CHAPTER I

CONCEPTUAL FRAMEWORK

1.0 Introduction

The process of thinking is a fluid mechanism resulting into evolving structures due to unique amalgamations of knowledge, experiences and myriad interpretations (Smith & Kosslyn, 2008). These thinking processes are analogous to the ones described by Bloom within the cognitive domain (Anderson, Krathwohl, & Bloom, 2001); according to which the learner intakes the content in its presented form and acquires knowledge; reflects further to trudge the mind to comprehend it; inquires by applying the content in analogous situations or contexts. This allows the content to get assimilated and accommodated within the mental structures through mental processes described by Jean Piaget. An appropriate schema of the content thus formed in the layers of the mind yields limited short-term productivity. The content must further undergo the critical thought process that is analysis and synthesis to create a new schema or a new content. This new content can be acceptable only if it is evaluated with standard elements. Thus, in order to internalize knowledge, the learner has to undergo certain mental processes which can be termed as thinking or learning (Crawford, n.d.). Thinking cannot be unmeshed from learning, though learning can be considered as an outcome of thinking (King et al., 2017). Learning that occurs as a product of the use of lower order thinking skills is limited only to the prescribed context, whereas the practice of higher order thinking skills generate learning outcomes that can be utilized in variable contexts, for challenging problems and for real life situations (Wheeler & Haertel, 1993). Learning outcomes that result from the practice of higher order thinking skills happen to be the major goal of education (National Curriculum Framework [NCF], 2005). Thus, teaching strategies that encourage the use of these skills among students are required to be designed and practiced. Mathematics is a subject that includes content matter that offers unlimited scope to its learners for higher order thinking (Goethals, 2013). The cognitive processes involved while learning Mathematics is termed as 'mathematical thinking' that pursues clear thinking with perseverance that leads to logical conclusions through logical processes, along with the capacity to handle abstractions (NCF, 2005; [National Council of Teachers of Mathematics [NCTM], 2000). But this mathematical thinking is not always a natural consequence of the formal Mathematics education offered in schools. Infact there is a need to consciously develop and implement such pedagogies that target mathematical thinking. The previous century acknowledged the pedagogical shift in transaction of mathematical content from pure procedural forms to forms that target such thinking. The present century is witnessing the transition with most of the countries adopting the same as their educational goals.

NCF (2005) of India emphasizes on pedagogic processes in Mathematics like - formal problem solving, use of heuristics, estimation and approximation, generalization, visualization, representation, reasoning and proof, making connections, mathematical communication (NCF, 2005, p. 42,43). Most of the countries of U.S.A that follow the NCTM Standards document (*Principles and Standards for School Mathematics*) in their K-12 education (Reys, 2014, p. 37), include Process Standards like - problem solving, reasoning and proof, communication, connections, and representation (NCTM 2000, p. 29). The F-10 Australian Curriculum 2011 includes proficiency strands like - *understanding, fluency, problem-solving* and *reasoning* – that highlight thinking and doing of Mathematics. The Mathematics Curriculum for secondary schools of Malaysia offers Mathematics as a tool to develop higher order problem solving and decision making skills to its students (Ministry of Education of Malaysia, 2004). Mathematics teaching is thus expected to train minds for higher order thinking.

1.1 Higher Order Thinking Skills

In 1987, the *National Research Council* sponsored a project that attempted to synthesize all the many theories about higher order thinking. The express goal of the project was to make recommendation about how to foster higher order thinking skills in students. While lower order thinking skills are 'the skills required for 'mastering facts or completing a task with specific steps', the *National Research Council* study described higher order thinking as "Thinking that is : *Non-Algorithmic* - Involving paths of action for solving problems that are not specified in advance; *Complex* - Involving problem solving where multiple solutions are possible; *Effortful* - Involving considerable mental energy directed toward problem solving; *Nuanced judgments* - Involving subtle, less-than-obvious decisions about strategies; *Application of multiple criteria* - Involving transferal of some (sometimes conflicting) criteria to the problem solving process"[4]. The mental skills required to practice higher order thinking can be termed as higher order thinking skills.

Brookhart (2010), identified definitions of higher order thinking as falling into three categories: (1) those that define higher order thinking in terms of *transfer,* (2) those that define it in terms of *critical thinking,* and (3) those that define it in terms of *problem solving.* The definition in the transfer category is: 'Two of the most important educational goals are to promote retention and to promote transfer (which, when it occurs, indicates meaningful learning) … retention requires that students remember what they have learned, whereas transfer requires students not only to remember but also to make sense of and be able to use what they have learned', according to Anderson and Krathwohl (as cited in Brookhart, 2010, p. 63).

The critical thinking category includes this definition given by Norris and Ennis: 'Critical thinking is reasonable, reflective thinking that is focused on deciding what to believe or do' (as cited in Brookhart, 2010, p. 3).

Another example in this category comes from Barahal (2008), who defines critical thinking as "artful thinking", including the elements of observing, reasoning, investigating, comparing, connecting, describing, finding complexity, and exploring viewpoints.

In the problem-solving category Brookhart (2010) provides the following definition: 'A student incurs a problem when the student wants to reach a specific outcome or goal but does not automatically recognize the proper path or solution to use to reach it. The problem to solve is how to reach the desired goal. Because a student cannot automatically recognize the proper way to reach the desired goal, she must use one or more higher order thinking processes. These thinking processes are called problem solving' by Nitko and Brookhart (as cited in Collins, 2014).

King et. al. (2017) in the publication for *Center for Advancement of Learning and Assessment*, described 'higher order thinking skills' as: 'The skills that include critical, logical, reflective, metacognitive, and creative thinking. One naturally gets indulged in such thinking in the state of uncertainties or while encountering unfamiliar problems or questions. But the pre-requisite of such indulgence is the existence of lower order thinking skills such as the cognitive abilities to remember and recall subject matter related to previous knowledge and the ability of simple application and analysis of the same. Appropriate teaching strategies, conducive learning environments and students' attitude to excel, facilitate the growth of higher order thinking skills in them'.

