CHAPTER 2

REVIEW OF RELATED LITERATURE

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2.0 Introduction

The focus of a Literature Review is to summarize and synthesize ideas, research findings and intellectual contributions made by others to facilitate the investigator to ideate, validate, organize, justify, contradict or support the Study. It is a compilation of different research studies related to the present research area that can be critically analyzed to identify gaps and choose appropriate approaches to address them. Analysis of the methodologies used in the past studies can facilitate the organization of the steps required to conduct the present research. The review can also prove useful to decide on the data analysis, interpretation and the reporting aspects. Interpretation of the available research results can be collated to support certain findings or contradict certain ideas that may evolve from the present research. The researched and proved facts can be cited to validate the ideas used to frame the conceptual background of the Study; to prove the need of the Study or to justify the findings in the discussion section.

For getting acquainted with the type of research work to be undertaken by the researcher a serious attempt was made to review studies done in India and abroad. The researcher specially focused on the studies related to Mathematics education and higher-order thinking skills. Around seventy-six studies were then selected to be included in the present Study. Studies were then classified under seven broad categories, as per the requirement of the present Study. The seven major categories are shown below.

1. Studies that reflect the present scenario of Mathematics education
2. Studies on student-characteristics imperative for higher mathematical achievement
3. Studies on effective strategies for enhancing Mathematics achievement
4. Studies on strategies to develop different mathematical and higher order thinking skills
5. Studies that prove the importance of guided instructions in Mathematics
6. Studies on teacher-related difficulties in using effective strategies in Mathematics teaching
7. Studies highlighting the difficulties involved in the teaching and learning of 'Real Numbers'

2.1 Studies that Reflect the Present Scenario of Mathematics Education

Rubenstein (1985) conducted the Study, *'Computational estimation and related mathematical skills'*. In this Study several computational estimations were explored with respect to other mathematical skills and gender. 144 girls and 165 boys of seven schools were

given some estimation tasks to be performed. The findings of the Study indicated that with reference to different types of estimation tasks, the open-ended estimation was the most difficult, that was followed by estimation relative to a reference number and then estimation within an order of magnitude. Numerical tasks were difficult than verbal tasks. Decimals were more difficult than Whole numbers, and quotients were more difficult than products, which was more difficult than sums and differences. Boys scored higher than girls on the total estimation test and on the order of magnitude scale.

Montague and Garderen (2003) conducted a Study, *'A cross-sectional Study of Mathematics achievement, estimation skills, and academic self-perception in students of varying ability'*. This Study investigated Mathematics achievement, estimation ability, use of estimation strategies, and academic self-perception of 135 students from 4th, 6th, and 8th grade with learning disabilities (LD), average achievers, and gifted students. The results indicated that the students with LD and the average achievers performed significantly lower than the gifted students on all estimation measures, they differed significantly from one another only on estimation strategy use. Even the gifted students did not have a well-developed understanding of estimation. The accuracy of their estimates improved when they were asked open-ended questions about the strategies they used to estimate. The results underscore the need to teach estimation from an early age.

Patel (2007) conducted a Study titled *'Development of a programme for enhancing achievement of the students of class X in Mathematics'*. A multi scale sampling technique was used and 30 Gujarati medium schools, 70 students and 30 teachers teaching in selected schools were selected as samples. Different tools like information sheet, questionnaires and unit tests were used to collect the data, which resulted into the following findings: 91% of the students felt difficulty while learning Mathematics in classroom. The reasons were lack of concentration due to weak base, frequent transfer of teachers in the class, the method of explanation differing from teacher to teacher, speedy teaching, less practice in geometry, lack of proper guidance. 59% of the students did not like the teaching method of teachers. 53% told that their teachers did not give attention towards their mistake in calculation. 76% of the teachers test the prerequisite knowledge for particular Chapter not in a much planned manner, with a few recall type questions.

Bhagowati (2011) conducted a Study titled *'Present scenario of Mathematics and science learning in secondary schools of Morigaon district – A Study'*. A survey method was used to Study the status of Mathematics and Science learning in secondary schools of Morigaon district. Data was collected from 300 secondary students from 10 schools. The

conclusions revealed that the present science and Mathematics learning was not satisfactory, students lacked sufficient knowledge in almost all the concepts of Mathematics which resulted into students' failure in Xth Board exams. The reason for this situation was the insufficient knowledge of teachers in the subject and their inability to teach the same in lower classes.

Shakila (2011) conducted a Survey research titled, *'Learning of mathematical concepts in relation to spatial ability and problem solving skill among secondary pupils'*. A sample of 620 ninth standard students was administered with three tests - The Mathematical Ability test (on topics- mensuration, sets, & symmetry); Spatial Ability test and Problem Solving Skill Scale (developed by Dr. G. Vankatesan). The data was then analysed to find the mean, standard deviation, percent of mean, analysis of covariance, multiple correlation, t-values and F-values. Findings of the research indicated that half of the sample had average and two-fifths of them had low levels of learning for the respective Mathematics concepts. Most of the pupils had average spatial ability, only two-fifths had high spatial ability.

Arundhati (2012) conducted Research titled, *'Factors affecting Mathematics education at school level with special reference to Bongaigaon district'*. The findings of this survey revealed that 'the average conceptual knowledge and Mathematics ability of students were lower than their scholastic achievement in exams; they were learning the steps and methods for working out the solutions to the problems instead of making sense of the concept behind the problem. They were unable to use prior knowledge and apply to the problem at hand; teaching process in classrooms were traditional; there was a strong positive correlation between the family income & parental education with Mathematics achievement; teacher and students attitudes also affected the Mathematics scores.

Rampal and Subramaniam (2012) in the Article, *'Transforming the elementary Mathematics curriculum: Issues and challenges'* in the book 'Mathematics Education in India: Status and Outlook' state that even though the latest NCERT textbooks of primary classes, include exercises and tasks that provide opportunities to students to discover new patterns and devise different strategies to deal with numbers, make quick comparisons and estimations, review one's answer using processes of reflection. But this has not lead teachers to shift away from the mechanical algorithmic approach in classrooms. Assessments still followed the conventional mode and only focused on the answer or use of the right algorithm rather than focusing on the process of thinking or doing the task- for which the teachers are not trained. The upper primary curriculum is also designed to enable students to explore, experiment and acquire reasoning skills, it is not implemented in large numbers of classrooms

in India. This indicated that bringing about change in the instructional and the assessment processes need a longer and a more difficult struggle for India, with sustained work with teachers, education officials and even parents. The only State in India where a positive change in Mathematics education has been observed is Kerala, where teachers use activity-based approaches more effectively.

Dewan et. al. (2012) examined the Mathematics teaching and learning of the secondary levels of India. They found that at this level the biggest challenge is to change the attitude of teachers towards Mathematics in the way it should be taught. Teachers transacted Mathematics in a way that it could be used only for calculations and contextual problem solving and not for developing thinking processes that are needed to solve problems. Teaching was restricted to text book, practice or guide book examples with strong focus on short cuts and memory devices. The teaching is de-linked from experiences of students and participation by children is minimal. Teachers themselves are often afraid of Mathematics considering the subject only for intelligent ones. Activity based teaching and student centred teaching got restricted only to use of concrete materials for few concepts in primary classes. Thus, concluding that Mathematics classrooms, in spite of the NCF and redesigned NCERT curriculum remain didactic and assessments test only calculations, algorithms, definitions and answers to 'difficult questions'.

Morgan (2013) conducted a Study, *'A Study of college students' estimation skills with mathematical computations'*. This research Study was done to examine the computational estimation capabilities and the strategies used by college students in doing so. The research was based on two hypothesis – (1) students will struggle while performing computational estimations involving operations with numbers that are not Integers as opposed to operations involving Integers. (2) students will most often use the strategy of compensation in contrast to other strategies. This hypothesis was tested on 59 students within two sections of a Liberal Art State University. Each student was given an eight-problem assessment and a five-question survey. The first was used to evaluate students' ability to perform computational estimation and the latter to find out the strategies they used. Results showed that the students were not proficient in using computational estimations, implying that there was a need to develop estimation skills in students in earlier stages of their education.

Sarma and Ahmed (2013) conducted a Study titled *'A Study on the difficulty of teaching and learning Mathematics in under graduate level with special reference to Guwahati city'*. For this Study, data was collected from 10 colleges from 25 students and 25 teachers. The findings revealed that 52% of students found Mathematics difficult and 60%

found it scary. Other problems like misappropriate curriculum, crude assessments, lack of preparation, less use of technological tools were also identified.

Tularam et. al. (2015) conducted a Study titled *'A Study of students' conceptual, procedural knowledge, logical thinking and creativity during the first year of tertiary Mathematics'*. The purpose of this Study was to examine as to why many students fail in mathematical courses in the first year environmental science degree program, in spite of a less demanding curriculum in Mathematics. This was done by examining 133 students' conceptual thinking patterns and strategies. The students were asked to prepare a focus sheet that summarized their knowledge of topics learned. The sheets were examined in detail to reveal the average levels of procedural and conceptual competence and low levels of logical and creative competence. The focus sheets also indicated a lack of structure, planning, preparation and organization. It demonstrated lack of deeper levels of understanding of the topics learned.

Yasoda (2015) conducted a Study titled *'An investigation to the problems relating to teaching learning Mathematics at secondary level'*. Samples for this survey were collected from 80 schools from Chittoor, Kadapa and Nellore districts. 644 students of class VIIIth and IXth classes and 161 teachers provided data for the Study. The findings revealed that students of class VIII found 'commercial Mathematics' and 'mensuration' difficult and teachers found 'circles' and 'concurrent lines and triangles' difficult to teach. In the secondary classes, 'linear programming', 'progression' and 'geometry' were difficult for students and 'polynomials' were difficult for the teachers to teach. Students were finding difficulty in solving problems and understanding mathematical symbols and language.

Sikdar and Poddar (2015) conducted a Survey titled '*To Study the implementation of CCE in Mathematics Assessment in GSHSEB schools of Vadodara city'*. 35 upper primary Mathematics teachers of the GSHSEB schools were taken as samples. The Study concluded that 'the teachers who were transacting higher-order thinking skills through their instructions and assessments were very limited as they were not trained on those lines and the textbooks offered limited scope for the same. Apart from that, teachers revealed a number of difficulties that they faced in teaching Mathematics in terms of class strength, expansive syllabus, time limitation, lack of resources, and management of class during group activities and problem in grading group activities. The interview of DIET officials and responses of Mathematics teachers revealed that no training workshops were conducted to strengthen the content or HOTS teaching strategies or assessments were taken up.

ASER, Beyond Basics (2017) reported the findings based on the survey of 26 rural districts across 24 states of India, mainly focusing on the youth age 14-18. Information was collected for four domains – activity, ability, awareness and aspirations. Results revealed that within the ability domain, more than half of the students still struggled with basic arithmetic operations; majority of them could perform simple daily activities that involved counting money, adding weights and telling time. It was observed that the students who could do basic Mathematics operations were also better at application type questions. Boys faired better than girls in all aspects of the Study.

