as it is a fraction
0	W, Z, Q	0 does not belong to N as it cannot be used for counting. It belongs to W and Z as it can be used to represent absence of a quantity. It belongs to Q due its fractional form 0/1
1		
3.5	Q	It does not belong to N, W & Z as it does not represent whole quantities. It belongs to Q as it is a fraction 35/10
-2×10^7	N, W, Z, Q	
2.3×10^{-5}	Q	
0.00005	Q	
1 crore	N, W, Z, Q	

Slide 12



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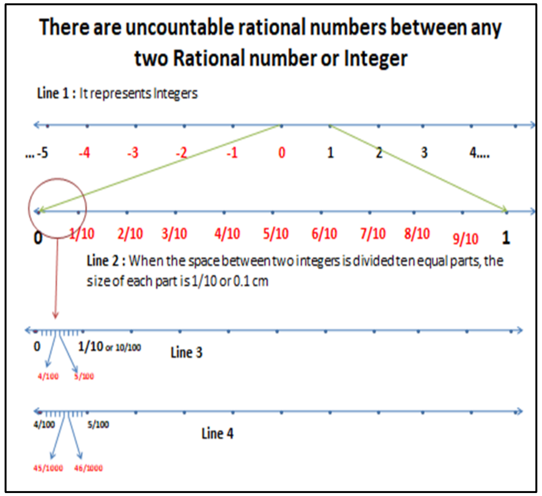

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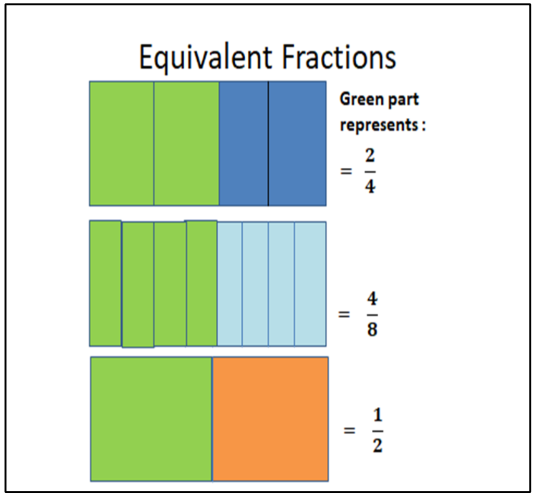

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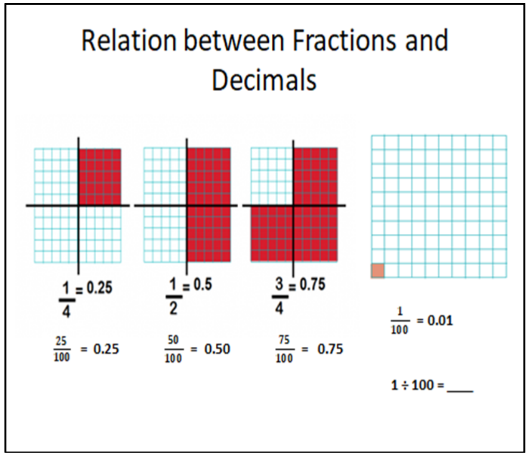

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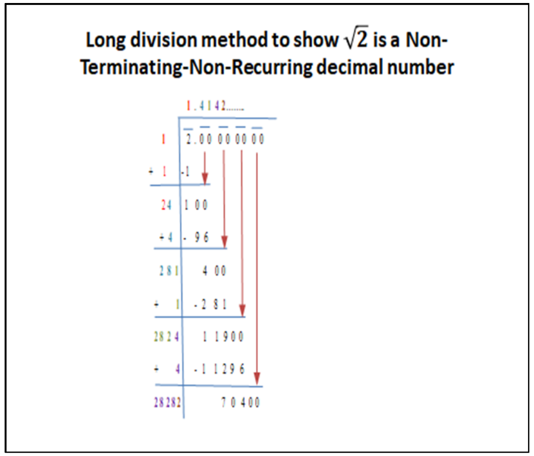

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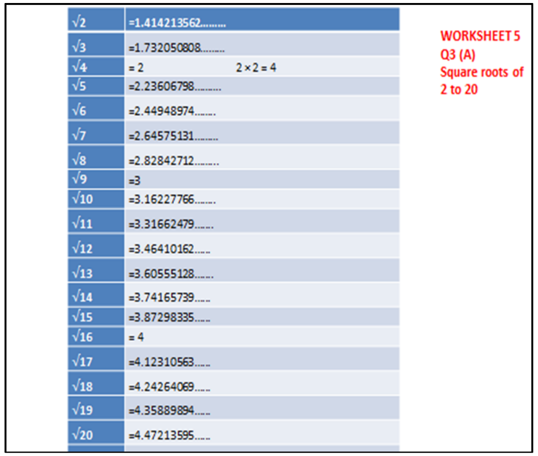

