DEVELOPING, IMPLEMENTING AND ASSESSING AN INSTRUCTIONAL PACKAGE FOR HIGHER-ORDER-THINKING-SKILLS IN MATHEMATICS

A

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CHAPTER VI

SUMMARY, MAJOR FINDINGS, DISCUSSION AND SUGGESTIONS

6.0 Introduction

According to John Dewey (1933), "the productive process of thinking is a sequenced chain of events, it begins with reflection, moves to inquiry, then to critical thought process, leading to a conclusion which is further substantiated with processes that go beyond personal beliefs and images" (King, Goodson, & Rohani, 2017). These thinking processes are analogous to the ones described by Bloom within the cognitive domain. Thinking cannot be unmeshed from learning, though learning can be considered as an outcome of thinking (King, Goodson, & Rohani, 2017). Learning that occurs as a product of the use of lower order thinking skills is limited only to the prescribed context, whereas the practice of higher order thinking skills generate learning outcomes that can be utilized in variable contexts, for challenging problems and for real life situations. Learning outcomes that result from the practice of higher order thinking skills happen to be the major goal of education. Thus, teaching strategies that encourage the use of these skills among students are required to be designed and practiced.

Mathematics is a subject that includes content matter that offers unlimited scope to its learners for higher order thinking. The cognitive processes involved while learning Mathematics is termed as 'mathematical thinking' that pursues clear thinking with perseverance that leads to logical conclusions through logical processes, along with the capacity to handle abstractions (NCF, 2005; NCTM, 2000). But this mathematical thinking is not always a natural consequence of the formal Mathematics education offered in schools. Infact there is a need to consciously develop and implement such pedagogies that target mathematical thinking. The NCF (2005) proposed pedagogical processes like *formal problem solving, use of heuristics, estimation and approximation, generalization, visualization, representation, reasoning and proof, making connections, mathematical communication* for Mathematics classrooms. Mathematics teaching is thus expected to train minds for higher-order thinking. The previous century acknowledged the pedagogical shift in transaction of mathematical content from pure procedural forms to forms that target such thinking. The present century is witnessing the transition with most of the countries adopting 'development of higher order thinking skills' as one of their educational goals.

6.1 Rationale of the Present Study

It is high time now that Mathematics education in India should focus on its actual goal of using the subject of Mathematics to develop higher cognitive abilities of students rather than just focusing on scores. Countries like U.S.A., Australia, Malaysia, Finland had their

Mathematics curriculum designed to develop problem solving, reasoning and proof, communication, connections, and representation skills. The national curriculum of India NCF (2005) and NEP (2019) also propagates Mathematics education as a tool to develop higher order thinking skills in individuals. It indicates clear effort to pull out Mathematics from the drudgery of structure, procedure and algorithms into more real, soluble and relevant forms with its suggested pedagogies.

The literature reviewed includes a number of strategies that have proved successful in developing higher order thinking skills through Mathematics instructions. Cognitive strategies like organizing and presenting materials in small steps, checking student understanding, eliciting student participation; identifying and integrating mathematical connections; visualization techniques; estimation techniques; mathematical connections using concept mapping have proved effective in developing higher order thinking skills in students.

Constructivist strategies like active engagement of students in doing Mathematics, posing challenging problems, making interdisciplinary connections, using multiple representations, using heuristics; presenting real-world cases, guiding for short inquiry type experiment; asking higher-order questions to help students to conjecture, invent and solve problems through instructions; and formative assessments that target deeper conceptual understanding in students- have been researched and proved to be successful in developing higher order mathematical thinking abilities in students. Generalization techniques can be integrated with classroom instructions to promote higher order thinking. Many research studies strongly emphasize the need of guided and informed instructions by teachers to aid students towards investigations to discover the concepts and do higher order thinking.