2.1.1 Implication of the 'studies that reflect the present scenario of mathematics education' on the present study

Thus, the above stated studies, articles and reports indicate that though the curriculum for the elementary and the secondary school education is structured with explicit aims of developing mathematical thinking in students, the pedagogies actually practiced inside the classrooms are transacting only procedural knowledge. Students at the secondary as well as the undergraduate level face difficulty in Mathematics; lack sufficient knowledge in almost all the concepts; have average levels of procedural, logical and creative competencies and spatial-abilities (Patel, 2007; Bhagowati, 2011; Arundhati, 2012; Sarma & Ahmed, 2013; Tularam et al., 2015; Yasoda, 2015; Shakila, 2011). Students of all levels do not have a proper understanding regarding estimation tasks, especially computational estimations, which has to be an integral mathematical outcome (Rubenstein, 1985; Montague & Garderen, 2003; Morgan, 2013).

Teachers are not trained to implement the activity based and student centred approaches effectively (Sikdar & Poddar, 2013; Rampal & Subramaniam, 2012) and majority of the teachers lack teaching competencies in terms of content and pedagogical knowledge and lack positive attitude towards teaching (Bhagowati, 2011; Surender, 2015; Dewan et al., 2012). Assessments are limited to lower order thinking questions assessing only procedural and computational efficiencies of students (Sikdar & Poddar, 2013; Rampal & Subramaniam, 2012). Though the textbooks (NCERT) are designed to enable students to experiment and acquire reasoning skills (Rampal & Subramaniam, 2012; Dewan et al., 2012), the systemic limitations in terms of class strength, vast syllabus, insufficient resources, lack of preparation, and time restrictions are causing impediments for equivalent transaction (Sarma & Ahmed, 2013; Sikdar & Poddar, 2015; Rampal & Subramaniam, 2012).

Thus, the implications for the present Study from the available research facts and findings are presented ahead. In case of students, lack of conceptual understanding of basic

mathematics topics have been reported in Patel (2007), Bhagowati (2011), Arundhati (2012), and Tularam et al. (2015). This established the main focus while designing the contents in the Instructional Package, which was to target Conceptual level teaching along with the procedural requirement. Also, lack of cognitive abilities like numerical estimating skills, spatial abilities and solving HOTS problems among students was reported by Rubenstein (1985), Montague & Garderen (2003), Morgan (2013), and Tularam et al. (2015) in their studies. The present Study aspired to provide a way to this gap as well. With respect to mathematics teachers, Patel (2007), Rampal & Subramaniam (2012), Dewan et al. (2012) and Sikdar & Poddar (2015) reported the lack of appropriate knowledge regarding higher order thinking skills, teaching strategies that can be used to facilitate the same, activities besides hand-on (included in NCERT textbooks), and designing HOTS questions for assessment. The present Study could act as an exploration for the researcher to find a remedy for the same and disseminate the experience to help teachers in this direction. An important finding of the Study by Arundhati (2012) was – 'there is a strong positive correlation between family income and education with mathematics achievement'. This lead the researcher to place importance on the equivalence of the different samples. Thus, the Schools where the initial try-out and the final implementation happened were equivalent in terms of the Trust they belonged to and the socio-economic backgrounds of the students. Similarly, the experimental and control group were also chosen from the same School to maintain equanimity.

The next section highlights on some of the required characteristics in students to be better achievers in Mathematics.

2.2 Studies on Student-Characteristics Imperative for Higher Mathematical Achievements

Booth & Thomas (1999) conducted a Study, *'Visualization in Mathematics learning: Arithmetic problem-solving and student difficulties'*. In this research Study, a group of students were divided into two groups based on their visuo-spatial abilities. They were assessed on standard Mathematics tests. No difference in the mathematical performance was found in both the groups. The individuals were then interviewed, along with which they were given arithmetic word problems of three different presentations: orally; with a picture; and with a diagram' to be solved. The result showed that the group with higher visuo-spatial skills performed significantly better on the problems.

Dolma (2002) in the Study explored the relationship between estimation skill and computational ability for Whole and Rational numbers. Quantitative and qualitative data was collected form 91, 77 and 73 students respectively from classes 5, 7 and 9. Group-

administered tests and interviews were used as instruments to collect data. Two tests with identical items, one pair estimation-related and the other computation-related, was administered to all the students. Interviews were conducted on a selected number of 18 students based on the criteria of them being slightly above the average and slightly below the average. The findings of the Study revealed that performance in estimation was positively correlated to written computations. Thus, students with good estimation skills are also good in mathematical computations. Students could deal better with Whole numbers than with other topics. There was more disparity between estimation and computational ability in topics like division and multiplication of fractions and decimals. Students who were slightly above average chose sensible strategies to solve problems; while students with slightly below average chose rote-learned algorithms more. This Study highlighted the importance of integrating estimation skill in teaching and learning Mathematics.

Zohar and Dori (2003) conducted a Study titled *'Higher-order thinking skills and low-achieving students: Are they mutually exclusive?'* The Study was to examine the view of the teachers that HOTS can be fostered only in high achievers and not in low achievers. Four different studies were examined; each addressed a different project with goals to teach HOT in science classrooms. The findings revealed that at the end of the four programs, high achievers had gained higher thinking scores, and students of the average and the lower groups had made good progress with respect to their initial scores. The Study further makes strong suggestion that teachers should engage students of all academic levels in tasks that involve HOTS.

Meece (2003) in the Study *'A Study of teacher and student perceptions of learner-centered practices'* carried out survey on 109 middle school teachers, 2200 middle school students in urban, suburban and rural communities to assess the use of learner-centered teaching practices that stress higher-order thinking. Teachers and students both completed surveys for the same. Findings of the Study revealed that for teachers, the only ratings correlated with student motivation and achievement were related to teachers' reported support for higher-order thinking. For students, rating on all learner-centered practice including practices that support higher-order thinking were correlated with motivation and achievement. Higher-order thinking practices were the only practices found to be related to motivation from both teachers' and students' perspectives.

Atallah (2003) conducted a Study, *'Mathematics through their eyes: Students conception of Mathematics in everyday life'*. The researcher conducted an exploratory Study and a pilot Study to finally prepare a questionnaire having both open-ended and choice items.

This tool was administered to 238 female students in an institute of higher learning in the Middle East. The Study revealed some broad conceptions of Mathematics among the participants. Most of them believed that Mathematics was a form of mental activity useful for developing intellectual and problem solving skills. 81% of the student responses pointed towards the development of intellectual and practical skills as an outcome of Mathematics learning.

Garderen (2006) conducted a Study on sixty-six grade six students from four urban South Florida primary and middle schools. They were asked to solve mathematical problems and were interviewed along with that. The students were classified into three groups- students with learning disability, average-achieving students and gifted students. The findings of the Study revealed that students with high spatial-visualization ability tended to produce images that were schematic in nature, while those with low spatial-visualization abilities produced mostly pictorial images. The Study suggested that with increased aptitude in spatial visualization, students will have the ability to use more sophisticated schematic imagery to solve Mathematics problem.

Jacob (2012) conducted the Study *'Mathematical achievement and critical thinking skills in asynchronous discussion forums'*. The relation between critical thinking (CT) skills and Mathematics achievement of students of an engineering Mathematics unit was examined in this Study. Two batches of students (40 from one batch and 60 from the other) from the Swinburne University of Technology were the participants. The discussion postings from two online discussion forums were analysed using the CAIS model to score their CT scores. The achievement scores for the final examination of Mathematics subject was then examined against the CT scores of each student. The results indicated a linear relationship of the CT skills and Mathematics achievement.

Neetu (2013) conducted a Study titled *'A Study of socio-psychological correlates of learning - thinking style and creativity of secondary school students'*. This Study attempted to Study the 'socio-psychological' correlates in relation to learning and thinking style and creativity of secondary school students. A descriptive survey method was used and findings were: There exists positive and significant relationship between socio-economic status and learning- thinking- style of students. A significant difference was found between creativity of boys and girls and between urban and rural. Girls faired better than boys and the urban was better than the rural.

Kamalam (2015) conducted a Study on 1000 high school students of class IX and X in Tamilnadu to check the relationship between Mathematics achievement and academic self -

regulation. The Study revealed a significant relationship between external regulation, introjected regulation and academic self-regulation; and Study skills like listening, note taking, reading and writing skills; with the Mathematics achievement in high school students. It also revealed that there is no significant relationship between intrinsic motivation and Study skills like time scheduling, concentration, examination skills with achievement in Mathematics.

Madhusudan (2015) conducted a Study titled *'Interaction effect of mathematical creativity, intelligence and problem solving ability on achievement in Mathematics of IX standard students of Banglore district'*. The objective of the Study was to find the relationship between achievement in Mathematics and mathematical creativity, intelligence and problem solving ability of IXth standard students. 600 students Studying in English medium schools in Bangalore district were taken as samples and subjected to standard tools used for data collection. The findings showed that students with better mathematical creativity, intelligence and problem solving abilities were better achievers in Mathematics compared to the moderate and the low ones. It also revealed that the students of ICSE Board did better than students of the CBSE in problem solving, mathematical creativity and general intelligence tests.

Sreeraj (2015) used a normative survey method to find the relationship between Multiple Intelligence and achievement in Mathematics. The survey was conducted on 1500 secondary students of three district of Kerala. The data collected from the survey revealed that there was positive correlation between Mathematics achievement with the different components of multiple intelligence like verbal/linguistic; logical-mathematical; visual-spatial; interpersonal; intrapersonal intelligence.

Simms et. al. (2016) conducted a Study, *'Explaining the relationship between number line estimation and mathematical achievement: the role of visuomotor integration and visuospatial skills'*. This Study mainly investigated what exactly does performance on the number line estimation task measure. The Study thus examined the relationships among visuospatial skills, visuomotor integration, number line estimation and mathematical achievement. Standardized tests for each of the measures and number-line estimation task were administered to 77 students of 8-10 years. Majority of the measures were significantly correlated. Findings thus suggest that visuospatial and visuomotor integration skills are important for number line-estimation tasks, which is significant predictor of mathematical achievement.

Tanujaya et. al. (2017) conducted a Study with a purpose to find out the relationship between higher order thinking skills and academic performance of student in Mathematics instruction. A correlational research method was used with 41 students of Mathematics education of the University of Papua as participants. The instrument used to measure HOTS had two indicators; critical thinking skill and creative thinking skill. Another instrument was that of an achievement test with 9 HOTS questions. The data thus collected was analysed using the correlation factor and regression equation. Both the statistics showed a significant relationship between HOTS and students' academic achievement.

Mulbar et. al. (2017) conducted a qualitative Study, *'Students' mathematical connection based on levels of mathematical abilities: Qualitative Study in SLETV'*. For this research three VIII grade students from SMP Negeri 3 Watansoppeng were selected as subjects. Two instruments, a math ability test and a mathematical connection ability test, was given to them on the topic 'system of linear equations with two variables'. The results of the Study revealed that the mathematical ability of students were positively correlated to the ability to make mathematical connections. Also the quality of answers varied for the different levels of students.

Ndiung and Nandi (2018) conducted a Study titled, *'Mathematics connection ability and students Mathematics learning achievement at elementary school'*. This Study suggested that education should fulfil the 21st century requirement of individuals who can think originally. To explore the same, the researcher examined the effect of students' ability to make mathematical connections on their Mathematics learning achievement. The Study was conducted on 35 students at Watu Weri state elementary school. An 'Ex-post facto' research design used was used in this Study and tests and documentations were used for collecting data. The data was analyzed by linear regression method. The result of the research revealed that there was a positive effect of students' Mathematics connection ability on their Mathematics achievement.