It is defined as – "The processes – analyze, evaluate and create" by Anderson and Krathwohl (2001). According to Lopez and Whittington (2001), "Higher order thinking occurs when a person takes new information and information stored in memory and interrelates and/or rearranges and extends this information to achieve a purpose or find possible answers in perplexing situation" (as cited in Goethals, 2013). According to Weiss (2003), "Higher order thinking skills are used when one indulges in collaborative, authentic, ill-structured and challenging problem solving" (as cited in Goethals, 2013). "It is a powerful way of thinking about things in the world - logically, analytically, quantitatively, and with precision" (Devlin, 2012).

A more specific description of higher order thinking skills is provided by the Bloom's Taxonomy,1956 (cited in Collins, 2014); according to which, mental skills involving Application, Analysis, Synthesis and Evaluation are higher order cognitive skills. '*Application* is the ability to use information, methods, concepts, theories in familiar situations and solve problems using required skills or knowledge. *Analysis* is the ability to see patterns, organize parts, recognize hidden meaning and identification of components. *Synthesis* is the ability to use old ideas to create new ones, generalize from given facts, relate knowledge from several areas, predict and draw conclusions. *Evaluate* is the ability to compare and discriminate between ideas, make choices based on reasoned argument and verify value of evidence' (Collins, 2014).

These higher level cognitive skills cannot be achieved only by teaching strategies like drill or rote work, repetitive practice, use of incentives, verbal reinforcements, and establishment of rules (Kelly, 2012) which fall under the behaviorist paradigm. Nor can only the cognitivist strategies like classifying or chunking information, linking concepts or new content to previously known ones, providing structure or designing the lesson in efficient and meaningful ways, using real world examples, conducting discussions, problem solving, using analogies, providing visuals, using mnemonics (Kelly, 2012) work. It is the perfect combination of behaviorist, cognitivist and constructivist strategies that can yield the required result of 'development of higher order thinking skills'.

The following section highlights on some specific constructivist instructional strategies that can be used in Mathematics classrooms to refurbish the afore-said goal.

1.2 Instructional Strategies to Develop Higher Order Thinking Skills in Mathematics

Teaching-learning process in Mathematics is basically made up of three phases. First is the teaching phase, wherein the teacher uses different strategies to develop conceptual knowledge and procedural knowledge with respect to the specific topic. The second phase is the practice and exploration phase for students: *practice*, to attain basic procedural and computational efficiency and *exploration*, to attain mathematical thinking. The third phase is the assessment or the evaluation phase to check student abilities to recall, generalize or transfer what they have learned (Dixon et al., 1998). Feedback and remedial may follow to maximize efficiency.

1.2.1 Strategies for the teaching phase

Higher order thinking skills (HOTS) are grounded in lower order thinking skills (King et al., 2017), which makes in-depth understanding as well as basic computational (algorithmic and calculation) skills to be an essential pre-requisite to develop HOTS.

Educators and policy makers, thus place increased emphasis on teaching the realistic conceptual background of the content-matter in parallel to the respective mathematical background to develop conceptual knowledge in conjunction to the required procedural knowledge (NCTM, 2000; Lawson, 2007; Protheroe, 2007). While procedural knowledge thrives on specific, clear, logically connected explanations of algorithms and calculations using behaviourist strategies, access to conceptual knowledge would require more realistic approaches. In Psychology, the term 'Concept' is defined as "a generalized idea about a thing, person or event" and the process of 'Concept formation' is carried out through the processes of perception, abstraction and generalization – where 'perception' is formation of a mental image of the 'topic'; abstraction is the observing of similarities with the 'topic'; and after going through abstractions a number of times generalizing the common properties and coming down to the general idea about the respective topic (Mangal, 2009). 'Concept formation', 'Conceptual knowledge' or 'Conceptual understanding' carries the same meaning in the context of this Study. According to National Research Council (2001) and the NCTM (n.d.), "Students demonstrate conceptual understanding in Mathematics when they provide evidence that they can recognize, label, and generate examples of concepts; use and interrelate models, diagrams, manipulatives, and varied representations of concepts; compare, contrast, and integrate related concepts and principles; recognize, interpret and apply the signs, symbols and terms used to represent concepts."

1.2.1.1 Cognitivist strategies to develop conceptual understanding of a concept

Thomas and Thome, 2009 (as cited in Collins, 2014) suggested a multi-step process for teaching learning concepts, which includes naming critical, additional, false features of the concept; comparing the new to the already known concept; giving best examples and non-examples; and identifying other similar or connected concepts, classifying or chunking information, using real world examples, using analogies, conducting discussions, and providing visuals (Kelly, 2012; Rittle-Johnson & Schnieder, 2014; Bruner, 1967 cited in Mangal, 2009; Marzano, 1998). The strategy of providing the contexts and then varying the contexts in which students can use a newly taught skill is also effective in developing conceptual understanding. Hines, Cruickshank and Kennedy; and Snyder et. al. (all cited in Kauchak & Eggen, 1998) suggested that teachers should provide clear and specific instructions, taking care that no ambiguity or confusion may arise among students in order to improve students' attitude towards thinking tasks. Mathematics teachers can transact concept clarity by 'helping students construct a deep understanding of mathematical ideas and processes by engaging them in doing Mathematics: creating, conjecturing, exploring, testing, and verifying' (Lester, 1994). Few more strategies that can be incorporated with instruction to enhance conceptual understanding and encourage mathematical thinking among students with an underlying motive to develop HOTS are given below.