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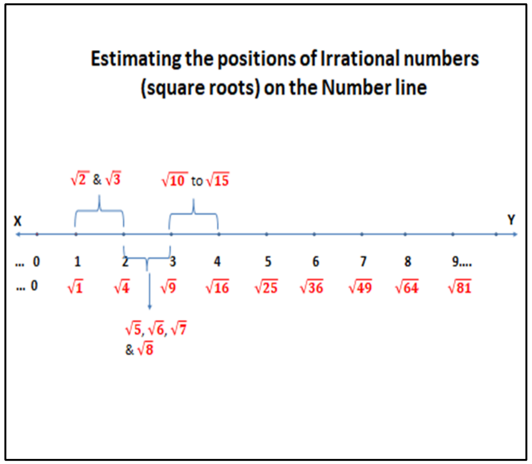

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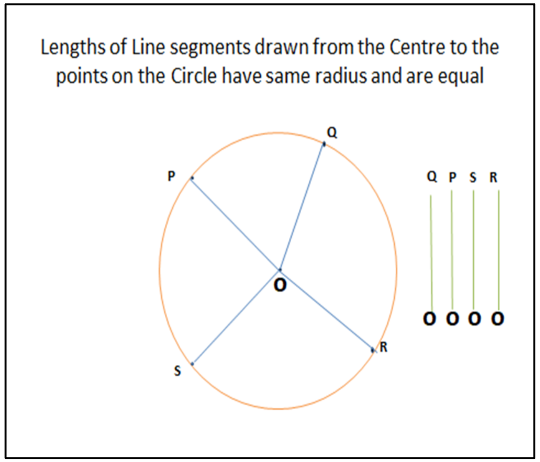

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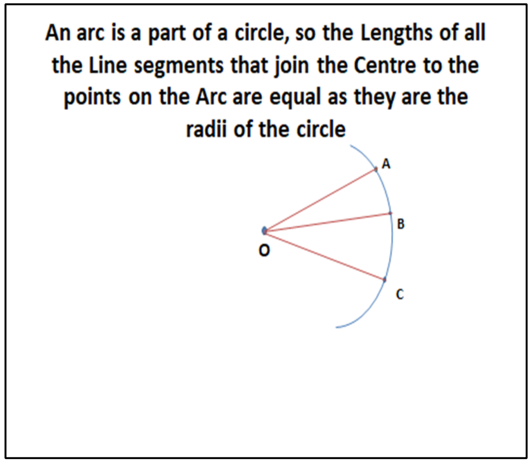

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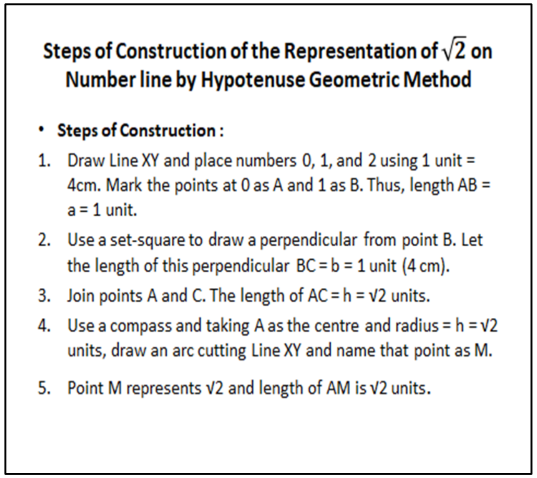
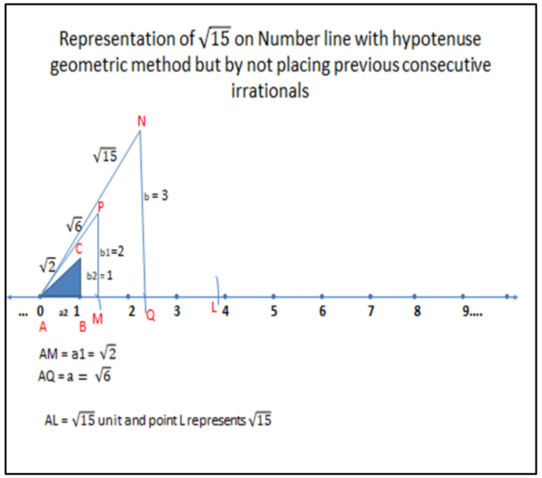
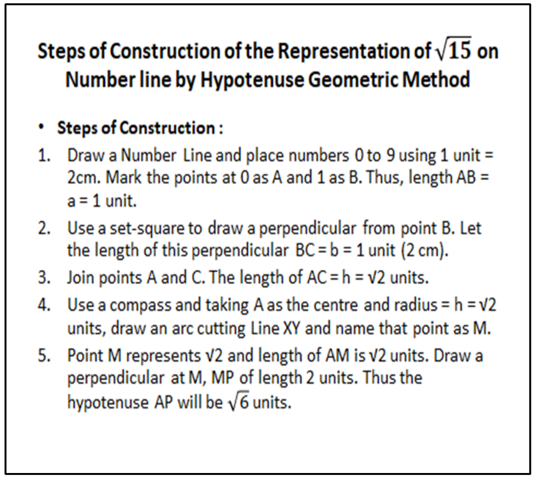