In India, a number of studies indicate that only in Kerala some activity-based and student-centred approaches are being used effectively. This is evident from the Class X State Board exam results 2018, which showed 97.84% pass rate in Kerala against a mere 51.47 % in Gujarat. Also the NCERT textbooks are designed to help teachers execute the pedagogies mentioned in the NCF 2005 to some extent. These textbooks which were used in all CBSE schools in India in the secondary and higher-secondary classes; are recently mandated to be used for the secondary classes in the Gujarat State Board schools. Inspite of this, the teaching inside the classroom has hardly come out of the mechanical algorithmic approach. However, the teachers are not to be blamed for the non-execution of effective teaching strategies; lack of enough knowledge/training at the B.Ed. level, lack of an extensive in-service programme, lack of required competencies and content mastery are some of the reasons. Systemic limitations in terms of class strength, vast syllabus, insufficient resources, lack of preparation, and time restrictions are causing impediments in the transaction of higher level instructions in Indian classrooms.

From the literature reviewed, the researcher could find very few studies that made an attempt to deal with the topic ‘Real numbers’. Most of the Indian studies on Mathematics education used standardized models to redesign several Mathematics topics. Most of the Studies on Mathematics education conducted out of India, used ‘teaching strategies’ instead to develop higher order thinking skills in students. This inspired the researcher to use specific instructional strategies to redesign the topic ‘Real numbers’ of Class IX, rather than taking up a standardized model for the same. The topic ‘Real numbers’ was investigated by a few researchers with respect to the difficulties it poses for the students, there were none that attempted to design instructions for teaching the concept.

The researcher thus, developed a package which aligned to the processes envisaged in the NCF 2005, tailored as per Indian classroom conditions. It could be used as a tool by Mathematics teachers to transact instructions in forms that encourage students for higher-order thinking with respect to the content ‘Real Numbers’. The package included well-structured lesson plans; ready-made teacher and student resources; assessment sheets. It was prepared meticulously on the basis of appropriate and effective instructional methods taking into consideration the practical feasibility of Indian classrooms. The Instructional Package thus developed was implemented on IX standard students, who fall under an age group that has maturity to handle abstract ideas and can be guided towards higher levels of thinking, self-learning and self-assessment techniques.

The techniques and methods used to conduct the research Study is elaborated in the following sections.

6.2 Statement of the Problem

The Study was titled as “Developing, implementing and assessing an instructional package for higher order thinking skills in Mathematics”.

6.3 Objectives and Hypotheses of the Study

1. To develop an Instructional Package on the content ‘Real Numbers’ in Mathematics for class IX students.
2. To implement the Instructional Package on class IX students.
3. To study the effectiveness of the developed Instructional Package over the Conventional method of teaching on the acquisition of higher order thinking skills in the content ‘Real Numbers’ in class IX students.

3.1 To study the effectiveness of the developed Instructional Package over the Conventional method of teaching on the acquisition of higher level competencies in the content 'Real Numbers'.

3.2 To study the effectiveness of the developed Instructional Package over the Conventional method of teaching on the acquisition of basic level competencies in the content 'Real Numbers'.

3.3 To study the effectiveness of the developed Instructional Package over the Conventional method of teaching in terms of the Mean Achievement scores for HOTS questions at specific levels - Comprehension, Application, Analysis, Synthesis and Evaluation in the content 'Real Numbers'.

3.4 To study the effectiveness of the developed Instructional Package over the Conventional method of teaching in terms of the Mean Achievement scores for HOTS questions including all levels in the content 'Real Numbers'.

4. To study the reaction of students on the developed Instructional Package and its implementation.

- **Hypotheses**

The hypotheses constructed for the Study (for Objectives 3.3 and 3.4) are as follows.

1. There is no significant difference between the Mean Achievement scores of the class IX students exposed to the Instructional package over the ones exposed to the Conventional method of teaching for HOTS questions at the Comprehension level in the content 'Real Numbers'.
2. There is no significant difference between the Mean Achievement scores of the class IX students exposed to the Instructional package over the ones exposed to the Conventional method of teaching for HOTS questions at the Application level in the content 'Real Numbers'.
3. There is no significant difference between the Mean Achievement scores of the class IX students exposed to the Instructional package over the ones exposed to the Conventional method of teaching for HOTS questions at the Analysis level in the content 'Real Numbers'.
4. There is no significant difference between the Mean Achievement scores of the class IX students exposed to the Instructional package over the ones exposed to the Conventional method of teaching for HOTS questions at the Synthesis level in the content 'Real Numbers'.
5. There is no significant difference between the Mean Achievement scores of the class IX students exposed to the Instructional package over the ones exposed to the Conventional

method of teaching for HOTS questions at the Evaluation level in the content ‘Real Numbers’.