- *Making mathematical connections.*

NCTM (2000, p. 64) defines mathematical connections in *Principles and Standards for School Mathematics* as the ability to “recognize and use connections among mathematical ideas; understand how mathematical ideas interconnect and build on one another to produce a coherent whole; recognize and apply Mathematics in contexts outside of Mathematics” (Pitkaniemi & Hakkinen, 2012). According to Perry and Docket (2008), Mathematical connection means, the connection that exists (1) between the new concept with the previously learnt concepts; (2) within and between different content areas in Mathematics; (3) between Mathematics and the real-life scenarios of the child (as cited in Ndiung & Nendi, 2018). The study of Ndiung and Nendi (2018) proved the fact that there is a significant effect of students' mathematical connection ability to their mathematical achievement. *Principles and Standards for School Mathematics*, NCTM (2000) states that students with the ability to connect mathematical ideas have deeper and a more lasting understanding, which help them further to comprehend the connections across mathematical topics; relate Mathematics to other subjects and learn to utilize Mathematics to solve problems even in contexts outside Mathematics. Thus examining and discussing mathematical connections need to be a regular classroom experience of students.

Position Paper National Focus Group on Teaching of Mathematics (2006) is the document that provides curricular guidelines for teaching of Mathematics in India. It emphasizes the inclusion of pedagogical processes that focus on mathematical connection. This needs carefully designed lessons that help students to see connections among content areas, among mathematical processes and within their own thinking. It is the teachers' responsibility to guide students towards this proficiency by continuously looking out for additional connections to be made explicit in their instructions. Thus, it is important that teacher should be well acquainted with the relationships between different mathematical ideas across all levels to design instructions, tasks, and examples that facilitate student learning through connections.

Concept maps are the best way to show the relationship that exists between the new concept in-hand to the variable known and unknown contexts. It helps in creating mental readiness for the new knowledge and appropriate hooks to see its relevance across the Chapter, subject, syllabus, course and real applications (Hafiz et. al., 2017). Concept Maps can aid teachers to transmit information about the specific concept with respect to other known and unknown concepts; and students to construct an understanding of the specific concept (Baroody & Bartels, 2000, p. 606). These kinds of visual aids along with teacher-instructions that emphasize the relatedness of mathematical ideas help students to learn and more importantly use Mathematics (NCTM, 2000, p. 64; Monila, 2012; Collins, 2014; Tout, et al., 2015 cited in Corovic, n.d.). Bartels (1995) suggested hand-outs or review sheets given to students to help them see connections among concepts. Thus, at times the connections can be explicitly shown to students, while at other times, opportunities are to be given to them to indulge in and discover the connections.

1.2.1.2 Constructivist strategies to develop content-specific higher order thinking skills.

Constructivism is becoming the dominant learning theory in education, as per Wittrock (as cited in Marzano, 1993). According to Clark and Clark, the constructivist theory states that, "meaning is *constructed* by the learner via the interaction of *new* information with *old* information existing in long term memory" (as cited in Marzano, 1993). Becker and Varelas stated, constructivist strategies used to teach a particular concept would focus on methods that would emphasize students to construct the understanding of the concept based on their own experiences, with teacher playing the role of a facilitator providing appropriate situation, tasks, and conditions (as cited in Nagappan, 2002). Constructivism basically evolves from Piaget's theory which states that, "each time one prematurely teaches a child something he could have discovered himself, the child is kept from inventing it and consequently from understanding it completely (Nagappan, 2002). Some of the constructivist techniques which were more on lines of 'guided discovery approaches' were used in the present Study are explained further.

- *Questioning and probing skill.*

NCTM (1991) suggests that teachers should orchestrate the teaching of Mathematics by "posing questions and tasks that elicit, engage, and challenge each student's thinking" (Crowl et al., 1997). Presentation of unfamiliar questions, probing for better understanding, encouraging creativity, stimulating critical thinking and enhancing self-confidence (Tofade et al., 2013) are some tough measures to be practiced by teachers regularly. Phrasing questions that align to Bloom's taxonomy higher-challenge areas that address analysis, synthesis and evaluation can spur higher order thinking (Ged Gast, p. 3). Question cues used decisively based on respective cognitive functions can be used during and after instructions to elicit higher order thinking in students.

‘Focused questioning’ is another strategy that can be used when students struggle to answer more complex questioning. Here teacher can frame Focus-questions that lead student through the steps of thinking (Ged Gast, p. 3). Higher-order questions are often more open-ended encouraging students to think beyond textbook-based structured answers (Barwell, 2011; Hoffman & Brahier, 2008; Suurtamm et al., 2015). Thinking skills of students can be elevated by “asking students to clarify and justify their ideas orally and in writing” (NCTM, 1991; Bautista, n.d.). Questions that target mathematical thinking needs pre-planning with conscious and targeted teacher efforts (King et al., 2017; Ahmad and Platero cited in Dowden, 2014; Bautista, n.d.). Thus, it is important that lesson plans are made meticulously with each verbal and written set of questions articulated with the envisaged learning and thinking goals.

- *Generalization techniques.*

Sriraman (2004) defines Generalization as ‘the process by which one drives or induces from particular cases’. Mason (1996) states that he process of generalization is considered to be one of the most powerful thinking processes (as cited in Dutrascu, 2017). The mental processes that are working while generalizing are analysis, synthesis and abstraction (Rubinshtein, 1994). ‘Analysis’ is defined by Krathwohl (2002) as the process of breaking of material into its components and identify how they relate to one another and to the overall structure or purpose; while according to Davydov (1990) ‘Analysis’ is the method or logical technique by which objects are represented by observed common attributes, these attributes are identified by operations of delineating similar or identical characteristics of given objects.

‘Synthesis’ according to Krathwohl (2002) is putting the elements together to form a novel, coherent whole or to make an original product; and according to Davydov (1990), it is the method or logical technique that uses the attributes observed through analysis to create a new system. Dubinsky (1991) described the process of ‘Abstraction’ to happen in three levels after the process of Synthesis. They are: (1) Empirical abstraction, which means deriving knowledge from the external properties of the given data; (2) Pseudo-empirical abstraction, which consists of deriving new properties by transforming the initial data; (3) Reflective abstraction, which involves drawing properties from mental or physical actions and project these actions at a higher level of thought. Mason (1999, p. 9) explains generalization as a process of noticing patterns or properties similar in different situations. Seeing patterns is an activity that requires the analyzing and evaluating skill of selecting the important component and ignoring the unwanted ones across situations and decide on the use of mathematical forms to come down to a conclusion-which involves synthesis skills (Goodwin, 1994; Latour, 1987; Verran 2001).