Slide 22



Slide 23

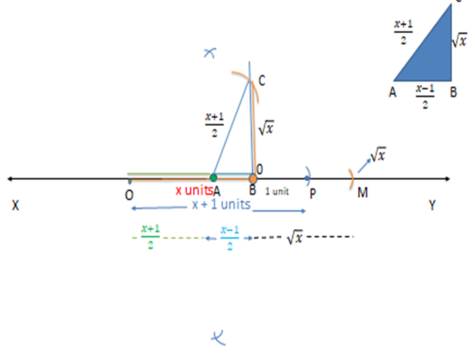


Slide 24

- Taking A as the centre and AP as the radius, draw an arc on Number line and name it Q. Thus AQ = $\sqrt{6}$ units.
- Draw a perpendicular from Q, QN of length 3 units. Join AN. Thus AN = $\sqrt{15}$ units.
- Taking A as the centre and AN as the radius draw an arc and name the point L.
- Thus point L represents $\sqrt{15}$ on the Number line and AL = $\sqrt{15}$ units

Slide 25

Representation of Irrational number on Number line by the Perpendicular Geometric Method



Slide 26

Steps of Construction of $\sqrt{7}$ by the Perpendicular Geometric Method

1. Draw a Number Line $\overleftrightarrow{XY}$.
2. Take a point **O** on $\overleftrightarrow{XY}$ and draw **OB = 7 units (x units)** on it.
3. Place a point **P** on $\overleftrightarrow{XY}$, such that **BP = 1 unit.**
4. Construct the **mid-point A** of **OP**, to get **OA =** $\frac{x+1}{2}$ **units.** Thus, **AB =** $\frac{x-1}{2}$ **units.**
5. Draw a perpendicular from **B** on the upper side of $\overleftrightarrow{XY}$ and call it **BS**.
6. Take **A** as **centre** and **OA** as **radius** and draw an arc on **BS** at point **C.** Thus, **BC =** $\sqrt{x}$ **=** $\sqrt{7}$ **units.**
7. Take **B** as **centre** and **BC** as **radius** and draw an arc on $\overleftrightarrow{XY}$ at **M.**
8. Thus, If **B** is considered **zero**, then **M represents** $\sqrt{7}$ **on Number Line.**

Slide 27

APPENDIX B

(3) BLUEPRINTS OF EVALUATION 1 AND 2

EVALUATION 1 – BLUEPRINT

	TOPIC	**Understanding**			**Application**			**Analysis**			**Synthesis**			**Evaluation**			**Total Marks**
		1(4)	**2(4)**	**3(4)**	**4(4)**	**5(4)**	**6(4)**	**7(4)**	**8(4)**	**9(4)**	**10(4)**	**11(4)**	**12(4)**	**13(4)**	**14(4)**	**15(4)**	
1.	Rational numbers as fractions							1/2(2)				1(4)				1(4)	**10**
2.	Relationship between the numbers that lie in the numbering systems N, W, Z and Q		1(4)	1(4)		1(4)		1/2(2)									**14**
3.	Finding Rational numbers between two given Rational nos.				1(4)		1(4)										**8**
4.	Identification of the position of a given Rational numbers with respect to given Rational numbers									1(4)	1/4(1)						**5**
5.	Concept of Rational nos. as an infinite set								1(4)								**4**
6.	Relation of Rational nos. with Decimal nos.	1(4)									1/4(1)		1(4)	¼(1)			**10**
7.	Types of Decimal expansions										½(2)			¾(3)	1(4)		**9**
	No. of Questions(Marks)	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**15(60)**

EVALUATION 2 – BLUEPRINT

	TOPIC	Understanding			Application			Analysis			Synthesis			Evaluation			Total Marks
		1(4)	**2(4)**	**3(4)**	**4(4)**	**5(4)**	**6(4)**	**7(4)**	**8(4)**	**9(4)**	**10(4)**	**11(4)**	**12(4)**	**13(4)**	**14(4)**	**15(4)**	
1.	Representation of Decimal numbers on Number line	1(4)	1(4)														**8**
2.	Difference between Rational numbers and Irrational numbers			1(4)			1(4)								1(4)		**12**
3.	Finding Irrational numbers between given numbers								1(4)			1(4)					**8**
4.	Use of Pythagoras theorem on Irrational numbers				½(2)					½(2)	1(4)			1(4)			**12**
5.	Representation of Irrational numbers on Number line				½(2)	1(4)				½(2)							**8**
6.	Properties of Rational numbers							1(4)					1(4)			1(4)	**12**
	No. of Questions(Marks)	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**1(4)**	**15(60)**