2.2.1 Implication of the 'studies on student characteristics for higher mathematics achievement' on the present study

The previous section put together some studies to reveal the status of Mathematics education. The compilation indicated the problems, limitations and difficulties faced by students and teachers with the subject. The present section highlights on some internal as well as external parameters that influence mathematical achievement in students. Studies of Tanujaya et. al. (2017), Jacob (2012) indicate that students with higher order thinking skills performed better in mathematical tasks and show high achievement and according to Zohar

and Dori (2003) HOTS can be fostered not only in high achievers but also in those with moderate and low achievers. Cognitive skills such as mathematical creativity, intelligence and problem-solving skills (Madhusudan, 2015) and Multiple intelligence in terms of verbal, logical-mathematical and visual-spatial influence mathematical achievement (Sreeraj, 2005). Booth & Thomas (1999) and Garderen (2006) indicate the importance of visuo-spatial skills for problem-solving; Visuo-spatial processing and visuo-motor integration skills are required for number line estimation tasks, which again is associated to high achievement in Mathematics (Simms et al., 2016). The ability to make mathematical connections is positively correlated to one's mathematical ability (Mulbar, et al., 2017; Ndiung & Nandi, 2018). Students with good estimation skills also are good in mathematical computation (Dolma, 2002) which is an essential requirement for getting higher scores in Mathematics. Students who are slightly above the average choose several strategies to solve problems while those who are below the average rely on routine algorithms. Apart from these cognitive requirements, psychological traits like motivation (Meece, 2003); external regulation, introjected regulation, academic regulation and study skills like listening, note-taking and reading-writing skills (Kamalam, 2015) also play important roles in mathematical achievement. Socio-economic status and learning-thinking styles also affect achievement (Neetu, 2013). Thus, Mathematics offers mental activity to develop intellectual and problem solving skills and thus external characteristics like a dynamic classroom environment, rich lesson design ingrained with real-life learning activities and multidimensional views of concept are essential elements of Mathematics education (Atallah, 2003).

The studies reported in this segment place several implications for the present Study. The present Study aspired to design implementable teaching strategies to develop higher cognitive abilities among mathematics students. This requirement was substantiated by the studies of Booth & Thomas (1999), Jacob (2012), Madhusudan (2015), Sreeraj (2015), Mulbar et. al. (2017), and Tanujaya et. al. (2017); which indicated the importance of cognitive skills like visuo-spatial skills, estimation skills, critical thinking skills, mathematical creativity, mathematical connectivity, problem solving ability and logical-mathematical ability on mathematics achievement. The study of Zohar & Dori (2003) revealed that tasks involving HOTS not only benefitted the high achievers and also showed positive results for the average and low achievers. Thus, the need for the development of higher order thinking skills is to some extent established for enhancing mathematical achievement. 'Number line estimations' was a topic included within the Instructional Package due to its correlation with visuo-spatial skills (Simms et. al., 2016). Dolma (2002)

stated that ‘above average students chose sensible strategies to solve problems while below average students chose rote-learned algorithm’; This understanding substantiated the need for using blended teaching strategies rather than using only constructivist approaches in the Package, so that the benefit is not limited only to the high achievers. Garderen (2006)’s Study noted that low achievers used pictorial diagrams to solve problems. Thus, in the present Study, researcher tried to make the contents in the Instructional Package more visual in form.

2.3 Studies on Effective Strategies for Enhancing Mathematics Achievement

Patel (2009) conducted a Study that focused on development and implementation of a Laboratory Teaching Programme for VIII standard students following the Gujarat board syllabus. Topics like Set theory; Expansion & Factorization; Basic geometry (point, line, line-segment, ray, plane, angle, parallel lines); Analysis of data; Construction; Circle and Surface area & Volume were used for designing the LTP. This experimental Study employed the 2^4Factorial Design and was implemented on 220 students in two schools of Ahmedabad, Gujarat. The experimental group and the control group were equated based on the I.Q. of the students. The achievement scores of the experimental group who were subjected to the Laboratory Teaching Programme and the control group with the traditional classroom chalk and talk method, were subjected to ANOVA, F-test and t-test. The LTP proved to be significantly effective on the achievement of the students and was successful both on high and low I.Q. students.

Shah (2011) developed a Creative Teaching Programme (CTP) based on the William’s three dimensional model for implementing cognitive-affective behavior. The dimension one consisted of the subject matter (topics of Mathematics form standard VII, Gujarat Board syllabus). Dimension two was that of Teacher behavior or Modes of teaching (comprising of elements like organized random search, examples of change, habits, visualization skills, analogies, provocative questions, discrepancies, attributes, tolerance & ambiguity, adjustment to development). Dimension three consisted of four cognitive (fluent thinking, flexible thinking, original thinking, and elaborative thinking) and four affective (curiosity, risk taking, complexity, and imagination) student behavior. Fifteen lesson plans were developed by the investigator using the three dimensions. An experimental research with the 2^4Factorial Design was employed with the experimental group given the CTP and the control group taught with the traditional method. ANOVA, t-test and F-test were used for data analysis. The results revealed that the CTP had significant effect on Mathematics achievement of high I.Q. students, while having no significant effect on the low I.Q. students.

Pitkaniemi and Hakkinen (2012) conducted a Study titled *'The instructional quality of classroom processes and pupils' mathematical attainment concerning decimal fractions'*. This Study was conducted on 72 students of Vth grade with the experimental group subjected to teaching of decimal fractions, with an emphasis on instructional coherence, cognitive activation and emotional support. The control group was subjected to same parameters but in an average level. The conclusion of the Study was that 'teaching course including the above parameters supported students produce better mathematical attainment'.

Gururajan (2013) conducted a Study titled *'Development and implementation of computer assisted instruction in Mathematics for standard VIII students'* The main objective of the Study was to check the effectiveness of Computer Assisted Instruction (CAI) alone; and CAI with Discussions over Conventional method of teaching on class VIII students of Vadodara. A post-test only control group design was employed and Arithmetic units including Percentage, Profit & Loss and Simple & Compound Interest were converted into computer assisted instructions. The Study concluded that the technique of CAI with discussions result into better achievements in Mathematics. The Study also emphasized the role of teacher's guidance during the implementation of self-instructional strategies.

Jayasree (2014) conducted a Study titled *'Effect of direct instruction model on achievement in selected mathematical skills of upper primary pupils of Kerala'*. A pretest-posttest equivalent group design was used for the Study. A total of 180 samples from Kerala were divided into 3 groups – experimental group 1 was subjected to 'Direct Instruction Model of teaching by Engelmann (2001), experimental group 2 was subjected to 'Objective Based Instruction' and the third group was the control group. The Direct Instructional model was based on a systematic method of presenting materials in small steps, pausing to check student understanding and then eliciting participation for students; and basically used to help the slow learners- was designed using the primary class topics of Fraction, Decimals and Percentage. A comparison of all the three methods proved that the 'Direct Instruction Method' was more effective in terms of student achievement in comparison to the other two methods.

Sunitha (2015) conducted the Study titled, '*Effectiveness of Cognitively Guided Instructional Strategy on Mathematics anxiety and achievement in Mathematics of upper primary students*'. The first phase of the Study was a survey and in the second phase a quasi-experimental method was employed with pretest-posttest non-equivalent group design. Samples for the first phase of the Study comprised of 400 upper primary students from seven schools of Malappuram and Palakkad districts of Kerala. In the second phase, the Cognitively

Guided Instructional strategy was implemented; for which 128 students of VI standard of four intact classrooms of two schools were samples with 68 students in the experimental group and 62 students in control group. Cognitively Guided Instructional strategy was structured for Class VI topics, 'Volume of Rectangular Prisms' and 'Decimal numbers'. The Survey results proved that the level of Mathematics anxiety of upper primary students was below scale average value. The experiment proved that the Cognitively Guided Instructional Strategy was effective in reducing Mathematics anxiety and also enhancing Mathematics achievement. The phases included in the Cognitive Guided Instructional Strategy were presentation of problem, finding solution to the problem, discussion of solution strategies with its sub-steps.

Haji et. al. (2016) in the article, *'Developing students' ability of mathematical connection through using outdoor Mathematics learning'* reported a research Study. The purpose of the Study was to find the effect of outdoor mathematical learning on the achievement and the mathematical connectivity. A quasi-experimental design was employed with 64 students of two schools as sample for the Study. The findings of the Study indicated that the students in the experimental group had higher achievement and also had higher ability to make mathematical connections compared to the control group taught by the conventional method of teaching.

2.3.1 Implication of the 'studies on strategies to enhance mathematics achievement' on the present study

The above section comprised of studies that examined the effect of some teaching strategies on mathematical achievement of students in upper primary classes. Some of the studies targeted on absolute cognitive processes; while some on hands-on/psychomotor processes to enhance mathematical achievement. Cognitive processes like instructional coherence, cognitive activation, and emotional support (Pikaniemi & Hakkinen, 2012); direct instructional model (Jayasree, 2014); cognitively guided instructional strategy (Sunitha, 2015) and creative teaching programme (Shah, 2011) and psychomotor processes like laboratory teaching programme, computer assisted instruction, outdoor mathematical learning; were successfully tested to record enhancement in mathematical achievement.

Apart from the understanding gained from the Studies regarding the cognitive processes included within the Interventions to enhance mathematical achievement, there were other implications for the present Study. All the reported Studies in this section had similarity with the present Study with respect to the methodology, research design, intervention design, effectiveness of the intervention on mathematical achievement and also the statistical

procedures used for data analysis. A review of the same, helped the researcher to pave the design of the Study structure and the Intervention.

2.4 Studies on Strategies to Develop Different Mathematical and Higher Order Thinking Skills

Niemi (1996) conducted a Study, *'Assessing conceptual understanding in Mathematics: representations, problem solutions, justifications, and explanations'*. In this research Study measures of fractional knowledge was developed and that was administered to 540 Vth grade students in 22 classrooms. They were given tasks for which they had to represent their conceptual knowledge for different contexts and formats. The performance of the students was compared across different tasks. The findings revealed that the level of representational knowledge had a positive correlation with problem solving, justification, and explanation tasks. It was also found that the explanations and justifications could be efficiently and reliably judged, which provided the information regarding individual students' understanding. Thus, this research proved the effectivity of an assessment design strategy that can be used for assessing written assessments for classroom diagnosis and for instructional improvement.

Jurrow (2004) in his Study, '*Generalizing in Interaction: Middle school Mathematics students making mathematical generalizations in a population-modelling project'* examines the processes involved in the task of making generalization with regard to a mathematical content. A project that involved creating and analysing mathematical models of population growth was given to a group of middle school Mathematics students, who worked for 8 weeks to complete the task. The analysis of students' work revealed implications for teachers to engage students in mathematical generalizing in classroom. It suggested that, even though students spent a good span of time engaging in several higher level activities like creating models, examining aspects while building and altering those models, analysing the sensibility of different models, investigating the same in different situations using different representations, and discussing and comparing their models with their peers'; it was not enough to see the connections across activities or to begin to make mathematical generalizations. Students also needed guided reflection and multiple scaffolded opportunities to represent what is general in and across situations.