Krutetskii (1976) discovered two ways in which school children can learn Mathematics through generalization. The first method is called Empirical generalization: It is a gradual process of analyzing a series of concrete examples in which the non-essential attributes are systematically changed. This method can be effectively used for students for average learners, helping them to understand general mathematical knowledge. The second method called Theoretical generalization is effective for the more successful students. It involves generalizing a solution from just a single example by identifying the internal connections and relationships involved in the task. This extends to generalizing of methods and approaches to reach to solutions rather than just generalizing for a particular case.

Mason (1996) identified the need of teachers to be aware of the importance of helping students to generalize mathematical ideas to develop mathematical thinking. Teachers role include presenting appropriate examples that can help students to generalize (Zakis et al., 2007); encourage students to explain and justify their mathematical claims and revoice student contributions to draw out similarities across findings (Leher et al., 2003); and offer enough opportunities to students to generalize during classroom interactions (Zakis et al., 2007). Teaching strategies would need to engage students into inductive reasoning i.e. to observe or work with given set of data, analyse it in the process and identify the pattern or the relationship that exists within the components and synthesize them to infer a mathematical rule, property, law, formula or definition (Sriraman, 2004). Ley (2016) indicated that teaching through generalization may not need entirely new methods of instruction. He proposed a blend of 'discovery learning' and 'direct instruction' used appropriately to promote generalization (Alfieri et al., 2011). 'Discovery learning' indicates figuring out something oneself which requires deep and active mental processing helping learners to explore and invent novel solutions (McDaniel & Schlager, 1990). 'Direct Instruction' approaches promote generalization by efficiently helping learners to build deep domain-specific knowledge and to focus on core concepts (Ley, 2016).

- *Estimation techniques.*

According to Smart (1982), "*to estimate* means to form an approximate opinion of size, amount, or number that is sufficiently exact for a specified purpose". It is one of the basic and useful parts of Mathematics that helps one to connect the subject to real life (Berry et al., 2014). There has been an increased emphasis recently on conceptualization of numbers, according to Booth and Siegler (2006), which requires estimation skills. Numeric estimations require a deep understanding of the value of numbers, mental thinking and computation, mathematical operations and contextual verification (Cochran et al., 2013; Van de Walle and Folk, 2005 cited in Berry et al., 2014). Thus, Estimation involves making a judgement based on very general consideration, but demanding cognitive abilities of analysis, synthesis and evaluation; thus making it a higher-level thinking skill (Van de Walle and Folk, 2005 cited in Berry, et al., 2014). Poyla's (1945) problem-solving model with the "Looking Back" approach indicates judging the reasonableness of the answer obtained- a function of estimation. Mathematics does offer innumerable scope to learners to exercise their estimation skills, but it needs specially designed instructions to identify the occasions, tasks, concepts that are needed to be manipulated for that purpose (Leutzinger et al., 1986).

Different instructional strategies that can be used by teachers during their classroom instructions to facilitate estimating skills are:

- Posing a problem with known backgrounds and encouraging students to guess the answer (numeric value/ shape/ distance/ position/ etc.).
- Probing further to modify the guess to a reasoned guess. This can be done by having students estimate an unknown quantity by either comparing it to a known quantity or partitioning it into known quantities or by using mental computation (Leutzinger et al., 1986).
- Strengthening estimation skills by having students verbalize the thinking they used to derive their answers.
- Finally conducting task-based verification by either using algorithmic or experimental procedures.
- Apart from hypothesising and validating the reasoned estimate, making quick informal estimates, 'rule of thumbs' can be generously used during content delivery. 'Rule of thumb' operations mean applying mental hooks to a situation (Mitchell et al., 1999).

    - *Visualization techniques.*

Visualization in Mathematics can be described in three distinct ways: (1) Visualization Object - it involves interpretation of physical objects like illustrations, animations, computer-generated displays etc. with the purpose to understand a mathematical idea (2) Introspective Visualization - it is an imaginative construction of some visual experience without a visualization object. Mental objects picturized in the mind. (3) Interpretive Visualization involves interpretation of meaning from visualization objects or introspective visualizations, which are cognitive functions (Philips et al., 2010, p. 26). These distinctions are important to understand for establishing effective applications of visualization in the Mathematics classroom

(Macnab et al., 2012). Guzman (n. d.) describes visualization as a process to attend to the possible concrete representations of the objects to understand the abstract relationships underlying it. These processes as indicated above lead cognitions towards new discovery, which is a function of higher order thinking.

‘Visualizing means summoning up a mental image of the content in hand to understand it better. The image may be of some geometrical shape, or of a graph or diagram, or it may be some set of symbols or some procedure’ (The Open University, 1988, p.10). The basic ideas of Mathematics – order, distance, operations with numbers etc. – are born from concrete or visualizable situations. Even while dealing with abstract ideas, mathematicians need to explore the corresponding concrete ideas. This exploration can be termed as mathematical visualization. Majority of the Mathematics content at the secondary and higher secondary levels need students to carry out Isomorphic visualization. In which, the process begins at a concrete level, moving gradually toward the respective abstract connections and further towards generalizing and manipulating the abstract dimensions (Guzman, n.d.).

Teachers generally can make use of as many visual aids as possible while content transaction, for which appropriate diagrams, pictorial forms, graphs, power point presentations, videos etc. can be used, trying to establish a perfect concrete visual schema. This solid background then has to be explored further to probe and lead students to carry out mental functions of analysis, synthesis and evaluation and come out with interpretations and conjectures. To make the task more complex, students further can be lead to discover relationships or conjectures from mental images.