6. There is no significant difference between the Mean Achievement Scores of the students exposed to the Instructional Package over the ones exposed to the Conventional Method of teaching for HOTS questions of all levels in the content ‘Real Numbers’.

6.4 Explanation and Operationalization of Terms

1. Effectiveness

For the present Study, ‘Effectiveness’ is the degree to which the developed Instructional Package is successful in developing higher order thinking skills in students who were exposed to the Package over the ones exposed to the Conventional method of teaching.

2. Instructional Package

For the present Study, the Instructional Package refers to a systematic instructional design involving effective teaching strategies and assessment procedures. It will include Student Learning Materials, Worksheets, Practice sheets, Evaluations with HOTS questions; Content- Chart, Lesson plans, Power-point presentations, Scoring criteria and Rubrics – all systematically structured for the selected Mathematics content ‘Real Numbers’ of class IX.

3. Effective teaching strategies

Different Teaching strategies that were integrated with the content were –

Cognitivist Teaching Strategies; Use of Mathematical Connections; Use of Questioning and Probing skill; Use of Generalization techniques; Use of Estimation techniques; Use of Visualization techniques.

4. Conventional method of teaching

Conventional method of teaching Mathematics specifically indicates teacher-centred teaching with domination of the Lecture method. This method of teaching is commonly used in most of the schools and is characterized by the following features:

- Content is limited to the text books.
- Role of the teacher is to teach algorithms by providing clear, step-by step demonstrations of each procedure, recapitulating the same, providing adequate opportunities to students to practice the procedures, and offering specific corrective measures when necessary (Smith, 1996)
- The procedures to all mathematical problems are known, contexts are not changed in practice work and in assessments.

- Students are expected to memorize facts, follow rules, execute procedures, and plug in formulas (Hiebert, 2003).

5. Higher order thinking skills in Mathematics

Higher order thinking skills as defined in Bloom's Taxonomy (2001) is used for the present Study. The below mentioned cognitive skills are evaluated for the content 'Real Numbers', Class IX Mathematics (GSHSEB, CBSE).

- *Comprehension* is the ability to understanding information; grasp meaning; interpret facts; compare and contrast; order, group, infer causes
- *Application* is the ability to use information, methods, concepts, theories in familiar situations and solve problems using required skills or knowledge.
- *Analysis* is the ability to see patterns, organize parts, recognize hidden meaning and identification of components.
- *Synthesis* is the ability to use old ideas to create new ones, generalize from given facts, relate knowledge from several areas, predict and draw conclusions.
- *Evaluate* is the ability to compare and discriminate between ideas, make choices based on reasoned argument and verify value of evidence (Collins, 2014).

The assessment questions were devised to evaluate the above skills in the present Study. Higher order thinking skills considered here was the total score obtained by the students on comprehension, application, analysis, synthesis and evaluation level questions in the Posttest on the content 'Real Numbers' developed by the investigator.

6. Basic level competencies

Competency is a set of defined behaviors or skills that provide a structured guide enabling the identification, evaluation and development of the behaviors in students. In the present research the term 'Basic Level Competencies' refers to cognitive skills of:

- identification and application of concepts, theories and rules in known contexts;
- calculations (application of mathematical operations); and
- algorithmic procedure used in a mathematical problem from the content 'Real Numbers' at Class IX level.

7. Higher level competencies

For the present Study, 'Higher Level Competencies' refers to:

- comprehension of information, grasping of meaning, interpretation of facts, compare, contrast, order, group (in case of Comprehension level questions).

- use of information, use of methods, concepts, theories in new situations or unknown contexts to solve problems or make inferences (in case of Application level questions).
- identification of components, organisation of the components, recognition of hidden meaning to solve problem (in case of Analysis level questions).
- use old ideas to create new ones, generalise from given facts, relate knowledge from several areas, and draw conclusions (in case of Synthesis level questions).
- comparison and discrimination between ideas, making choices based on reasoned argument and verification of value (in case of Evaluation level question).