Volkova (2005) conducted the research Study, *'Characterizing middle school students' thinking in estimation'*. The goal of this research was to develop a framework for describing students' thinking in situations involving computational estimation. Case Study methodology was used to construct the descriptors of the framework. Eight high-achieving students of the

8th grade were the participants of the Study. The students were interviewed with 19 tasks designed to assess students' ability to estimate Whole numbers, fractions, percent, and decimal fractions. The responses of the students were analyzed and used to develop the Estimation Thinking Framework with four developmental levels – pre-dimensional, unidimensional, bidimensional, and integrated bidimensional. Apart from that, the data also revealed that children of the same age can be on different levels of thinking with regard to computational estimation. The progress of children to a higher level depends more on the content and methods of instruction received by the student, than on their age.

De Sousa (2007) studied, *'The effect of formative assessment on student achievement in Mathematics'*. This thesis examined the implementation of formative assessment in a Mathematics class and determined its effectiveness in improving student achievement in Mathematics. An experimental Study was conducted on students studying in 10th, 11th& 12th grades over a three-week period time on topics of Algebra. Selected types of formative assessment like open ended questions, think-pair share and self & peer assessment were utilized in the experimental group and traditional instruction without recurring formative assessment was given to the control group. The results indicated that the students of the experimental group scored higher on all assessments throughout the unit. They demonstrated much deeper conceptual understanding of the topics in the unit. They were better able to articulate their mathematical thinking in a precise manner compared to the control group.

Carraher et. al. (2007) in the Study, '*Early algebra and mathematical generalization'* examined issues that arise in students' making generalizations. Fifteen 8thgrade students were examined as they produced and represented generalizations during the implementation of two lessons on algebra. This Study concluded with an important finding that elaborated on the successive process of learning that can guide teachers to practice the generalization process. Students must first be shown how to see different patterns; then note down relations and structures that evolve; then gradually guided to formulate mathematical generalizations; followed by (even more gradually) derivation of new information by reflecting on known concepts.

Rivera (2007) interviewed twenty-two ninth grade students in a beginning Algebra course to assess how they established generalization for a tiling squares problem. Interviews were conducted to find out the strategies used by ninth grade students to develop algebraic generalization. Rivera concluded that if algebraic generalizations are promoted through visualizations then patterns must be seen as mathematical objects rather than everyday

objects; thus highlighting the importance of providing opportunities of visualization in form of Mathematics terms and operations on a regular basis to students.

Ho Siew Yin (2009) in his research Study provided insights into the use of visualization in mathematical problem solving among primary school students. Fifty primary grade 5 and grade 6 students were given word problems with high degree of visuality and difficulty. The researcher documented five processes and seven roles of visualization in their problem solving. The processes used were-Understanding (the spatial relations of the elements in the problem), Connecting (to a previously solved problem), Constructing (a visual representation in mind, on paper, through tools), Using (the visual representation to solve the problem) and Encoding (the answer to the problem). The Study highlights the importance of visualization in problem solving and emphasizes teachers and students roles in giving due importance to the same.

Bills and Rowland (2009) in the Article, '*Examples, generalization and proof: Advances in Mathematics Education'* displayed the relation between generalizations and the examples provided. With help of classroom activities the researcher demonstrated that examples do not always fulfil their intended purpose to point to the generalization. Generic examples were needed to support structural generalizations. This Study emphasizes the choice of 'examples' to be made by Mathematics teachers in order the lead students to generalize Mathematics facts, formulae, or definitions.

Adams (2011) conducted a Study on, *'Teaching problem solving skills in math by engaging higher-order thinking'* in a High School in Maryland. The Study was guided by the research question: Will asking students higher-order questions, having students develop their own strategies for solving problems, and requiring students to prove their results improve their acquisition and understanding of the content, their ability to problem solve, and their disposition towards problem solving? Students of 9^{th} and 10^{th} grade with a mixture of 11^{th} and 12^{th} grade were included as samples for the Study. To test the research question, the researcher implemented a strategy which included asking higher-order questions and providing opportunities for students to solve problems. The results of the Study were: (1) The action strategy improved students' acquisition and understanding of the content. (2) The strategy had mostly a positive impact on the students' ability to problem solve.

Goethals (2013) in his Study '*the pursuit of higher-order thinking in the Mathematics classroom: A review*' reviewed many studies and claimed that 'research suggests that teachers do not fully understand how to test, analyze, or even assess higher-order learning

concepts.' His Study thus made an attempt to advance the knowledge of higher-order thinking in mathematical concepts. Excerpts of this Study are given below:

- NCTM (2012) suggested some questioning strategies to help students to conjecture, invent, and solve problems; They are 'What would happen if? … Do you see a pattern?....Can you predict?... Wetzel (2013), William (1999), Yee (2000), Foster (2004), Mok et. al. (2008), Ratner and Epstein (2009), Emerson (2010), Wichelt (2009) are the many researchers who suggested questioning strategies to develop HOTS.
- Collis (1998), Paloff (1999), Spriger et. al. (1999) Mkhize (1999), Artzt et. al. (2008) suggested the teaching strategy of students working in group settings for the development of HOTS.
- Protheroe (2007) suggested that a Mathematics classroom must do the following things to achieve effective environment for higher-order thinking: 'Actively engage in doing Mathematics, solve challenging problems, make interdisciplinary connections, share mathematical ideas, use multiple representations to communicate ideas, use manipulatives or other tools.'
- Miri et. al. (2007) proposed three teaching strategies for generating HOTS : 'Present real-world cases and encourage students to cope with relevant situation; direct class discussions and encourage students to ask questions and present their own solutions; and guide short inquiry type experiments in groups.'
- Use of visualization techniques was suggested in many studies to promote higher order thinking. Gianquinto (2007) and Rodd (2010) emphasized the use of imagery in instructions to enhance conceptual understanding. Rosen and Saloman (2007); O'Dwyer et. al. (2008); Weiss and Bordelon (2011) stressed on technology-driven environments for better student performance. Gomen-Chacon (2013) reviewed a large number of visualization studies to propose the use of interactive images to achieve higher order thinking.
- Newmann et. al. (2001) used state-based-assessments as tool to delineate between 'didactic questions' that required procedural and factual knowledge as base to respond and 'HOTS questions' that required problem solving and reasoning skills to respond. Wenglinsky (2004) conducted a similar Study using national standardized assessments taken at large scale to find a positive correlation of instructions that emphasize on higher order thinking with student performance. Thompson (2008) conducted a Study in U.S. to examine the Mathematics teachers on the interpretation of Bloom's taxonomy, to find that

teachers did not understand completely the meaning of higher order thinking and thus had difficulty in creating such assessment questions.

Eli et. al. (2013) conducted a Study *'Mathematical connections and their relationship to Mathematics knowledge for teaching geometry'*. The Study backed the importance of higher order thinking skills in Mathematics teachers. The purpose of this concurrent exploratory mixed method Study was to examine the knowledge of teaching geometry of the prospective middle grades Mathematics teachers with respect to the mathematical connections they could make on a given task of Geometry. The Study revealed that the knowledge for teaching geometry of the teachers was below average. It was also found that curricular connections and the Mathematics knowledge for teaching geometry had a positive correlation.

Montague et. al. (2014) in the Study *'The effects of cognitive strategy instruction on math problem solving of middle-school students of varying ability'* replicated a research-based cognitive strategy intervention called *Solve It!* on 7^{th} grade students (which was earlier successful on 8^{th} standard students). Thirty-four 7^{th} grade Mathematics teachers participated in this Study. They implemented *Solve It!* on 644 students in forty different schools in Miami. *Solve It!* was based on the problem solving steps like Read, Paraphrase, Visualize, Hypothesize, Estimate, Compute, and Check. Curriculum based Mathematics problems were given as pretest and posttest to the experimental and the control group (n = 415). Findings of the Study revealed that the intervention was effective across all ability groups and can be used in inclusive classrooms.

Chinedu et. al. (2015) in the Paper titled, *'Strategies for improving higher order thinking skills in teaching and learning of Design and Technology Education'* presented an in-depth review of literature. The paper revealed the fact that teachers in Design & technology education faced difficulty in preparing and teaching for develop higher order thinking skills in students. The paper critically examined the existing practices in the same area and extracted out some methods like, (1) teaching the meaning of HOT (2) teaching the concept of concepts (3) naming and categorizing concepts (4) moving from concrete to abstract and back (5) teaching inference and connecting concepts (6) teaching question-answer relationships (7) including brainstorming activities in the lessons.

Ormond (2016) conducted a research Study, *'Scaffolding the mathematical "connections": A new approach to preparing teachers for the teaching of Lower Secondary Algebra'*. This Study mainly focused on the requirement of the Australian Curriculum on the goal to transact Mathematics concepts with a broader, connected understanding of the subject

of Mathematics. It was a three-year mixed methods Study for the pre-service primary education students and in-service teachers. The purpose was to prepare novice teachers through modelling a more coherent approach to Mathematics teaching with a focus on Number and Algebra strand, highlighting on the inter-connections within the topics. Results of this Study indicated that the new approach used to help teachers to teach Mathematics to suffix one of the goals of Australian Mathematics curriculum i.e. seeing the relationships and connections in Mathematics, was effective in enabling teachers to teach the concepts more efficiently.

Ley (2016) in his conceptual Paper, *'Generalization: Making learning more than a Classroom Exercise'* refers to generalization as the ability to transfer concepts and skills in novel or different contexts. He provided strategies to incorporate generalization in classroom instructions by blending different approaches like 'Discovery learning with leading probe questions' and 'Direct instructions'. Discovery learning approaches require learners to perform deep and active mental processing and Direct instruction approaches help learners to build deep domain-specific knowledge and focus on core concepts; thus promoting generalization.

Sunitha (2017) conducted an experimental Study on 153 ninth standard students of four schools of Kerala to check the effectiveness of the RBC Model of Instruction (developed by Hershkowitz, Schwarz and Dreyfus, 2001) on the achievement in Mathematics and other variables such as life skills in Mathematics and thinking skills of students. The Chapters 'Polynomials' and 'Circular Measures' of class IX of the Kerala State syllabus was used to develop the RBC model. This Model was designed to address the abstract nature of secondary school Mathematics. It included different phases such as Recognizing, Building-with, Constructing, and Consolidation. The RBC Model of Instruction was implemented on the experimental group and the traditional method of teaching on the control group. This model was found to be effective in enhancing achievement of students based on the objectives such as remembering, understanding, applying, analysing, evaluating and creating. It was also effective in enhancing life skills in Mathematics like financial, statistical, geometric, measurement and numerical skills. It proved effective in enhancing thinking skills like spatial and algebraic thinking skills.