Piggott and Woodham (2011) put forth some strategies to use visualization during problem solving:

1. Visualizing to step into the problem: The problem needs to be visualized in the first phase to get a real or a deeper understanding, so getting a clear mental image of the situation helps in further generalization.
2. Visualising to model a situation: This is useful in case of situations that are abstract or physically unattainable, like considering a case involving a very large number.
3. Visualising to plan ahead: This involves using visualising during the problem-solving process to anticipate. In other words, asking yourself: 'What will be the consequence if I do this?'.

1.2.2 Strategies for practice/exploration and assessment phase.

Certain mathematical formulae, symbols, properties, definitions, procedures, algorithms form basis for individual explorations or problem solving. The practice phase should offer students with enough opportunities to attain those basic competencies. Most of the Mathematics classes often focus on drills and procedural understanding (Sinay & Nuhornick, 2016) and hardly offer time for problem solving. More scope for problem solving, through which students can be given opportunities to develop conceptual understanding and to connect mathematical ideas; and those which allow students to come up with original thoughts (Poyla, 1945) help to develop higher order thinking.

Pegg (2010) indicated that, “for the successful development of higher order thinking skills, activities of instruction and assessment need to be closely intertwined.” Questions or tasks, though unfamiliar in terms of contexts but familiar in terms of subject-matter knowledge are needed to be posed. Blooms taxonomy provides a proper structure to meet this requirement. It can be used to design questions for the different levels: Comprehension, Application, Analysis, Synthesis, and Evaluation based on the set of competencies (Bloom, 1956). Students need a greater and a frequent exposure to questions that ensure responses allowing them to exercise their higher order thinking skills. Thus, Formative and Summative assessments need to be designed to help students shape their learning that goes beyond textual algorithms towards Bloom’s higher levels creating mental readiness for more analysis, synthesis and evaluation level tasks.

1.2.3 Classroom environment for mathematics teaching-learning.

One more important pre-requisite for ensuring that students cognitively slog to attain knowledge of higher levels, is to maintain a conducive psychological climate in the classroom. A climate in which students are in a state of mental well-being: are self-confident, motivated, interested and mentally charged up to enjoy Mathematics. While it is widely accepted that learning in general is inherently emotional and affective, there seems to be something about Mathematics that commonly sees it as less interesting and less enjoyable than other school subjects (Radisic et al., 2014). This is particularly concerning, and if understanding of mathematical learning is to be advanced, then it is crucial that both cognitive and affective factors be explored in an integrated and orchestrated manner (Leder & Forgasz, 2002). Also there is a positive correlation between students’ understanding and the teacher’s knowledge of students’ thinking and solution strategies when working on mathematical problems (Vacc & Bright, 1999). Thus, teachers need to consciously understand and learn how their students think in Mathematics classrooms, reflect on those experiences and take it ahead to refine their teaching. Teacher training institutions as well as in-service teacher training programs needs to cater to this requirement.

Thus, Teacher innovativeness and creativity (Hargreaves, 2003), expertise in content, implementation of appropriate pedagogical practices (Fullan, 2014), knowledge of students' mathematical thinking process, and application of good assessment methods (Chai et al., 2011) are pre-requisites to inculcate higher order thinking skills in students. Instructions in the classroom need to be well planned and structured around the parameters that have proved to be successful in promoting higher order thinking skills in students and an affective climate need to be maintained. The onus lies on the teachers, who can be considered as the major medium to transact the concrete policies, curricula and knowledge forms into palpable fluid forms palatable for the students.

1.3 Goals of Mathematics as Envisaged by the Indian Education System

The Position Paper National Focus Group on Teaching of Mathematics (2005) begins with the statement, "The main goal of Mathematics education in schools is the mathematisation of the child's thinking….". The kind of thinking that is expected to develop through Mathematics learning is the ability to deal with abstraction and to solve problems. Mathematics at the secondary stage differs drastically in its ideologies with respect to the primary stages. Secondary level Mathematics primarily is an application of all the basic concepts learnt at the prior school levels and is purer and theoretical in nature. Concepts are complex and belong more to the abstract unseen paradigm, challenging the cognitive structures of students. They are expected to achieve mathematical competence way beyond computational and procedural ways of problem solving; towards discovering new solution approaches; generalizing known and unknown mathematical facts, using mental skills of visualizing and estimating to unravel facts, solutions, and approaches; seeing several mathematical connections within the subject and the real world; unraveling the concept behind mathematical representations; and getting a strong hold on the mathematical language.

The secondary stage in school education is demarcated as classes IX-X comprising of children ranging from ages around 14 to 15 years. This stage is characterized by Jean Piaget's 'Formal Operational Stage'. At this point, the person is capable of hypothetical & deductive reasoning and is able to think about abstract concepts which are often required in science and Mathematics. Metacognition and problem-solving skills also develop during this phase, although all these abilities vary from person to person.

Some of the broad objectives of teaching Mathematics at these stages are (Central Board of Secondary Education [CBSE], 2014) are to help learners to:

- Acquire knowledge and understanding by way of motivation and visualization of basic concepts, terms, principles, symbols and underlying processes and skills.

- Feel the flow of reasons while proving a result or solving a problem.
- Apply the knowledge and skills acquired to solve problems and wherever possible, by more than one method.
- To develop positive ability to think, analyse and articulate logically.
- To develop interest in Mathematics as a problem-solving tool in various fields for its beautiful structures and patterns.

Thus the pedagogic processes need to contain multi-dimensional strands – some directed to develop computational skills and procedural skills and most of them directed to develop mathematical thinking skills.