APPENDIX B

(4) SELF LEARNING MATERIALS WITHIN THE INSTRUCTIONAL PACKAGE

SLM 1 : NUMBERING SYSTEMS

Numbers are used in our real lives to represent, measure or count things. We have been learning these numbers as groups or sets one by one in the previous standards. First, we studied about the Natural Numbers, then the Whole numbers, after we understood their properties and their operations completely, we proceeded to understand the Integers. Fractions and Decimals which we had been learning right from the class 4 level got included in the set of Integers to form the set of numbers called Rational Numbers which you studied in detail in class 8. Before we proceed forward to understand one more set of numbers called Irrational Numbers, let us revise and polish all the previously learnt Numbering Systems.

I. Natural Numbers

- Natural Numbers is the set of numbers starting from 1 and including all positive numbers. It never ends, as we can always add 1 to get the next number. This set is an uncountable set.
- This set is represented mathematically as N : {1,2,3,……..}
- Natural numbers are generally used for counting and measuring **whole or unbroken quantities.**

Example :10 men, **2,34,345** rupees, **55** km, **3**kg, **10** litres, **67** sq. m., etc.

- Prime numbers and Composite numbers belong to this set of Natural numbers.
- **Prime and Composite Numbers :** A prime number is a Natural number that has exactly two distinct Whole number factors (or divisors), namely 1 and the number itself. Composite number is a Natural number that has more than two distinct whole factors.

Example : We can build up 36 by multiplying two numbers as follows :

36 = 1 × 36
36 = 2 × 18
36 = 3 × 12
36 = 4 × 9
36= 6 × 6

→ **1, 2, 3, 4, 6, 9, 12, 18, and 36 are factors of 36. Thus, as it has more than two factors, it is a composite number**

13 1 × 13 → **13 has only two factors 1 and 13. Thus, as it has only two factors 1 and itself, it is a prime number**

II. Whole numbers

- Whole numbers is the set of numbers starting from 0 and continuing, with uncountable numbers in it. This set contains all positive numbers along with zero.
- This set is represented mathematically as W : {0, 1, 2, 3, 4, ………..}
- Whole numbers are also used to count and measure **whole or unbroken quantities**. It also includes zero, underlying the importance of 'nothing'.

Example :50 dollars, **0** students are absent on a particular day, bank balance after paying the medical bills is **zero** for Mr. A.

III. Integers

- Integers is the set of all negative and positive Whole numbers. It is an infinite set.
- This set is represented mathematically as Z : {……, -3, -2, -1, 0, 1, 2, 3, ……}
- As is evident from the above notation that the set Z includes both the sets N and W.
- Integers do not include fractional and decimal numbers.
- Integers are used to count and measure **whole or unbroken quantities** which might be more than zero or less than zero

Example : Room temperature is around **22°** C. Freezing is any value below **0°** C, thus **-3°** C is an example of Integers. Velocity of a ball thrown upwards is negative **(-9** m/s or **-19** m/s) when it is moving upwards, as it is moving against gravity. The acceleration of a bike decreases after the application of breaks, which may be **-30** m/s^2.

- **Even and Odd Numbers :** Integers which are divisible by 2 are even numbers and those not divisible by 2 are odd numbers.

Example : -100, -24, 0, 36, 231578 etc. are even numbers
-317, -9, 7, 35 etc. are odd numbers

Even and Odd numbers belong to the set of Integers (Z)

IV. <u>Rational Numbers</u>

- Rational Numbers is the set of all positive and negative Whole numbers and all fractions. It is an infinite set.
- This set is represented mathematically by the symbol Q.
- This indicates that any number that can be expressed in the form of fractions like 2/3 or -13/7 or 387/100 etc. are Rational Numbers.
- Natural numbers, Whole numbers and Integers like 7, 0, -12, ….. can be written as 7/1, 0/1, -12/1 (in fractional forms) respectively. Thus, the set of Rational numbers includes Natural numbers, Whole numbers, as well as Integers.
- All the Decimal numbers that can be written as fractions like 23.4 is same as 234/10; also 0.0043 is same as 43/10000, also belong to this set of Rational numbers.
- Thus, Rational numbers can be used to count and measure whole as well as broken quantities.
- This set includes all the Integers as well as all the numbers that lie between two Integers.
- **Example :7** girls; a chocolate bar is broken into **4** pieces and one piece given to a child, who gets **¼** of the chocolate that is less than 1; the worker worked for **2.5** days, which means 2 whole days and half of the third day; the size of a micro-organism is **0.01**cm, which means the length of 1cm (as shown below as line segment AB) is divided into 100 parts and the length of one of those 100 parts is the size of the micro-organism.