Paul (2017) conducted an experimental Study to check the effect of Brain Based Learning Strategy (BBLS) and Circles of Learning Strategy (CLS) on achievement in Mathematics and self-efficacy of standard VII students. The BBLS is a learning strategy designed by Jensen, 2008 and is a seven staged brain based learning strategy. CLS is a Co-

operative learning strategy outlined by Johnson & Holubec, 1994. In the first phase of the Study a survey was conducted on upper primary school Mathematics teachers to gather information regarding the instructional strategies they used. In the second phase, the experimental Study with the non-equivalent pretest posttest control group design was used to implement the BBLS and CLS on two experimental groups and the Activity based teaching method on the control group. Each group consisted of 40 students as samples in 3 intact classrooms of a school in Kerala. The interaction effects of the three strategies and learning styles on achievement in Mathematics and self-efficacy was found. The topics of class VII Kerala state syllabus – 'Unchanging relations', 'Repeated multiplication' and 'Area of a Triangle' were used to design all the Instructional strategies used in the Study. The findings of the first phase revealed that problem solving method; assignments & projects and activity oriented methods of teaching were the mostly used strategies in the upper primary schools by Mathematics teachers. It also revealed that though most of the teachers were aware of the expected outcomes of different strategies like logical thinking, reasoning etc. they were reluctant to use them due to constraints like time, difficulty in classroom management, unwanted discussions in group activity, difficulty in evaluating activities and disinterest of students. The results of the experiment revealed that BBLS showed significantly higher levels of achievement in Mathematics in comparison to CLS and activity method. The BBLS was also significantly effective in case of all the objectives of remembering, understanding, applying, analysing and creating.

Samo et. al. (2017) conducted a Study, ***'Developing contextual mathematical thinking learning model to enhance higher-order thinking ability for middle school students'***. The main purpose of this Study was to develop contextual mathematical thinking-learning model. This was a R & D Study with three phases: investigation, development and implementation. The investigation of different theories that support to enhance higher-order thinking ability was used to develop the model. The model had different dimensions like, (1) the contextual problem was presented, (2) critical and analytical questions were asked based on that (3) that was followed by individual and group investigation (4) presentation of solution and discussion on the same (5) reflection and (6) higher-order thinking test. The implementation phase involved an experiment consisted of 78 Junior High School students who were divided into experimental and control group. This phase concluded that the contextual mathematical thinking learning model can be effectively used to enhance higher-order thinking ability of students.

Apino and Retnawati (2017) conducted the Study, '*Developing instructional design to improve mathematical higher order thinking skills of students'*. The purpose of this Study was to develop an instructional design that would be helpful to improve higher order thinking skills (HOTS) of students in learning Mathematics. The instructional design was developed on the basis of three main components: (1) providing non-routine problems to students involving them in problem solving (2) Facilitating students to develop their creative thinking and critical thinking abilities of analysis and evaluation (3) encouraging and probing students to construct their own knowledge. The design was implemented on teachers and X grade students in a region in Indonesia. Focus group discussions and tests were conducted, the data thus obtained was analyzed by quantitative descriptive method. The results of the Study revealed that the developed instructional design was effective in improving the higher order thinking skills of students in Mathematics.

Kadir and Fatra (2017) conducted a Study, '*Concept mapping learning strategy to enhance students' mathematical connection ability'*. This research was conducted on 72 senior high school students in Jakarta to check the mathematical connection ability of students and the effect of using concept mapping learning strategy on the same. A quasi-experimental method was used with randomized control group design. Data obtained from the post-test after the application of treatment on the content- trigonometry was analyzed. Results of the research revealed that that the concept mapping learning strategy worked to enhance the students' mathematical connection ability, relative to the conventional learning strategy.

Zhu et. al. (2017) conducted a Study, *'Numberline estimation predicts mathematical skills: Difference in grades 2 and 4'*. The purpose of this Study was to examine the role of number line estimation on two mathematical skills- calculation fluency and math problem-solving. 148 students of grade 2 and 4 from Shanghai, China participated in this Study. They were assessed on number line estimation, non-verbal cognitive ability, working memory, attention, and mathematical skills. The results showed that after controlling the effects of age, non-verbal cognitive ability, working memory, and attention – in grade II number-line estimation correlated significantly with math problem-solving and with calculation fluency; while in grade 4 number line estimation correlated significantly with math problem-solving but not with calculation fluency. The Study concludes that number line estimation could be a useful indicator for teachers to identify and improve children's mathematical skills.

Dumitrascu (2017) in the Article, '*Understanding the process of generalization in Mathematics through activity theory'* created a theoretical framework for the generalization

activity using Leontiev's activity theory and Rubinshtein's description of the generalization process. He elaborated on the psychological processes involved during the activity of theoretical Generalization; which were the cognitive activity of analysis, synthesis and abstraction. The framework also provided guidelines to Mathematics teachers to design instructions using generalization as an strategy to develop the above mentioned cognitive processes.

2.4.1 Implication of the 'studies on strategies to develop different mathematical and higher order thinking skills' on the present study

Wenglinsky (2004) in Goethals (2013) indicated a positive correlation of instruction that emphasize on higher order thinking with student performance in Mathematics. Most of the studies included in Goethals (2013) suggested cognitivist teaching strategies like active engagement of students in doing Mathematics (Protheroe, 2007 and Miri et al., 2007), posing challenging problems, making interdisciplinary connections, using multiple representations, using heuristics (Protheroe, 2007); presenting real-world cases, guiding for short inquiry type experiment (Miri et al., 2007) to promote higher order thinking. Asking higher-order questions to help students to conjecture, invent and solve problems during transaction of instructions (NCTM, 2012; Wetzel, 2013; William, 1999; Yee, 2000; Foster, 2004; Mok et al., 2008; Ratner & Epstein, 2009; Emerson, 2010; Wichelt, 2009; Adam, 2011) and during formative assessments (De Sousa, 2007) were found to cause deeper conceptual understanding in students enhancing their mathematical thinking skills. Use of visualization techniques was suggested in many studies Gianquinto (2007); Rodd (2010); Rosen and Saloman (2007); O'Dwyer et. al. (2008); Weiss and Bordelon (2011); Gomen-Chacon (2013); Ho Siew Yin (2009) to promote higher order thinking. Also the importance of estimating skills, especially number-line estimations for better mathematical thinking was highlighted in the studies of Volkova (2005) and Zhu et. al. (2017). Niemi (1996) indicated that level of representational knowledge had a positive correlation with problem solving, justification and explanation tasks which are also the ideal requirements of responses to HOTS questions. The Study also provides a framework to assess such responses. Ormund (2016) and Eli et. al. (2013) proved the importance of identifying and integrating mathematical connections by Mathematics teachers for effective teaching. The concept-mapping learning strategy worked well and was suggested to enhance the mathematical connection ability of students (Kadir & Fatra, 2017).

A number of Studies (Samo et al., 2017; Apino & Retnawati, 2017; Sunitha, 2017; Montague et al., 2014) reported the use of teaching models integrating several Mathematics

topics using sequential phases on lines similar to: (1) the contextual problem was presented (2) critical and analytical questions were asked based on that (3) that was followed by individual and group investigation (4) presentation of solution and discussion on the same (5) reflection and (6) higher-order thinking test. The brain based learning strategy used by Paul (2017) based on logical thinking and reasoning skills proved to be more effective than the cooperative learning strategy (circles learning strategy) and the activity-based learning in improving achievement for objectives of remembering, understanding, applying, analysing, evaluating and creating.

The cognitive processes involved during mathematical generalization involve psychological processes of analysis, synthesis and abstraction, is indicated by Dumitrascu (2017) who also created a theoretical framework on the same. Ley (2016) also indicated generalizing skill as a higher-order thinking skill as it involved the ability to transfer concepts in different contexts. Generalizing can be incorporated in classroom instructions by blending 'discovery learning' and 'direct instructions' (Ley, 2016); use of generic examples (Bills & Rowland, 2005); and use of visualization (Riveria, 2007). Carraher et al. (2007**)** elaborated on the successive process of learning (seeing patterns to creating new concepts) that can guide teachers to practice the generalization process. Jurrow (2004) proved in his research Study that even if students are actively involved in specially designed complex tasks, they are not able to see the mathematical connections or make mathematical generalization on their own. They need guided reflection and multiple scaffolded opportunities to represent what is general in and across situations.

Thus, several cognitivist strategies that target conceptual clarity through showing mathematical connections and multiple representations, using estimations & visualizations and constructivist strategy of guided discovery by using the strategy of generalization are reported in the different studies as described in the above paragraphs to develop higher order thinking in students. The understanding of the above stated strategies were useful while deciding on the teaching strategies and the cognitive approaches that were adopted in the Instructional Package, which was the main core of the present Study.

2.5 Studies that Prove the Importance of Guided Instructions in Mathematics

Hammouri (2001) in the Study, *'The problem solving cognitive strategies used by high achievers of the Hashemite University'* investigated the cognitive strategies used by sixteen high achievers of higher education system while trying to reconstruct six geometrical pieces to get a plane figure. The results revealed that most of the students used the trial &error strategy to solve problems. The Study also claimed that even the high achievers are

dependent learners who have to be guided and informed about what to do and how to do. The Study also revealed several geometrical misconceptions.

Kirchner et. al. (2006) conducted a Study, *'Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching'*. This Study provides evidence to prove the fact that unguided or minimally guided instructional approaches which are very popular and intuitively appealing ignore the structures that constitute human cognitive architecture. It reports several empirical studies over the past half-century that indicated that minimally guided instruction was less effective and less efficient than instructional approaches that strongly emphasize on guidance of the student learning process. Some instructional research and models that are created on these lines were described in this Study.

Varghese (2009) in his Paper *'Teaching Mathematics with a holistic approach'* outlined an inclusive instructional strategy that emphasized the fact that, to be more inclusive, Mathematics educators needed to take up a balanced instructional approach including both teacher-centered as well as student-centered modes of teaching. The Study highlighted the fact that, adopting exclusive student-centred approaches were not appropriate for some students belonging to some ethnic groups. Thus, to meet the best needs of all the students a balanced teacher-centred as well student-centred approach needs to be adopted by Mathematics teachers.

McCarthy (2016) conducted a Study titled *'A jaworskian analysis of four senior class primary teachers endeavouring to teach Mathematics from a constructivist compatible perspective'*. This Study was conducted in Ireland where schools have similar conditions as India: large class size, pressure to cover a set of curriculum, domination of teacher-talk as methodology used. The Study addressed the question 'to what extent will an exposure to constructivism and its implication for the classroom impact on teaching practices in senior primary classrooms in both short and long term'. Experiences of four senior class primary teachers while they used constructivist compatible approaches in Mathematics classrooms, were considered for analysis. Views of students were also considered for the Study. The data obtained from interview was analysed from a Jaworskian perspective i.e. using the teaching triads (1) management of learning (2) mathematical challenge (3) sensitivity to students. Findings revealed improvement in pupil's attitude when teachers adopt an investigative group-centred approach. It also stressed on the need for cognitive challenge in activities presented to pupils. With the limitation of class size and time, investigative problem-based (constructivist) maths work if expedient and efficient use of time is done by providing

relevant information to students that aid investigation. But the dilemma for teachers remains what and when to tell. The investigator concluded that once the project ends teachers would shift back to their non-constructivist approaches but the awareness and the knowledge about student-thinking and learning styles would stay with them and enrich teaching further.

Teachers mentioned their reasons for not taking up constructivist approach in classroom as: Heavy workload; unavailability of suitable problem solving activities; the approach that helped students score well in standardized tests dominated more; time available to teach Mathematics was less.