1.4 The Mathematical Unit of Numbering System: Real Numbers

The first unit in class IX (CBSE and Gujarat Secondary and Higher Secondary Education Board [GSHSEB]) Mathematics is Real Numbers within the category of Numbering system. It is an important unit encompassing of the previously learnt concepts of Whole numbers, Integers and Rational numbers with the comprehensive understanding of a number of concepts related to Irrational numbers. The different sub-units included in the unit are as follows:

1. Review of representation of natural numbers, Integers, Rational numbers on the number line. Representation of terminating / non-terminating recurring decimals, on the number line through successive magnification. Rational numbers as recurring/terminating decimals.
2. Examples of non-recurring / non-terminating decimals. Existence of non-Rational numbers (Irrational numbers) such as $\sqrt{2}$, $\sqrt{3}$ and their representation on the number line. Explaining that every real number is represented by a unique point on the number line and conversely, every point on the number line represents a unique real number.
3. Existence of $\sqrt{x}$ for a given positive real number x (visual proof to be emphasized).
4. Definition of nth root of a real number.
5. Recall of laws of exponents with integral powers. Rational exponents with positive real bases (to be done by particular cases, allowing learner to arrive at the general laws.)
6. Rationalization (with precise meaning) of real numbers of the type $1/(a+b\sqrt{x})$ and $1/(\sqrt{x}+\sqrt{y})$ (and their combinations) where x and y are natural number and a and b are Integers.

The sub-topics reveal the abstract nature of the concepts. All the other units in class IX align to the same structure. The analysis of the National Council of Education Research and Training [NCERT] Textbook (followed presently in GSHSEB) of the geometrical portion reveals that 'the instructional content includes opportunities of higher order thinking in terms of hands-on-applications, conceptual knowledge and problem solving'; whereas the instructional content included in the textbook for the unit 'Real Numbers' have very limited scope for the same. The sub-units have gaps in logical sequencing and emphasize more on procedural knowledge rather than conceptual knowledge- which is the base for higher order thinking.

The studies conducted by Voskoglou and Kosyvas (2012), Yilmaz and Ay (2018), Merenluoto and Lehtinen (2002) elaborate on the several difficulties faced by students while dealing with this unit of Real Numbers. Students face problem in comprehending the multiple representations of several numbers in different numbering systems; incomplete understanding of Rational numbers causing impediment in internalizing the concept of Irrational numbers; problems in visualizing the abstract concept of density of real numbers in given intervals; geometric representations of Irrational numbers; and the mathematical operations on the square root and the exponential forms of Irrational numbers. Belin and Akar (2017) proved that special training needs to be given to prospective teachers to develop their understanding on Real numbers and design pedagogies to justify the same.

The conceptual framework thus created attempts to furnish a relevant backdrop for the present Study. It provides the base to make the following conclusions.

1.5 Need of the Study

Higher order thinking skills are cognitive skills, which have to be transacted to our children not because they are current global trends, but because they are the ultimate aims of Mathematics education. Moreover, twenty-first century, attributed for the advent of the technological revolution, naturally demands human capitol with higher order thinking skills. These cognitive skills are ingrained in the subject of Mathematics, but can be acquired only if teaching-learning procedures are specifically designed to cater to the mentioned aim. The National Education Policy [NEP] recommends to "encourage rational, logical, analytical and quantitative thinking in all aspects of the curriculum" (NEP, 2019, p. 88). It also recommends that 'assessments' need to be based on testing the "understanding of core concepts and higher order capacities" (NEP, 2019, p. 104).

Mathematics teaching in Indian classrooms has been widely acknowledged to be devoid of the afore-mentioned qualities. In most of the cases procedural and algorithmic processes dominate the teaching and learning. In very few cases, where some efforts are seen, rely upon activity-based teaching to align to the idea of quality education. Unfortunately, this method can result to be more disastrous if the concrete or real life understanding gained through the 'activity' is not skillfully and effectively connected to the mathematical representations. Moreover, schools are bombarded with new progressive pedagogies like 'constructivism', 'multiple intelligence plans', 'flipped classroom' and other such models which are tough for teachers to incorporate for all topics. Ultimately, they are left out more confused.

Mathematics teachers of 21st century are aware of the importance of teaching higher order thinking skills. However, according to the study of McMillan et.al., when teachers are surveyed about how often they think they assess application, reasoning, and higher order thinking, both elementary and secondary teachers claim they assess these cognitive levels quite a bit, but in fact they do not (as cited in Brookhart, 2010). They state of using technology; conduct activities; give projects; conduct group-work etc. as expected from them; but what they need to understand is the fact that highest quality of education does not mean providing education with expensive gadgets, world famous technologies or best physical infrastructure but to impart pure knowledge in forms absorbable by the students, with proper connections and logical sequencing, enriched with different dimensions and most importantly in forms catering to different cognitive levels and learning styles of students.

Most of the pedagogies mentioned in the NCF (2005) for transacting Mathematics contents (use of heuristics, estimation and approximation, generalization, visualization, representation, reasoning & proof, making connections, mathematical communication) are hardly being practiced in the Mathematics classrooms. Instructions, especially in middle and secondary classes still align to the traditional mechanically structured forms with no efforts to make the mathematical and the real meanings of the concepts explicit. This hinders the acquisition of conceptual knowledge among students which is a pre-requisite to solve problems that require higher order thinking. Moreover, opportunities on regular basis needs to be given to students in the classroom to indulge in mathematical thinking; to discover mathematical properties and processes; to explore the concept; to discover several relationships and connections; to strive to achieve in-depth understanding of concepts; to see patterns and generalize; and to evaluate with correct reasoning and justify.

Mathematics teaching inside the Indian classrooms need to be refocused towards two main aims – use of appropriate strategies to develop conceptual knowledge and designing approaches to encourage students for mathematical thinking. There are inadequacies among Mathematics teachers to develop and deliver instructions with the aforesaid focus.

Thus, in the present Study the investigator raises a few research questions and seeks to address them through the present Study:

- How can Instructional Plans be created that have the right blend of 'Constructivist strategies' like generalization, visualization, estimation, higher order questioning, establishing mathematical connections and 'Cognitivist strategies' to teach Mathematics topics at school level?
- What is the possibility that such Instructional Plans be effectively implemented in Mathematics classrooms of India?
- To what extent are such Instructional Plans effective in the development of higher order thinking skills?
- How effective are such Instructional Plans in increasing the Mathematics achievement among students?
- How do students respond to such Instructional Plans being used in their classrooms?
- How can the classroom instructions in Mathematics be made efficient enough to lead the students to higher levels of thinking and help them address HOTS questions efficiently?
- How can the Mathematics teachers be better equipped to disseminate good quality teaching in the classroom?