8. HOTS questions

HOTS is the abbreviation used for Higher order thinking skills and HOTS questions are those questions that focus on thinking skills measuring students' abilities to reason, justify, analyze, process and evaluate information besides testing understanding of information. These questions seek answers that go beyond the textbooks, widening the horizons of students. The responses for these questions need students to undergo mental skills of comprehension, application, analysis, synthesis and evaluation.

- **Operationalization of terms**

1. Achievement scores for different level questions:

- *Comprehension:* Marks obtained by students for their ability to understand information; grasp meaning; interpret facts; compare and contrast; order, group, infer causes
- *Application:* Marks obtained by students for their ability to use information, methods, concepts, theories in familiar situations and solve problems using required skills or knowledge.
- *Analysis:* Marks obtained by students for their ability to see patterns, organize parts, recognize hidden meaning and identification of components.
- *Synthesis:* Marks obtained by students for their ability to use old ideas to create new ones, generalize from given facts, relate knowledge from several areas, predict and draw conclusions.
- *Evaluate:* Marks obtained by students for their ability to compare and discriminate between ideas, make choices based on reasoned argument and verify value of evidence (Bloom' Taxonomy 2001 cited in Collins 2012).

6.5 Methodology of the Study

The present Study aimed to find out the relative effectiveness of the developed Instructional Package on the students' Achievement scores for HOTS questions and Higher order thinking skills in terms of Basic level competencies and the Higher level competencies of the students for the topic ‘Real numbers’ in Mathematics, over the Conventional method of teaching Mathematics. In the present study the investigator selected class IX students as sample to carry out the research. Experimental method was adopted to conduct the research. The design for the study was ‘Equivalent-Posttest-Control Group Design’.

- **Design of the Study**

According to Best & Kahn (2012), the Equivalent-Posttest-Control Group research design is one of the most potential Experimental design. The Matching Group Technique to create equivalent groups was used in the present Study. The two class-IX intact groups were made equivalent in terms of mean and standard deviation of some other variable (previous year Mathematics achievement scores) and then one of the group was randomly selected as the Experimental group and the other as Control group. A total of 72 students were considered as samples, out of which 36 students belonged to each the Experimental group and the Control group. The Experimental group was taught by the developed Instructional Package and the Control group was taught by the Conventional method of teaching Mathematics. The Chapter taken for experimentation was ‘Real Numbers’ of class IX text book following the GSHSEB State syllabus.

- **Population of the Study**

The population of the Study consisted of all IX standard students of English medium schools of Vadodara city following the GSHSEB syllabus in the year 2017.

- **Sample of the Study**

IX standard students of one English medium school of Vadodara following the GSHSEB syllabus was selected purposively as sample for the Study. The selection of the school for this Study was done considering the projector facility in the classroom and the readiness of the school to share facilities required for the Study. Seventy-two secondary students were selected as samples for the present Study. Out of which thirty-six students belonged to the Experimental group and thirty six belonged to the Control group.

- **Tools and materials used for the Study**

Achievement tests were developed by the investigator for different purposes.

1. Pretest (to prove equivalence of Experimental and Control group)
2. Evaluation 1 (Formative Assessment within Instructional Package)
3. Evaluation 2 (Formative Assessment within Instructional Package)
4. Posttest (Tool for Data collection)

5. Scoring Rubric for Posttest
6. Lesson Plans based on effective teaching strategies in Mathematics
7. Reaction Scale

6.6 Data Analysis Procedure

In order to respond to the sub-objectives 3.1 and 3.2, the responses for each of the fifteen Posttest questions of the students exposed to Instructional Package were compared to the respective responses of the students exposed to the Conventional method of teaching. The Investigator-made-Soring Rubric for the Posttest, that described the Sample Responses for all the fifteen questions and the basis on which each answer was scored for the Basic and the Higher level competencies, was used for this purpose. Each of the fifteen responses of the Experimental group students and Control group students were checked. Number of students who scored 2, 1 and 0 at the Basic level and 2, 1, and 0 at the Higher level were recorded and each was converted into percentage and graphs to visualize the comparative data of the Experimental and the Control group.