2.5.1 Implication of the 'studies that prove the importance of guided instructions in mathematics' on the present study

Though constructivist student-centred teaching approaches are propagated to improve student attitude and develop higher order thinking skills, many studies like Varghese (2009); Kirchner et. al. (2006); Himmouri (2016); McCarthy (2016) and Gururajan (2013) investigated to prove that unguided or minimally guided instructional approaches are less effective and efficient in dispensing conceptual understanding even to the high achievers. The studies strongly emphasize the need of guided and informed instructions to aid students towards investigations to discover the concepts and do higher order thinking.

Since the present Study focused on exploring ways to facilitate higher order thinking in students, the major decision that the researcher had to make was – the selection of appropriate strategies or approaches to transact the mathematical content, that would fulfill the purpose. The Studies reported in this section clearly indicated that only including constructivist approaches to transact contents will not serve the purpose. With proof that such approaches bring in positive change in student attitude and improve their conceptual knowledge (MaCarthy, 2012), there are also enough proof that constructivist strategies are neither teacher–friendly due to the toll it takes in managing, monitoring and even conducting such strategies, especially in large class strength scenarios as in India (McCarthy, 2012), nor are much student-friendly (Kirchner et.al., 2006), in case of average and below average ones. A combination of effective learning of the content and effective mathematical thinking during the transaction can happen by adopting a balanced approach of teacher-centered and student-centered methods of teaching learning as suggested by (Varghese, 2009; Hammouri, 2001). Thus, blended cognitivist and constructivist approaches were used to transact content in the Instructional Package of the present Study.

2.6 Studies on Teacher-Related Difficulties in Using Effective Strategies in Mathematics Teaching

Tebbs (2000) conducted a Study titled *'Assessing teachers' efficacy towards teaching thinking skills'*. This exploratory Study addressed questions like: (1) What were the teachers' self-efficacy levels with respect to the teaching of thinking skills? (2) What difference existed with respect to levels of self-efficacy towards teaching of thinking skills? To address these questions 432 K-12 teachers in New England were surveyed using 5 point Likert-style, 26 items instrument. Analysis of data indicated: (1) Teachers were least efficacious towards teaching higher-order thinking and transfer of thinking. (2) Self-efficacy differed significantly depending upon subject taught or specialty; nature of training; degree to which training satisfied needs in terms of teaching thinking skills. This Study suggested that one possible way to deliberate development of thinking strength in all teachers was by pre-service and in-service training.

Ingvarson et. al. (2006) conducted a survey for the *Australian Government Department of Education, Science & Training, ACER.* The report of this survey reviewed some literature which conveyed the following : Wenglinskly (2000) used multilevel structural equation modelling with data from the *US National Assessment of Educational Progress (NAEP)* program, found that "students whose teachers had received professional development in higher-order thinking skills outperformed students whose teachers had not". Darling – Hammond and Ball (1998) in a review of research and practice in teacher education found that 'teachers who spent more time Studying teaching were more effective overall, and strikingly so in developing higher-order thinking skills".

Thompson (2008) conducted a Study titled, *'Mathematics Teachers' Interpretation of Higher-Order Thinking in Bloom's Taxonomy'*. Thirty-two high school Mathematics teachers from the southeast U. S. were asked to (1) define lower-and higher-order thinking, (2) identify which thinking skills in Bloom's Taxonomy represented lower-and higher-order thinking, and (3) create an Algebra I final exam item representative of each thinking skill. Results indicated that Mathematics teachers had difficulty interpreting the thinking skills in Bloom's Taxonomy and creating test questions for higher order thinking.

Tajudin (2015) conducted a Study titled *'Mathematical knowledge and HOTS for teaching algebraic problem solving'*. It was a Case Study of three Prospective Mathematics Teacher (PMT), who were chosen in order to get information related to what extent did they integrate HOTS in algebraic problem solving. The data obtained was analysed qualitatively with respect to parameters like 'Mathematical Knowledge for teaching', 'Heuristics used in

Problem Solving' and 'Ways PMT integrated HOTS in teaching'. Findings indicated, "group work method", "practical work method" and "questioning technique" was used by PMT and although the techniques aligned with the HOTS approach, the relationship between concept, representation and HOTS was not established properly.

Surender (2015) in his Study titled *'Study of teaching competencies in Mathematics among secondary school teachers'* used the survey method to Study the teaching competencies in Mathematics among secondary school teachers in terms of their content knowledge, pedagogical knowledge, classroom performance and attitude towards teaching. 300 secondary school teachers of the state of Telangana were the samples of the Study. The findings proved that majority of the teachers lacked the aforesaid teaching competencies.

Senol et. al. (2015) conducted a Study, *'Analysis of the relationship between estimation skills based on calculation and number sense of prospective classroom teachers'*. The purpose of the Study was to examine the relationship between estimation skills based on calculation, estimation strategies and number sense of prospective elementary teachers. 125 prospective teachers studying in the department of classroom teaching for elementary schools in a Turkish public university were the samples of the Study. They were given two tests- test developed by Tekinkur (2008) to test estimation skills and test of number sense. The prospective teachers scored low in both the number sense test and in the estimation skills test and thus their knowledge in this area were concluded to be insufficient. The Study suggested a need to restructure the course contents and train teachers in the mentioned skills which has been given a good amount of importance in the recent school curricula.

Alrwais et. al. (2016) in their Study,

'Implementation reality of Mathematics teachers of higher-order thinking skills at primary level' used an observation card on a random sample of 92 primary Mathematics teachers. The classroom teaching of the teachers was observed with regard to the transaction of higher-order thinking skills. Their opinions were also taken to report their perceptions on the extent to which they transacted higher-order thinking skills while teaching. The final findings revealed a disparity between the standpoint (perception) of the Mathematics teachers and the actual classroom observation. Thus, in reality Mathematics teachers are not transacting higher order thinking skills to the extent they perceive to do so.

Abdullah et. al. (2016) conducted the Study, *'Mathematics teachers' level of knowledge and practice on the implementation of higher-order thinking skills (HOTS)'*. The purpose of the Study was to identify the level of knowledge about higher order thinking skills among Mathematics teachers at a secondary school in Malaysia, and the practices they

followed to implement the same. The Study involved 196 participants. It mainly focused on the knowledge and the transaction of the curriculum, pedagogy and assessment by the participants. The data was compared with their demographic factors. The data was analyzed using various quantitative analysis techniques. The findings revealed that the level of knowledge and practice of assessment related to higher order thinking skills among the teachers was the weakest. Significant differences existed between demographic factors such as gender, location of school and exposure to the HOTS course.

Abosalem (2016) in his Paper *'Assessment techniques and students' higher-order thinking skills'*, tried to evaluate the teacher-made papers in Mathematics with respect to the cognitive skills as described in Bloom's taxonomy. Teacher-made tests constructed for the topic 'Fractions' consisting of five sections/content areas; was administered to grade 8 and 9 students. The results showed that all test items aligned and measured only the first three levels of Bloom's taxonomy. Around 57% of the test items were applications and 28% were recognition items. The research indicated that the tendency of the assessment methods used in schools was inclined more towards recall information and to do routine questions.

Cao (2018) conducted a Study, '*Teachers' capacity of instruction for developing higher-order thinking skills for upper secondary students – A case Study in teaching Mathematics in Vietnam'*. In this Study 60 secondary school Mathematics teachers of three provinces of Vietnam were surveyed to find their perceptions on developing HOTS in students; the difficulties they faced while implementing HOTS; and actually the extent to which they were implementing HOTS instructions. Questionnaires, classroom observations and discussions with teachers and students were used for collecting data. The findings revealed that majority of the teachers believed that implementing instructions for developing HOTS was necessary; but only 18% could do it. Fifteen percent of teachers accepted that they did not understand the methods to develop HOTS and thus never used them. Teachers' capacity to use higher order questions were not proportional to their knowledge of Mathematics. Problem-solving capacity of teachers was not good and they faced difficulty in using real-life examples to make lesson plans. The Study emphasized the need for training and retraining of instructions for HOTS for secondary school teachers.

2.6.1 Implications of the 'studies on teacher-related difficulties in using effective strategies in teaching mathematics' on the present study

Several Studies conducted in India as well other countries like England, Australia, U.S.A., Malaysia, Turk, Vietnam reveal the inefficiency of Mathematics teachers to transact effective teaching approaches that promote higher order thinking among students. Studies of

Tebbs (2000); Cao (2018); Surender (2015); Thompson (2008); Chinedu et. al. (2015) reveal, that teachers are least efficacious towards teaching HOTS and transfer of thinking tasks. Teachers face difficulty in interpreting thinking skills even from Bloom's taxonomy which is the most prevalent cognitive theory taught in teacher-training courses; thus making it difficult for them to create test items for HOTS (Thompson, 2008; Abdullah et al., 2016; Cao, 2018; Sikdar & Poddar, 2015; Goethals, 2013; Abosalem, 2016). Questioning included within instructions also is limited only to responses that are recall or routine type (Cao, 2018; Patel, 2007). Even the required mathematical knowledge in teachers of middle and secondary grades were found to be below average by Abdulla et. al. (2016); Eli et. al. (2013); Bhagowati (2011); Surender (2015), which is an essential pre-requisite to design HOTS instructions. Teachers faced difficulty in using real – life examples and lacked problem solving capacity (Cao, 2008). Majority of the teachers do not transact instructions that aligns to HOTS strategies, although some perceive that they do so (Alwaris et al., 2016; Tajudin, 2015; Cao, 2018), with majority transacting only procedural knowledge restricted to textbooks (Dewan et al., 2012). Majority of the teachers do not have a clear understanding of Bloom's taxonomy and are not able to use it effectively to create tests (Thompson, 2008; Alwaris et al., 2016; Abosalem, 2016). Prospective Mathematics teachers had below average estimation skills and number sense (Senol et al., 2015). According to Paul (2017) in his study in Kerala, Mathematics teachers are aware of the outcomes of transacting instruction that focus on logical thinking and reasoning, but are reluctant to use the same due to the difficulties involved in the implementation and other constraints (syllabus, time etc.). Tebbs (2000); Ingvarson et. al. (2006); Sikdar and Poddar (2015) emphasize the need of training the pre-service and the in-service teachers with the processes of transacting instructions to develop HOTS.

The understanding derived from the above set of Studies, mark great significance for the present Study, which was an exploration of methods for teaching mathematics effectively. The outcome would provide lighthouse to the researcher to frame ways to train the mathematics teachers for effective teaching.

2.7 Studies Highlighting the Difficulties Involved in the Teaching and Learning of Real Numbers

Merenluoto and Lehtinen (2002) conducted the Study, *'Change in Mathematics: understanding the Real numbers'*. The purpose of the Study was to find the difficulties faced by the students to transit conceptually from logic that they applied for natural numbers to the enlarged concept of Real numbers. A test was administered to 564 students from randomly

selected Finnish upper secondary schools. Majority of the students had just finished their first calculus class. The test included identification, classification and construction problems of Rational and real numbers. The results indicated that the changes in the number conceptions were not adequately carried out by majority of the students. While working with the Real numbers, students spontaneously used the logic and general rules of natural numbers. The number concept of majority of the students seemed to be based on the logic of natural numbers; moreover they seemed confident about their knowledge even though that was erroneous.