In order to address these questions, the investigator pursued the present Study which is titled:

Statement of the Study

Developing, Implementing and Assessing an Instructional Package for Higher Order Thinking Skills in Mathematics

1.6 Objectives of the Study

1. To develop an Instructional Package on the content 'Real Numbers' in Mathematics for class IX students.
2. To implement the Instructional Package on class IX students.
3. To study the effectiveness of the developed Instructional Package over the Conventional method of teaching on the acquisition of higher order thinking skills in the content 'Real Numbers' in class IX students.

3.1 To study the effectiveness of the developed Instructional Package over the Conventional method of teaching on the acquisition of Higher level competencies in the content ‘Real Numbers’.

3.2 To study the effectiveness of the developed Instructional Package over the Conventional method of teaching on the acquisition of Basic level competencies in the content ‘Real Numbers’.

3.3 To study the effectiveness of the developed Instructional Package over the Conventional method of teaching in terms of the Mean Achievement scores for HOTS questions at specific levels - Comprehension, Application, Analysis, Synthesis and Evaluation in the content ‘Real Numbers’.

3.4 To study the effectiveness of the developed Instructional Package over the Conventional method of teaching in terms of the Mean Achievement scores for HOTS questions including all levels in the content ‘Real Numbers’.

4. To study the reaction of students on the developed Instructional Package and its implementation.

1.7 Hypotheses of the Study

The hypotheses constructed for the Study are as follows.

1. There is no significant difference between the Mean Achievement scores of the class IX students exposed to the Instructional package over the ones exposed to the Conventional method of teaching for HOTS questions at the Comprehension level in the content ‘Real Numbers’.
2. There is no significant difference between the Mean Achievement scores of the class IX students exposed to the Instructional package over the ones exposed to the Conventional method of teaching for HOTS questions at the Application level in the content ‘Real Numbers’.
3. There is no significant difference between the Mean Achievement scores of the class IX students exposed to the Instructional package over the ones exposed to the Conventional method of teaching for HOTS questions at the Analysis level in the content ‘Real Numbers’.
4. There is no significant difference between the Mean Achievement scores of the class IX students exposed to the Instructional package over the ones exposed to the Conventional method of teaching for HOTS questions at the Synthesis level in the content ‘Real Numbers’.

5. There is no significant difference between the Mean Achievement scores of the class IX students exposed to the Instructional package over the ones exposed to the Conventional method of teaching for HOTS questions at the Evaluation level in the content ‘Real Numbers’.
6. There is no significant difference between the Mean Achievement Scores of the students exposed to the Instructional Package over the ones exposed to the Conventional Method of teaching for HOTS questions including all levels in the content ‘Real Numbers’.

1.8 Explanation of Key Terms

- **Effectiveness**

For the present Study, ‘Effectiveness’ is the degree to which the developed Instructional Package is successful in developing higher order thinking skills in students who were exposed to the Package over the ones exposed to the Conventional method of teaching for the content ‘Real Numbers’ in Mathematics.

- **Instructional package**

For the present Study, the Instructional Package refers to a systematic instructional design involving effective teaching strategies and assessment procedures. It will include Student Learning Materials, Worksheets, Practice sheets, Evaluations with HOTS questions; Content- Chart, Lesson plans, Power-point presentations, Scoring criteria and Rubrics – all systematically structured for the selected Mathematics content ‘Real Numbers’ of class IX.

- **Effective teaching strategies**

The teaching strategies that have been used in developing the Lesson Plans implemented as the research treatment, expected to cause effective learning in students were:

- Cognitivist Teaching Strategies
- Use of Mathematical Connections
- Use of Questioning and Probing skill
- Use of Generalization techniques
- Use of Estimation techniques
- Use of Visualization techniques

- **Conventional method of teaching**

Conventional method of teaching Mathematics specifically indicates teacher-centred teaching with dominance of the Lecture method. This method of teaching is commonly used in most of the schools and is characterized by the following features:

- Content is limited to the text books.

- Role of the teacher is to teach algorithms by providing clear, step-by step demonstrations of each procedure, recapitulating the same, providing adequate opportunities to students to practice the procedures, and offering specific corrective measures when necessary (Smith, 1996)
- The procedures to all mathematical problems are known, contexts are not changed in practice work and in assessments.
- Students are expected to memorize facts, follow rules, execute procedures, and plug in formulas (Hiebert, 2003).

- **Higher order thinking skills in Mathematics**

Higher order thinking skills as defined in Bloom's Taxonomy (2001) is used for the present Study. The below mentioned cognitive skills are evaluated for the content 'Real Numbers', Class IX Mathematics (GSHSEB, CBSE).

- *Comprehension* is the ability to understanding information; grasp meaning; interpret facts; compare and contrast; order, group, infer causes for (though this is not considered as a higher order thinking skill according to Bloom's taxonomy, 'comprehension' is the basic skill required to go ahead and exercise the other higher order thinking skills; thus this skill has been given attention in the present Study).
- *Application* is the ability to use information, methods, concepts, theories in familiar situations and solve problems using required skills or knowledge.
- *Analysis* is the ability to see patterns, organize parts, recognize hidden meaning and identification of components.
- *Synthesis* is the ability to use old ideas to create new ones, generalize from given facts, relate knowledge from several areas, predict and draw conclusions.
- *Evaluate* is the ability to compare and discriminate between ideas, make choices based on reasoned argument and verify value of evidence (Collins, 2014).

The assessment questions were devised to evaluate the above skills in the present Study. Higher order thinking skills considered here was the total score obtained by the students on comprehension, application, analysis, synthesis and evaluation level questions in the Posttest on the content 'Real Numbers' developed by the investigator.