The sub-objectives 3.3 and 3.4 were responded through the comparison of the final Posttest Achievement scores of the Experimental and the Control group, using 't' test.

The independent t-test was applied on the Posttest scores of the Experimental and the Control Group- Firstly, to check the effectiveness at individual cognitive levels: Comprehension, Application, Analysis, Synthesis and Evaluation; and Secondly, to check the effectiveness holistically at all levels- among the Control and Experimental group.

The responses of the students to the Reaction scale (Objective 4) were analysed using frequency and Intensity Index to further judge the effectiveness of the Instructional Package.

6.7 Major Findings of the Study

The analysis and the interpretation of the data obtained from the Posttest responses of the students of the Experimental and the Control group proves the effectiveness of the Instructional Package over the Conventional method of teaching with respect to 'higher order thinking skills' and 'achievement scores' of students of class IX for the content 'Real Numbers'. The same can be indicated from the following findings.

1. Students exposed to the Instructional Package has performed better than the students exposed to Conventional method of teaching for the HOTS questions of **Comprehension level,** as

a. Number of students who have achieved Higher level competencies like understanding of information, grasping of meaning, interpretation of facts, compare, contrast, order, group-

completely in Experimental group was 27.5% in comparison to 5.1% in Control group and *partially* was 37.3% in Experimental group in comparison to 32.4% in Control group.

b. Number of students who have achieved Basic level competencies like identification and application of concepts, theories and rules; computations (calculations and algorithmic procedure) - *completely* in Experimental group was 49.5% in comparison to 25% in Control group.

c. There was a significant difference between the Experimental and the Control group students in their Mean Achievement scores for HOTS questions of Comprehension level, with the obtained 't' value 3.68 greater than the table value 2.04 at 0.05 level ($p<0.05$).

2. Students exposed to Instructional Package has performed better than the students exposed to Conventional method of teaching for **Application level** questions, as

a. Number of students who have achieved Higher level competencies like use of information, use of methods, concepts, theories in new situations to solve problems or make inferences- *completely* in Experimental group was 13.5% in comparison to 1% in Control group and *partially* was 33.6% in Experimental group in comparison to 14.7% in Control group.

b. Number of students who have achieved Basic level competencies like identification and application of concepts, theories and rules; computations (calculations and algorithmic procedure) -*completely* in Experimental group was 38.6% in comparison to 9.3% in Control group.

c. There was a significant difference between the Experimental and the Control group students in their Mean Achievement scores for HOTS questions of Application level, with the obtained 't' value 3.56 greater than the table value 2.04 at 0.05 level ($p<0.05$).

3. Students exposed to Instructional Package has performed better than the students exposed to Conventional method of teaching for **Analysis level** questions, as

a. Number of students who have achieved Higher level competencies like identification of components, organisation of the components, recognition of hidden meaning to solve problem- *completely* in Experimental group was 9.7% in comparison to 1% in Control group and *partially* was 20.5% in Experimental group in comparison to 9.4% in Control group.

b. Number of students who have achieved Basic level competencies like identification and application of concepts, theories and rules; computations (calculations and algorithmic procedure) -*completely* in Experimental group was 24.7% in comparison to 5.2% in Control group.

c. There was a significant difference between the Experimental and the Control group students in their Mean Achievement scores for HOTS questions of Analysis level, with the obtained 't' value 3.18 greater than the table value 2.04 at 0.05 level ($p<0.05$).

4. Students exposed to Instructional Package has performed better than the students exposed to Conventional method of teaching for **Synthesis level** questions, as

a. Number of students who have achieved Higher level competencies like use old ideas to create new ones, generalize from given facts, relate knowledge from several areas, and draw conclusions- *completely* in Experimental group was 7.2% in comparison to 0% in Control group and *partially* was 32.5% in Experimental group in comparison to 6.1% in Control group.

b. Number of students who have achieved Basic level competencies like identification and application of concepts, theories and rules; computations (calculations and algorithmic procedure) - *completely* in Experimental group was 26% in comparison to 8.3% in Control group.

c. There was a significant difference between the Experimental and the Control group students in their Mean Achievement scores for HOTS questions of Synthesis level, with the obtained 't' value 3.80 greater than the table value 2.04 at 0.05 level ($p<0.05$).