Voskoglou and Kosyvas (2012) conducted the Study, '*Analysing students' difficulties in understanding real numbers'*. This Study is based on the understanding of Real numbers of high school students and technology students (prospective engineers and economists). A properly designed questionnaire was administered to 78 students of grade IX students in a school in Greece and to 106 students of Graduate Technological Educational Institute (T.E.I) of Patras. The results of this experimental data revealed that many students had incomplete understanding about Rational numbers which was an obstacle in the comprehension of Irrational numbers. The T.E.I. students showed complete weakness to deal with geometric constructions to represent Irrational numbers on real axis. Considerable number of students could not comprehend the density of Rational and Irrational numbers in a given interval. Further a qualitative Study was conducted to penetrate deeper and find the reasons that made the students respond to the questionnaire. Interviews of 10 of the high-school students and 10 of the T.E.I. students were taken, which revealed that students had a number of wrong beliefs regarding multiple representations of numbers and the operations and processes applied on them; and this was due to the formal and algorithmic accumulation of knowledge. In order to eliminate the misconceptions, the Study suggested that teaching should emphasize the use of multiple representations of real numbers and flexible transformations among the representations.

Belin and Akar (2017) conducted the Study, '*Prospective Mathematics teachers' making sense of the decimal representation of real numbers as Rational number sequences through quantitative reasoning'*. In this Study, classroom teaching experiments were conducted with 19 prospective Mathematics teachers in Istanbul about the decimal representation of real numbers with emphasis on quantitative reasoning. Video records and written artefacts were analyzed qualitatively. Results showed that thinking through quantities depicted in diagrams, once prospective teachers related long division with multiple representations of Rational numbers such as fractions, equivalent fractions and decimals

through the mental actions of equal partitioning, grouping and counting, they were able to deduce that all these representations corresponded to the same number. By squeezing the decimal representation of both Rational and Irrational numbers, prospective teachers were able to deduce that real numbers could be represented by the limits of Rational number sequences. This Study thus indicated that by providing task sequences and by showing how the difficulties regarding real numbers can be eliminated via focusing on quantities; the concept of real numbers can be taken up in education.

Obersteiner and Hofreiter (2017) conducted a Study, *'Do we have a sense for Irrational numbers?'*. The purpose of this Study was to investigate whether educated adults without training are able to assess magnitudes of Irrational numbers. Thus, a computerized experiment was conducted, in which mathematically skilled adults were asked to repeatedly choose the larger of the two Irrational numbers as quickly as possible. The responses needed reasoning about the Irrational number magnitudes. But the results suggested that participants hardly even tried to assess magnitudes of the Irrational numbers in their symbolic notation. Without practice, number sense seems to be restricted to Rational numbers.

Yilmaz and Ay (2018) conducted a Study, *'Exploring 8th grade students' skills and knowledge on Irrational numbers'*. The purpose of the Study was to determine the skills and knowledge of 8th grade middle school students on Irrational numbers. Ten 8th grade students from a public school in Ankara participated in the Study. Qualitative research design was used and the data was collected using clinical interviews. A number of concept-related (real numbers) efficiencies and difficulties of students got revealed from this Study. Analysis of the data revealed that students experience difficulty in defining Irrational numbers, identifying the relationships among set of numbers, explaining the difference between Rational and Irrational numbers, and performing operations with real numbers. They were relatively good in classifying numbers as Irrational numbers or not with proper explanations.

2.7.1 Implications of the 'studies highlighting on the difficulties involved in the teaching and learning of real numbers' on the present study

The Study conducted by Voskoglou and Kosyvas (2012) on around 180 students to examine their understanding of Real numbers could empirically inform us regarding a number of obstacles faced by students while comprehending different forms of 'numbers'. Multiple representations of numbers aroused confusion and incomprehension, as instructions focus only on their structural or mathematical meaning rather than their real-life meaning. Incomplete understanding of Rational numbers cause hurdle in the understanding of Irrational numbers. A good number of students could not understand the density of real numbers in a

given interval. Weakness in using geometric constructions to represent Irrational numbers on number line was also revealed in the Study Voskoglou and Kosyvas (2012). The Study of Yilmaz and Ay (2018) also revealed a number of difficulties that the students face in the topic of Real numbers. They faced difficulty in defining Irrational numbers; explaining the difference between Rational and Irrational numbers; identifying the relationship among different Numbering Sets; and performing operations with Real numbers (Voskoglou & Kosyvas, 2012). According to Merenluoto and Lehtinen (2002), students spontaneously used the logic and general rules of Natural numbers while dealing with Real numbers, specially while applying the mathematical operations, resulting into conceptual as well as computational errors. Obersteiner and Hofreiter (2017)'s Study with adults showed that while comparing Irrational numbers, participants did not use the strategy of comparison based on magnitudes; rather concluded on the basis of the symbolic notations of the given numbers. Thus teaching should emphasize on the use of multiple representations of Real numbers (Voskoglou & Kosyvas, 2012; Yilmaz & Ay, 2018; Obersteiner & Hofreiter, 2017). Belin and Akar (2017) needed to probe the prospective Mathematics teachers to use diagrams to see the quantified forms of fractions, equivalent fractions and decimal numbers, only then they were able to mentally use the strategies of partitioning, grouping and counting different numbers. Thus, to deal with the confusion of multiple representations of a number, students are needed to be equipped with strategies of either computing or estimating their magnitudes.

Thus, the present Study proposed to address the gaps and the problems that students generally face while studying Numbers belonging to different sets, specially the Rational and the Irrational numbers. The chapter 'Real Numbers' in class IX syllabus was thus considered for the content to be transacted in the present Study.

Thus, the literature reviewed in the above section, helped the researcher to identify the following more general or larger gaps; the more intricate ones are accounted and justified in the next section – Rationale of the Study. The broader gaps are:

(i) Students of the secondary and the undergraduate levels lacked higher level competencies.

(ii) Majority of the Mathematics teachers lacked competencies in terms of pedagogical transaction and positive attitude toward teaching.

(iii) Most of the studies conducted on Mathematics education in India so far, assessed Mathematics achievement in terms of scores instead of mathematical abilities.

(iv) A number of teaching strategies are identified to develop HOTS in students, but they are not practiced in a well integrated manner in Mathematics classrooms.

(v) The concept of Real Numbers is important to gain numerical efficiencies for students but, there are hardly any studies addressing this gap.

(vi) HOTS are primarily assessed in state-wide competitive examinations, but research proves the inefficiency of teachers and the school systems to promote the same among primary and secondary years of schooling.

2.8 Rationale of the Present Study

Mathematics has always been stigmatized to be the most difficult, complex and abstract subject to deal with. This is evident from the literature reviewed on the present status of Mathematics education in India, which indicates the teaching and learning of Mathematics to be unsatisfactory. Students face a number of difficulties to understand and excel in the subject and teachers face difficulty in finding out and implementing the right strategy effectively. Students at the secondary as well as the undergraduate level lack sufficient knowledge in almost all the concepts; have average levels of procedural, logical and creative competencies and spatial-abilities (Patel, 2007; Bhagowati, 2011; Arundhati, 2012; Sarma & Ahmed, 2013; Tularam et al., 2012; Yasoda, 2015; Shakila, 2011). Majority of the teachers lack teaching competencies in terms of content and pedagogical knowledge and lack positive attitude towards teaching (Bhagowati, 2011; Surender, 2015; Dewan et al., 2012). Teachers are not trained to implement the activity based and student-centred approaches effectively (Sikdar & Poddar, 2013; Rampal & Subramaniam, 2012). Even the required mathematical knowledge in teachers of middle and secondary grades were found to be below average by Bhagowati (2011); Surender (2015).

Mathematics achievement thus remains a concerned issue for the students of India. A number of teaching models and strategies have been devised and experimented through-out the years to improve the Mathematics scores. Cognitive processes like Direct Instructional model (Jayasree, 2014); Cognitively Guided Instructional strategy (Sunitha, 2015) and Creative Teaching Programme (Shah, 2011); RBC Model (Sunitha, 2017); Brain Based Learning strategy (Paul, 2017) and psychomotor methods like Laboratory Teaching Programme (Patel, 2009), Computer Assisted Instruction (Gururajan, 2013) were successfully tested to record enhancement in mathematical achievement.

But the goal of Mathematics education is not restricted to scores but to develop mathematical ability which comprises of both conceptual and procedural understanding of targeted concepts. Scholastic achievement is not a sure indicator of the actual mathematical ability of a student (Arundhati, 2012). Infact, cognitive characteristics like - mathematical creativity, intelligence, problem-solving skills (Madhusudan, 2015); multiple intelligence

(Sreeraj, 2005); visuo-spatial processing skills for problem-solving and estimation tasks (Sreeraj, 2005; Booth & Thomas, 1999; Garderen, 2006; Simms et al., 2016); ability to make mathematical connections (Mulbar et al., 2017; Ndiung & Nandi, 2018); mathematical estimation and computation skills (Dolma, 2002); and higher order thinking skills (Tanujaya et al., 2017; Jacob, 2012) – are found to be indicators of mathematical abilities, possession of which ensures better scores in standardized tests. Apart from these cognitive requirements, psychological traits like motivation (Meece, 2003); external regulation, introjected regulation, academic regulation and study skills like listening, note-taking and reading-writing skills (Kamalam, 2015); socio-economic status (Neetu, 2013) also play important roles in mathematical achievement. External characteristics like a dynamic classroom environment, rich lesson design ingrained with real-life learning activities; technology; use of imagery in instructions; and multidimensional views were also reported to be essential for enhancing mathematical abilities of students (Atallah, 2003; Haji et al., 2016; Rose & Saloman, 2017; O'Dwyer et al., 2008; Gianquinto, 2007; Rodd, 2010; Gomen-Chacon, 2013).

It is high time now that Mathematics education in India should focus on its actual goal of developing the mathematical abilities rather than just focusing on scores. Countries like U.S.A., Australia, Malaysia, Finland had their Mathematics curriculum designed to develop problem solving, reasoning and proof, communication, connections, and representation skills. The National Curriculum Framework of India NCF 2005 also propagates Mathematics education as a tool to develop higher order thinking skills in individuals. It indicates clear effort to pull out Mathematics from the drudgery of structure, procedure and algorithms into more real, soluble and relevant forms. It suggested pedagogies like formal problem solving, use of heuristics, estimation & approximation, use of patterns (generalization), visualization, representation, reasoning & proof, making connections, mathematical communication.

The literature reviewed includes a number of strategies that have proved successful in developing higher order thinking skills through Mathematics instructions. Cognitive strategies like organizing and presenting materials in small steps, checking student understanding, eliciting student participation (Jayasree, 2014); identifying and integrating mathematical connections (Ormund, 2016; Eli et al., 2013); visualization techniques (Gianquinto, 2007; Rodd, 2010; Rosen & Saloman, 2007; O'Dwyer et al., 2008; Weiss & Bordelon, 2011; Gomen-Chacon, 2013; Ho Siew Yin, 2009); estimation techniques (Volkova, 2005; Zhu et al., 2017); mathematical connections using concept mapping (Ormund, 2016; Eli et al., 2013; Kadir & Fatra, 2017) have proved effective in developing higher order thinking skills in students.