A B (1 cm length)

[***The ancient greek mathematician Pythagoras believed that all numbers were Rational, but one of his students Hippapus proved (using geometry) that you could not write the square root of 2 as a fraction, and so it was Irrational. But followers of Pythagoras could not accept the existence of Irrational numbers, and it is said that Hippasus was drowned at sea as a punishment from the gods!]***

Prime and Composite numbers belong to the set of Natural numbers (N).

Thus, Negative numbers, Zero, Fractional numbers, Decimal numbers cannot be termed as Prime or Composite numbers as none belong to the set of Natural numbers

Even and Odd numbers belong to the set of Integers (Z).

Thus, fractions and decimals cannot be termed as Even or Odd.

This is because every fraction and decimal number can have uncountable equivalent fractions or decimals. Eg. $\frac{3}{4}$may look like an Odd number, but it can always be written as $\frac{30}{40}$ or $\frac{300}{400}$, thus becoming Even.

Also Decimal numbers like 2.3, look like an Odd number, but can be written as 2.30 without the value being changed, thus making it Even.

****

SLM 2 :REPRESENTATION OF RATIONAL NUMBERS ON NUMBER LINE

We represent numbers on a Number Line because it helps us to understand where a particular number is placed or positioned with respect to other numbers. It is a way to bring down in concrete visible forms the specific position of a number on paper from our minds. Let us learn how to represent Rational numbers on Number Line. Rational Numbers comprises of (1) Integers (2) Fractional / Decimal numbers. We do a quick recap of representation of Fractions on Number Line and then we learn in detail how to represent Decimal numbers with one and more decimal places on the Number Line.

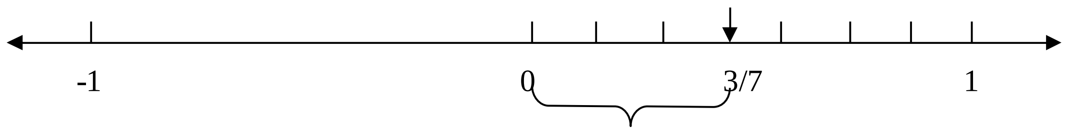
1. Representation of Fractional Numbers (Rational Numbers) on Number Line :

Fractions are of two types (1) Proper Fractional numbers and (2) Improper Fractional numbers

(1) **Proper Fractional Numbers:** Fractional numbers with **Numerators smaller than Denominators** Eg. **1**/6 , **3**/7, **12**/53, **-1**/3, **-5**/17 etc. are Proper Fractional numbers.

Positive Proper Fractional numbers **lie between 0 and 1** and Negative Fractional numbers **lie between (-1) and 0.**

Example : Represent 3/7 on a Number Line :



STEPS:1. 3/7 lies between 0 and 1
2. 3/7 means three equal parts out of seven equal parts.
3. So divide the region between 0 and 1 into seven equal parts
4. Then consider the first three parts from zero
5. Point A represents 3/7 and the distance between 0 and 3/7 is of length 3/7 units

(2) **Improper Fractional Numbers:** Fractional numbers with **Numerators larger than Denominators** Eg. 4/**3**, 10/**7**, 123/**12** etc. are Improper Fractional numbers. Positive Improper Fractional numbers are always **greater than 1**and **lie between two positive Integers**. Negative Improper Fractional numbers are always **less than (-1)** and **lie between two negative Integers.**

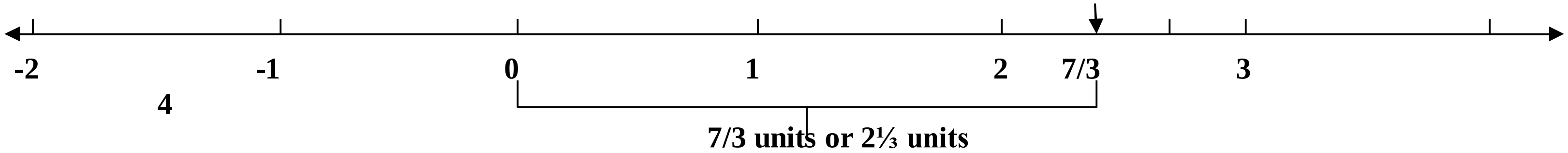
Example : $\mathbf{\frac{7}{3}}$ is an Improper Fractional number.