5. Students exposed to Instructional Package has performed better than the students exposed to Conventional method of teaching for **Evaluation level** questions, as

a. Number of students who have achieved Higher level competencies like comparison and discrimination between ideas, making choices based on reasoned argument and verification of value- *completely* in Experimental group was 7.5% in comparison to 1% in Control group and *partially* was 22.6% in Experimental group in comparison to 3.1% in Control group.

b. Number of students who have achieved Basic level competencies like identification and application of concepts, theories and rules; computations (calculations and algorithmic procedure) -*completely* in Experimental group was 16% in comparison to 2% in Control group.

c. There was a significant difference between the Experimental and the Control group students in their Mean Achievement scores for HOTS questions of Evaluation level, with the obtained 't' value 3.00 greater than the table value 2.04 at 0.05 level ($p<0.05$).

The results indicate that the students exposed to the Instructional Package performed better in Achievement test that focused on questions requiring higher order thinking skills, than that of the students exposed to the Conventional Method of teaching with respect to the Cognitive Levels – Understanding, Application, Analysis, Synthesis and Evaluation.

6. Students exposed to Instructional Package has performed better than the students exposed to Conventional method of teaching for all levels of HOTS questions, as

There was a significant difference between the Experimental and the Control group students in their Mean Achievement scores for all levels of HOTS questions in the Postest, with the obtained 't' value 4.53 greater than that of the table value 2.04 at 0.05 level ($p<0.05$).

7. The analysis and interpretation of the responses obtained from the Reaction Scale provided the following findings.

a. Students' reaction on the Instructional strategies implemented in the classroom:

The average Intensity Index was 4.25 for the statements related to the Instructional strategies like discussion of previous knowledge with reference to each sub-topic; detailed in-depth explanation of each concept; use of examples, counter-examples, contrasts, similarities in explanations; use of questioning technique to promote thinking; re-sequencing the topics for better links and holistic understanding; and enough time given for each sub-topic helped the students to understand the topic 'Real numbers' better than the usual form of teaching.

b. Students' reaction on their understanding on the different concepts and processes of the unit 'Real Numbers':

The average Intensity Index is 4.09 for the respective statements indicate that students have clearly understood all the concepts related to 'Real Numbers' along with the holistic meaning and structure of the Numbering system. They have understood the inter-connections between the different sub-topics of Real numbers. The Intensity Index is comparatively less for statement 11 (II = 3.84), indicating that some of the students have accepted that they understood complex aspects of Mathematics like estimation, proofs, verification and generalization with respect to the content 'Real numbers'.

c. Students' reaction on their feelings/perceptions towards the unit 'Real numbers' and towards the subject of Mathematics as a whole:

With an Intensity Index of 4.35, most of the students felt motivated and confident with the topic Real number and to go ahead with the further topics; but with an Intensity Index of 3.38, some students still feel Mathematics to be a difficult and complex subject.

d. Students' reaction on the Worksheets solved during the intervention period:

The average Intensity Index is 4.02 for the statements related to students' reaction on the Worksheets indicated that most of the students believed that the worksheets helped them to understand the topic and gave them chance to observe patterns and generalize.

e. Students' reaction on the Formative assessments - Evaluation1 and Evaluation 2:

The average Intensity Index is 3.92 for the statements related to the reaction of students related to the formative assessments indicated that the Evaluation 1 and Evaluation 2 gave scope to them to think at higher levels and motivated them to understand concepts rather than memorizing them.

f. Student's reaction on the overall Instructional Package and its implementation:

The Intensity Index is 4.35 for the statement regarding the reaction on the overall Instructional Package and its implementation indicated that most of the students found that the teaching, Worksheets and Evaluations helped them to look at Mathematics in a different way, which was logical, inter-connected and interesting.

6.8 Researcher's Observations during the Implementation of the Instructional Package

Although the Findings of the Study depict enhancement of the several competencies in students due to the Instructional Package, but there are certain facets and gaps in the content attainment in students that the researcher reflected on and hereby presents in this section. Some of the successes and failures while implementing the Instructional Package with reference to the different sub-topics of Real Numbers are included, which also gets reference in the Study of Voskoglou & Kosyvas (2012). Observations of the dynamics at the student level with regard to some of the sub-topics of Real Numbers are discussed below.