Constructivist strategies like active engagement of students in doing Mathematics (Protheroe, 2007; Miri et al., 2007), posing challenging problems, making interdisciplinary connections, using multiple representations, using heuristics (Protheroe, 2007); presenting real-world cases, guiding for short inquiry type experiment (Miri et al., 2007); asking higher-order questions to help students to conjecture, invent and solve problems through instructions (NCTM, 2012; Wetzel, 2013; William, 1999; Yee, 2000; Foster, 2004; Mok et al., 2008; Ratner & Epstein, 2009; Emerson, 2010; Wichelt, 2009; Adam, 2011); and formative assessments (De Sousa, 2007) that target deeper conceptual understanding in students- have been researched and proved to be successful in developing higher order mathematical thinking abilities in students. Mathematical generalization processes involve higher-order cognitive processes like analysis, synthesis and abstraction as indicated by Dumitrascu (2017); Ley (2016); Carraher et. al. (2007). Generalization techniques can be integrated with classroom instructions to promote higher order thinking (Ley, 2016; Riveria, 2007). Jurrow (2004) emphasized the need of teacher guidance to generalizing techniques for effective learning.

Varghese (2009); Kirchner, Sweller and Clark (2006); Himmouri (2016); McCarthy (2016) and Gururajan (2013) strongly suggested the Guided discovery method as the most successful method for transaction of Mathematics content. There are two methods suggested by Nagappan (2002) to teach 'thinking skills' to students; first – the direct method where thinking skills are taught without the support of subject-content and second – the integrated method where thinking is promoted by integrating it with subject-content. The second method was chosen over the first for the present Study as it not only abided to the curricular time-limits but also eased out the cognitive burden of first internalizing the thinking skills theoretically and then applying the same with content-matter.

The content-matter chosen for the integrated method to be designed and implemented for the Study was 'Real Numbers' of the class IX syllabus of the GSHSEB Board. This Chapter included the cumulative study of all the Numbering Systems studied so far by students and also led them to the abstract realm of Irrational numbers and its operations. It included use of algebraic functions and geometric representations. The Chapter also forms an important basis for the concepts to be studied under calculus in the future. From the literature reviewed, the researcher could find very few Studies that made an attempt to deal with the topic 'Real numbers'. Most of the Indian Studies on Mathematics education used standardized models to redesign topics for upper primary classes like Fractions, Decimals & Percentage (Jayasree, 2014); Volume of rectangular prisms & Decimal numbers (Sunitha,

2015). Shah (2011) used a model to design all the topics of class VII of the Gujarat Board syllabus and Paul (2017) redesigned the topics- Unchanging relations, Repeated multiplication, Area of triangle based on the BBL and CoL models. Sunitha (2017) used the RBC model to redesign the class IX topics – Polynomials and Circles. De Sousa (2007) studied the effects of formative assessment tasks for the topic Algebra on secondary and higher secondary students. Other studies reviewed were conducted out of India. Instead of using standardized models they used certain strategies with a goal of developing higher order thinking skills. Pitkaniemi and Hakkinen (2012) used strategies like instructional coherence, cognitive activation and emotional support for the topic Decimal fractions. Apino and Retnawati (2017) from Indonesia developed and tried an instructional design on class X topics using components like – involving students in non-routine problems; facilitating the ability in them to analyse, evaluate and create; and encouraging them to construct their own knowledge – to develop higher order thinking skills. This inspired the researcher to use specific instructional strategies (mentioned in NCF 2005) to redesign the topic 'Real numbers' of Class IX, rather than taking up an standardized model for the same.

From very few Studies that were conducted on the topic Real numbers, there were none that attempted to design instructions for teaching the concept. Voskoglou and Kosyvas (2012); Yilmaz and Ay (2018) and Mereluoto and Lehtinen (2002) were the only studies conducted to find the difficulties of students for the concepts of Real numbers. Studies of Belin and Akar (2017) on prospective teachers, and Obersteiner and Hofreiter (2017) on educated adults revealed that samples had limited comprehension of magnitudes of Irrational numbers and other representations of Rational numbers. Dolma (2002) conducted a Study to reveal that students were better in estimating and computing Whole numbers rather than the Rational numbers. Simms et. al. (2016), emphasized that number-line tasks not only enhanced the understanding of numbering systems but also enhanced visuo-spatial and visuo-motor integration skills. Most of the Studies conducted on generalization processing skills were done on the topic of Algebra (Jurrow, 2004; Carraher et al., 2007; Riveria, 2007). Thus, the present Study would be the first of its kind to address the challenges by offering solutions to teach the subject-content 'Real Numbers' by integrating opportunities that promote thinking skills.

After the designing and the implementing procedures for the Study were decided, the next issue was that of the 'Assessment'. While scores could be used to measure 'achievement'; the challenge was that of measurement of 'higher order thinking skills'. With variable interpretations of HOTS circulating in the educational scene, it was important to

choose the simplest, prevalent, and a well-known one. Bloom's taxonomy seemed to be the best option as this framework was taught in the teacher-training course (part of B.Ed. curriculum) to help teachers design learning objectives and blueprints for constructing test items. Generations of teachers have been using this taxonomy to design objectives and assessments. However, the extent to which these skills are taught and assessed continues to be an area of debate (Collins, 2014). Teachers still lack ability to create questions of the analysis, synthesis, evaluation and creation levels of Blooms taxonomy. This is an irony to the fact that students need to be trained to respond to HOTS questions which generally are a part of State-wide and Competitive examinations. Students need to be consistently exposed to such assessments to develop the right mindset and cognitive skills required to respond to HOTS questions.

This implies that teachers play a very important role in leading students towards mathematical efficiency and higher order thinking. The question is, are the teachers efficient enough for the task? Do we have the systems in place that would aid teachers with this task in India? A number of Indian and International studies respond to the first question with a negation (Tebbs, 2000; Cao, 2018; Surender, 2015; Thompson, 2008; Chinedu et al., 2015; Abdullah et al., 2016; Sikdar & Poddar, 2015; Goethals, 2013; Abosalem, 2016; Patel, 2007; Eli et al., 2013; Bhagowati, 2011; Alwaris et al., 2016; Tajudin, 2015; Dewan et al., 2012; Senol et al., 2015). These studies reveal that, teachers' efficiency levels in terms of mathematical knowledge, pedagogical transactions to teach HOTS and ability to transfer 'thinking' is below average. In India, a number of Studies (Paul (2017); Sunitha (2017); Sunitha (2016); Rampal and Subramaniam (2012) indicate that it is only in Kerala that some activity-based and student-centred approaches are being used effectively. This is evident from the Class X State Board exam results 2018, which showed 97.84% pass rate in Kerala against a mere 51.47 % in Gujarat.

In response to the second question, one of the steps taken by the Government is the designing of the NCERT textbooks to provide some ease to the teachers to execute the pedagogies mentioned in the NCF 2005 to some extent. These textbooks which were used in all CBSE schools in India in the secondary and higher-secondary classes; are recently mandated to be used for the secondary classes in the Gujarat State Board schools. In spite of this, the teaching inside the classroom has hardly come out of the mechanical algorithmic approach (Rampal & Subramaniam, 2012; Dewan et al., 2012). Even in Kerala where the situation is better, the Mathematics teachers are reluctant to use constructivist approaches in the classroom due to difficulties involved in the implementation and other constraints (Paul,

2017). Systemic limitations in terms of class strength, vast syllabus, insufficient resources, lack of preparation, and time restrictions are causing impediments (Sarma & Ahmed, 2013; Sikdar & Poddar, 2015; Rampal & Subramaniam, 2012) in the transaction of higher level instructions in Indian classrooms. Teachers mentioned their reasons for not taking up constructivist approach in classroom as: Heavy workload; unavailability of suitable problem solving activities; the approach that helped students score well in standardized tests dominated more; time available to teach Mathematics was less (McCarthy, 2016).

Based on – (1) the literature reviewed; (2) the researcher's teaching experience; (3) Mathematics classroom observations of five experienced and ten fresh Mathematics teachers for a period of one month; and (4) interviews with Mathematics teachers and school principals; the following statements in general can be made:

- Most of the teachers introduce their lessons with examples; hardly focusing on counter examples, comparisons or misconceptions.
- Most of the concepts are taught in haste, giving no time for the students to think, and introspect on their understanding.
- Previous knowledge of the students is generally checked only about facts learnt in the previous class through some direct questions or teacher statements. There seems a lack of preparation from the teachers to create a strong base before imparting new knowledge.
- Teachers hardly pose any higher-order open-ended questions to students.
- Teachers generally stick to the sequence of topics and subtopics present in the text books and focus only on the sums given in the text book.
- Connections within subtopics or other Mathematics topics are not very efficiently established through the Mathematics instructions in the classroom.
- Activities are done not to enrich instructions or complement student understanding but only for 'Formative Assessment' grading purposes.
- Students are hardly given any opportunity to conjecture or to discover a Mathematics property.
- Use of ICT in Mathematics is negligible.
- Teachers fail to give a holistic understanding or future implication of concepts taught.
- Assessment sheets generally have sums similar to the ones given in text book which can be solved in an algorithmic way.

- The content analysis of the topic Real numbers of the Class IX NCERT and the GSHSEB textbook revealed a number of loopholes in terms of sequencing of the sub-topics, lack of connectivity, lack of conceptual clarity, lack of pedagogical aid for teachers.

The above observations are amply supported by research done in this area. However, the teachers are not to be blamed for the non-execution of effective teaching strategies; lack of enough knowledge/training at the B.Ed. level, lack of an extensive in-service programme, lack of required competencies and content mastery are some of the reasons. Besides, there are various practical difficulties as well, as mentioned in Sarma and Ahmed, 2013; Sikdar and Poddar, 2015; Rampal and Subramaniam, 2012; McCarthy, 2016. Thus, the Mathematics teachers need help to transact good quality education in their classrooms.

This effectively justifies the fact that Mathematics education in Indian classrooms need a rigorous makeover from its conventional procedural format to forms that support acquisition of conceptual knowledge and promotion of mathematical thinking among students. The need becomes stronger at the secondary levels when Mathematics becomes more abstract in nature and assessments are more targeted to check the cognitive skills of application, analysis, synthesis, evaluation and creation. Students of this age group reflect maturity to handle abstract ideas and can be guided towards higher levels of thinking, self-learning and self-assessment techniques. Also they need to be trained for the forthcoming State-wide Board examinations and the National level Competitive examinations which ensure success to those with higher order thinking skills.

Thus, to sum up the ideas presented in the above sections, the present Study was designed to experiment with a proper blend of strategies that can be used by Mathematics teachers using guided discovery methods to transact a secondary level topic ‘Real Numbers’. The topic has been reported to arise a number of difficulties in students due to its structural format. The intervention would offer scopes for mathematical thinking to students throughout the journey and finally test whether it helped in developing mental competencies (Basic and Higher level) and achievement (scores) in tests made up of HOTS questions specific to the content ‘Real Numbers’. With this purpose, an Instructional Package was developed and implemented by the investigator.

An elaborate description of its development and methods adopted for its development and implementation is described in the next Chapter.