$\frac{7}{3}$is same as $2\frac{1}{3}$[Because mathematically ($7 \div 3 = 2\frac{1}{3}$) and in concrete forms $\frac{7}{3}$is same as 2 + 1/3]

In reality 7/3 means two full blocks and part of the third block as shown: 

Thus, its value will be more than 2 and less than 3, and it will lie between the Integers 2 and 3.

Example: Represent 7/3 on a Number Line:

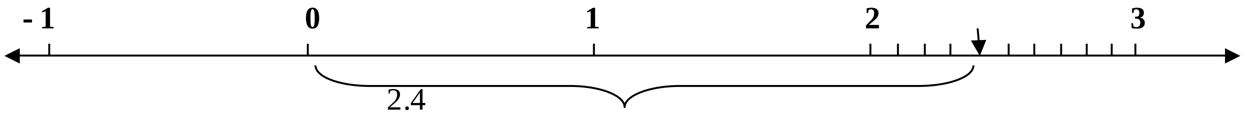


1. 7/3 means 2 units + 1/3 units, so it lies between 2 and 3
2. Consider the space between 2 and 3
3. Now represent 1/3 between 2 and 3 by dividing that region into three equal parts and then marking the first part (1/3)
4. Thus, the distance between 0 and 7/3 as shown on the Number Line represents the length of 7/3 units

2. Representation of Decimals on the Number Line

(1) Representing Decimals with one decimal place on a Number Line:

Example: Represent 2.4 on a Number Line



2.4 units

STEPS:1. The number 2.4 lies between 2 and 3.
2. So draw Number Line accordingly and place numbers equidistant from each other
3. 2.4 is same as 2 + 0.4, where 0.4 is same as 4/10.
4. Thus, the region between 2 and 3 is divided into 10 equal parts and the fourth point represents 0.4
5. The distance between the points marked 0 and 2.4 represents the length 2.4 units

(2) Representing Decimals with more than one decimal places on the Number Line

Example: Represent 2.435 on Number Line1.

1. We represent 2.4 on the Number Line first

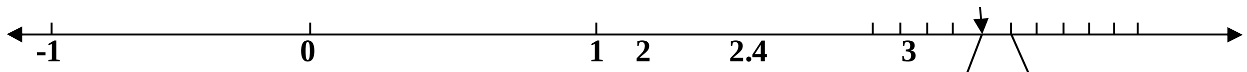


FIG. A

2. Now we represent 2.43 on the Number Line

FIG. B

3. Representing 2.435 on the Number Line

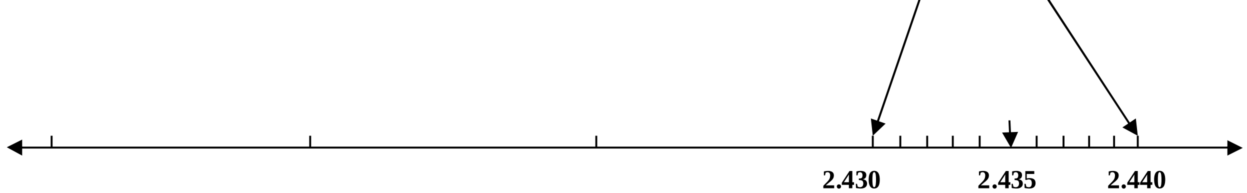
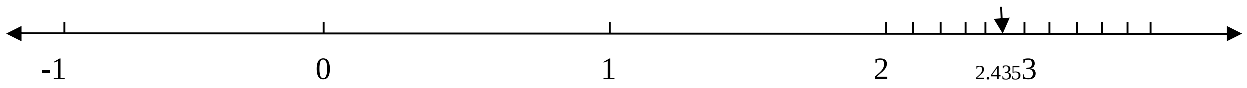


FIG. C

Finally this is where 2.435 is placed

FIG. D



STEPS:

1. 2.435 lies between 2.4 and 2.5

2. 2.435 is same as 2 + 0.4 + 0.003 + 0.0005

Or 2.435 = 2 + 4/10 + 3/100 +5/ 1000

3. To understand its actual position, we find **2.4** on the number line by dividing the region between 2 and 3 into 10 equal parts and marking the fourth point as 2.4 **(Fig. A)**

4. Further if we want to find **2.43** on the number line, it will be between 2.40 and 2.50. Now 2.4 = 2.40 and 2.5 = 2.50 (equivalent decimals). So the region between 2 and 3 is to be divided into 100 equal parts and the 43rd point represents 2.43. This is same as dividing the region between 2.4 and 2.5 into 10 equal parts and marking the third mark. **(Fig. B)**

5. To find **2.435** on the number line, the region between 2 and 3 is to be divided into 1000 equal parts, (this is not possible to be shown). But this is same as dividing the region between 2.43 and 2.44 into 10 equal parts and marking the fifth point, which represents 2.435 **(Fig. C)**

6. Fig. B and Fig. C are magnified to help you understand the concept.

7.Fig. D represents the position of 2.435 with respect to the other Integers.