- **Basic level competencies**

- Competency is a set of defined behaviours or skills that provide a structured guide enabling the identification, evaluation and development of the behaviours in students. In the present research the term 'Basic Level Competencies' refers to cognitive skills of:

- identification and application of concepts, theories and rules in known contexts;
- calculations (application of mathematical operations); and
- algorithmic procedure used in a mathematical problem from the content 'Real Numbers' at Class IX level.

- **Higher level competencies**

For the present Study, 'Higher Level Competencies' refers to the cognitive skills like:

- comprehension of information, grasping of meaning, interpretation of facts, compare, contrast, order, group (in case of Comprehension level questions).
- use of information, methods, concepts, or theories in new situations or unknown contexts to solve problems or make inferences (in case of Application level questions).
- identification of components, organisation of the components, recognition of hidden meaning to solve problem (in case of Analysis level questions).
- use of old ideas to create new ones, generalise from given facts, relate knowledge from several areas, and draw conclusions (in case of Synthesis level questions).
- comparison and discrimination between ideas, making choices based on reasoned argument and verification of value (in case of Evaluation level question).

- **HOTS questions**

HOTS is the abbreviation used for Higher order thinking skills and HOTS questions are those questions that focus on thinking skills measuring students' abilities to reason, justify, analyze, process and evaluate information besides testing understanding of information. These questions seek answers that go beyond the textbooks, widening the horizons of students. The responses for these questions need students to undergo mental skills of comprehension, application, analysis, synthesis and evaluation.

1.9 Operational Definition of Key Terms

- **Achievement scores for different level questions:**

- *Comprehension:* Marks obtained by students for their ability to understand information; grasp meaning; interpret facts; compare and contrast; order, group, infer causes
- *Application:* Marks obtained by students for their ability to use information, methods, concepts, theories in familiar situations and solve problems using required skills or knowledge.
- *Analysis:* Marks obtained by students for their ability to see patterns, organize parts, recognize hidden meaning and identification of components.

- *Synthesis:* Marks obtained by students for their ability to use old ideas to create new ones, generalize from given facts, relate knowledge from several areas, predict and draw conclusions.
- *Evaluate:* Marks obtained by students for their ability to compare and discriminate between ideas, make choices based on reasoned argument and verify value of evidence (Bloom' Taxonomy 200, as cited in Collins 2012).

1.10 Scope and Limitations of the Study

The present Study explores a complex concept - thinking skills - which are difficult to cater to in terms of development as well as measurement. Mathematics is the most efficient tool to develop higher order thinking skills, but the way it is promoted allows its intake only as a subject of formulae, definitions, properties and algorithms. This Study thus presents Mathematics with gears shifted towards conceptual understanding through guided discovery approaches and explores strategies mentioned in NCF 2005 (generalization, visualization, estimation, mathematical connections, higher order questioning) which have not been comprehensively taken up by any other researcher so far. The present Study has the scope to equip teachers to develop plans that offer opportunities to students for higher order thinking.

In the present Study, best efforts were made by the investigator to avoid errors while conducting the experiment and while sampling so that the outcome of the Study is reliable and valid. But there are always certain systemic and human limitations that are encountered during investigations in the field of education. In order to have samples belonging to similar classroom environment, socio-economic status, backgrounds and academic history, the Control and the Experimental groups were chosen from the same school. Due to the rigorous nature of the Study, only one Chapter 'Real Numbers' which included a large number of sub-concepts, was taken to design the Instructional Package.

The Study was restricted only to IXth class students of the GSHSEB (State Board). Higher order thinking processes of students were noted from their assessment sheets and worksheets given by the investigator and regular teacher's diary was maintained, as step by step solving of problems and justifications mentioned within were considered to be indicators of inculcation of HOTS.

Thus, video recordings and other methods were avoided. The investigator took all possible measures to overcome the limitations to make the result more generalized and acceptable to the education system.

1.11 Organisation of the Report

The report is organized under the following Chapters.

Chapter 1

This Chapter consisted of an Introduction to the Study, Elaboration of the different teaching strategies used in the Instructional Package, Need of the Study, Research questions, Statement of the problem, Objectives and Hypotheses of the Study, Explanation and Operationalization of key terms, Scope and limitations of the Study and the Organization of the report.

Chapter 2

This Chapter includes a collection of literature reviewed for the present Study. It includes Studies reflecting the present scenario of Mathematics education, Student characteristics imperative for higher mathematical achievement, Effective strategies used to enhance mathematical ability – achievement and higher order thinking skills, Importance of guided instructions in Mathematics, Teacher-related difficulties in using effective teaching strategies in Mathematics, and Difficulties involved in teaching and learning of 'Real Numbers'. Implication of each for the present Study is detailed respectively and an overall Implication is presented in form of the Rationale of the Study.

Chapter 3

This Chapter details out the Methodology used in the research in three phases. Phase 1 describes the Development of the Instructional Package including the teaching strategies used, Criteria for selection of the subject matter, Original and the Re-structured concepts, Design of the Lesson plans and the Worksheets. It also details out the Methods used to construct the Achievement tests and the Tools used in the Study. Phase 2 describes briefly the procedures used in the Initial try-out and the Final implementation of the Package in terms of sampling and the methodology adopted to conduct the research. Phase 3 detailed out the Procedures used to analyze the data.

Chapter 4

This Chapter includes the Instructional Package with twenty-six Lesson plans and fifteen Worksheets and two Formative Assessments that were implemented in the present research. It also includes the Plan by which the Instructional Package was implemented.

Chapter 5

This Chapter includes the detailed Analysis and the Interpretation of the analysed data. It also includes the Findings of the Study and Discussion.

Chapter 6

This Chapter includes the Summary, Conclusions based on the Major Findings of the Study. It also includes Suggestions for Mathematics teachers, policy makers, and for future researchers with context of the present Study.

The report is followed by comprehensive Bibliography and Appendices.