****

SLM 3: REPRESENTATION OF IRRATIONAL NUMBERS BY THE PERPENDICULAR GEOMETRIC METHOD

In this method, we need to construct a right-angled triangle on the Number line of the form as shown below in Figure 1

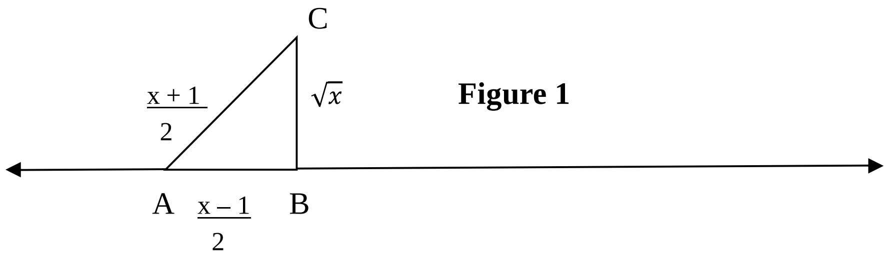


Figure 1

For the actual Construction, refer Figure 2

I) Constructing line segment of length $\frac{x+1}{2}$ on the Number line:

- Draw a Number line XY, construct a line segment OB of length x units on it.
- Construct the length x + 1 units, by placing the point P on the Number line such that BP = 1 unit. Thus OP = x + 1 units
- To construct the length $\frac{x+1}{2}$ units, we need to divide OP into half, or find the mid-point of OP. We use the construction of perpendicular bisector to find point A. Thus OA = AP = $\frac{x+1}{2}$ units.

II) Constructing line segment of length $\frac{x-1}{2}$ on the Number line:

- Check out the length of AB…
- AB = AP – BP

$$= \frac{x+1}{2} - 1$$

$$= \frac{x+1-2}{2}$$

AB = $\frac{x-1}{2}$ units

III) Constructing the right-angled triangle ABC, where AB = $\frac{x-1}{2}$ units, AC is the hypotenuse with length $\frac{x+1}{2}$ units and BC is the perpendicular side with the Irrational number $\sqrt{x}$ units:

- Draw a perpendicular from B, in the space above line XY.
- In order to construct AC (hypotenuse) of $\triangle$ ABC, use a compass and draw an arc, with A as the centre and OA as the radius i.e. $\frac{x+1}{2}$ unit, intersecting the perpendicular from B at C.
- Since the thus constructed right-angled triangle, has the length of its hypotenuse as $\frac{x+1}{2}$ unit, its base as $\frac{x-1}{2}$ unit, automatically the length of its perpendicular side will be $\sqrt{x}$ unit.

IV) Representation of $\sqrt{x}$ on the Number line:

- Since the length of BC is $\sqrt{x}$ units, taking BC as the radius and B as the centre, draw an arc on line XY with a compass at point M

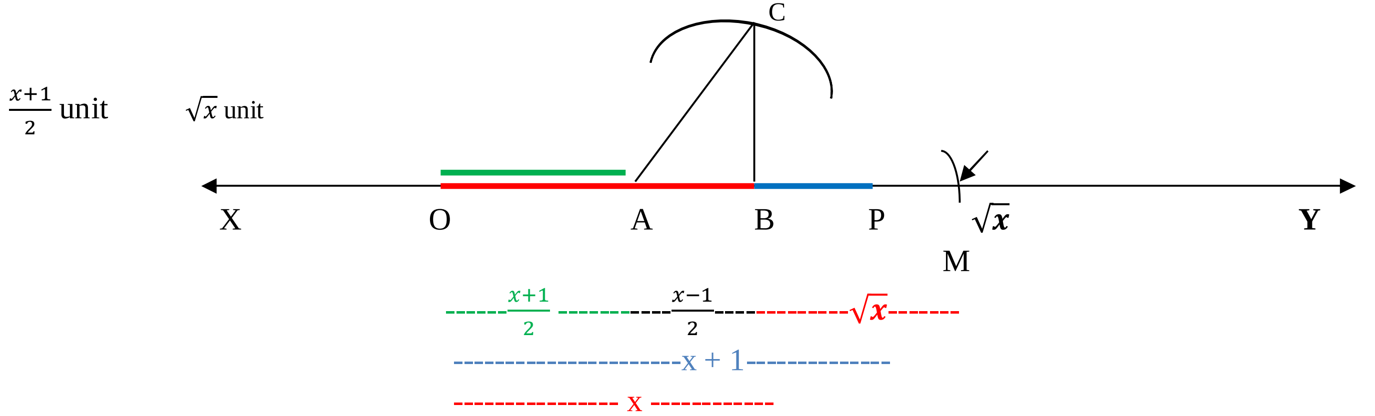


Figure 2

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APPENDIX C

SANCTION LETTERS

(1) Sanction Letter for the Pilot Study

KELAVANI ENGLISH MEDIUM SCHOOL
(Governed by : Kelavani Trust, Vadodara.) Index No. 65-526
Behind Akota Stadium, Old Padra Road, Vadodara-15. Ph. : 0265-2324852
Email : kems01998@gmail.com

Ref. No.: Date : 01-04-17

To Whom So Ever It May Concern

This is to declare that Ms. Sangeeta Poddar has conducted her pilot study for her Ph.D. Project titled " Developing, Implementing and Assessing an instructional package for higher order thinking skills in Mathematics" in our school ' Kelavani English Medium School'.

She conducted her pilot study from 24th March 2017 to 1st April 2017.

There was good response from students. We wish her all the best in the near future.

Thanking you.

Principal
KELAVANI ENGLISH MEDIUM SCHOOL
Baroda.

(2) Sanction Letter for the Final Study

Ph.: (0265) 2410668

Maharani English Medium School

Governed by : KELAVANI TRUST
Dandia Bazar, Vadodara.

INDEX No.: S.S.C.: 65.494
H.S.C.: 15.247 (G)

Outward No.: 1/2017-18
Date : 8/7/2017

To Whom so Ever It May Concern

This is to declare that ***Ms. Sangeeta Pramanik*** *has conducted her Ph.D Research study titled* ***"Developing, Implementing and Assessing an Instructional Package for Higher Order Thinking Skills in Mathematics"*** *in our school '****Maharani English Medium School****'.*

*She conducted her Ph.D Research work on Class IX students in the month of June- July 2017. She took 48 sessions to complete her experiment with the topic '****Real Numbers****'.*

There was good response from the students. We wish her all the best in the near future.

Fernandes

PRINCIPAL
MAHARANI ENGLISH MEDIUM SCHOOL
GOVN. BY THE KELAVANI TRUST, VADODARA

APPENDIX D

LIST OF EXPERTS

1.Dr. Lipika Majumdar

Assistant Professor

School of Engineering and Technology

Navrachana University

Bhayali, Vadodara, Gujarat

2. Dr. Pramila Ramani

Assistant Professor

Central University of Tamil Nadu

Thiruvarur, Tamil Nadu

3. Dr. Shilpa S Popat

Assistant Professor

School of Education

Central University of Gujarat

Gandhinagar, Gujarat

4. Mr. Snehal Soni

Head of Mathematics Department

Navrachana School, Sama

Vadodara, Gujarat

5. Ms. Mary Chako

Physics Faculty, (M.Sc. B.Ed. M.Ed.)

Navrachana International School, Vadodara

6. Ms. G. Sumangla

TGT, M.Sc. (Mathematics) B.Ed.

Secondary Mathematics Teacher

Delhi Public School, Kalali

Vadodara, Gujarat

APPENDIX E

PAPER PUBLICATIONS AND CONFERENCE PRESENTATIONS

(1) Papers Published in Journals

1. Poddar, S. & Sikdar, M. (2020). Cognitivist Lesson Plans: A tool for effective teaching for mathematics teachers. Interwoven: *An Interdisciplinary Journal of Navrachana University, 6*(2), 1-18, ISSN: 2581-9275)
2. Poddar, S. & Sikdar, M. (2019). Instructional strategies for higher level competencies in Mathematics. International Journal of Research and Analytical Reviews, Vol. 6, Issue 2, pp. 129-140, E-ISSN 2348-1269, P-ISSN 2349-5138.
3. Poddar, S. & Sikdar, M. (2019). Scope for higher order thinking through Mathematics instructions. International Organization of Scientific Research – Journal of Humanities and Social Science, Vol. 24, Issue 3,pp. 1-7, E-ISSN 2279-0837, P-ISSN 2279-0845.
4. Sikdar, M. & Poddar, S. (2016). Implementation of continuous and comprehensive evaluation for Mathematics assessment. Indian Journal for Teacher Education, Vol. 1, No. 2, pp. 1-18, P-ISSN 2349-6355, NCTE.

(2) Papers Presented in Conferences

1. Paper titled ‘Higher order thinking for employability through Mathematics instructions’ was presented in ‘National Seminar on Indian Economy: Performance & Prospects’ held on 5th Jan **2019** , organized by Department of Economics, Faculty of Arts, Maharaja Sayajirao University of Baroda.
2. Paper titled ‘Exploration of Mathematics classrooms: Sanctuary for higher-order thinking skills Vs Monotony of rote memorization’ presented in ‘National Conference on ‘Innovating for Development and Sustainability’ held on 5-6 October 2017, organized by Navrachana University, Vadodara.