- **Relationship between different numbering systems N, W, Z, Q**

The Pretest responses and Classroom interactions revealed lack in the understanding of students regarding the Numbering systems and their inter-relations. Some specific ones are pointed out below.

- Students could define Rational numbers correctly but could not apply it.
- The set 'Integers' was considered to include only negative numbers and not positive numbers.
- Fractional and decimal numbers were misunderstood by many as Integers.
- Students were not sure whether 'Fractions and decimals represent 'complete' or 'part' of a quantity in their real-life representations'.
- Rational numbers meant only fractional and decimal representations, i.e. numbers like 3 or -3 were considered as Integers and not Rational numbers.

- Numbers like $\frac{3}{1}$ or $\frac{-7}{1}$ were considered to be Rational numbers but not Integers.
- The set 'Rational numbers' was considered to include only positive numbers not negative numbers.
- Students were theoretically aware about the relationship of the sets $N \subset W \subset Z \subset Q$ but while dealing with elements alone they are not able to visualize the multiplicity.
- Students lacked the mental skill of mathematical perseverance; a long written question would easily put them off from striving to comprehend it and go ahead to solve it.

Thus, with so many student variables working like – unclear comprehension of mathematical language; mental attitude that leads intellect towards effortful memorizing but not effortful analytical thinking; mental conditioning that sets Mathematics as an algorithmic subject; and with a hoard of overlapped concepts and sub-concepts to deal with, especially for this topic '*Numbering systems N, W, Z and Q*'- it indeed was a difficult task to lead students towards complete concept clarity. The task should begin right from the levels (classes) when the individual Numbering systems are introduced. The investigator explicitly pointed out the mentioned errors to the students during the implementation phase to help them register the same and work with caution.

- **Irrational Numbers**

Lesson plan 8 and Worksheet 5 was used to deliver this concept by the guided discovery method. To check the effectivity of the above teaching strategies for the topic 'Irrational Numbers', a few HOTS questions on the same were included in Evaluation 2 test paper. Although most of the students could theoretically state the properties and the difference between Rational and Irrational numbers, only six students really had concept clarity on the same; and could apply this knowledge in different contexts.

Some of the conceptual errors that were revealed from the incomplete/incorrect answers of few students are:

- All numbers with the square root symbol were considered as Irrational numbers.
- Confusion with the number $\sqrt{9.5}$, indicating it as a Rational number, with a faulty reason 9.5 was a terminating decimal number.
- *9.7532…* was indicated as an Irrational number by many students because it seemed to be a non-recurring decimal number (confused with the opaque representation).

The conceptual error that students make in comprehending the periodicity of Real numbers in their decimal forms was also highlighted in the study of Voskoglou & Kosyvas (2012), which mentions such semiotic representations of Real numbers as obstruction in the students' comprehension in differentiating Rational and Irrational numbers. The investigator then addressed some of these errors in the classroom interaction with a lot of focused probing helping students to observe underlying aspects. The normal tendency of students is to respond from what is visible at the surface level. Probing students for in-depth exploration of the topic on a continuous basis brought in better understanding of the topic.

- **Density of the Real number system:**

Voskoglou & Kosyvas (2012) attributes '*the property of denseness of the set R*' as one of the major hurdles in the comprehension of Real numbers by students. The major concepts included within are 'the converging and diverging property of the set R' and 'identification and listing of Real numbers that lie between two Integers'.

The Pretest responses, Classroom interactions revealed important conceptual blocks. Students lacked understanding on the following concepts:

- Proper fractions (that they represent Rational numbers that lie between 0 and 1); Improper fractions (that they represent Rational numbers that are greater than 1); and Improper fractions (that they are needed to be converted into their Mixed fractional form to comprehend, visualize or estimate their positions with respect to the non-converging set of Integers).
- Fractional and Decimal numbers (that they are different representations of the same numerical value).

  [The above stated concepts are needed to identify uncountable Rational numbers between two given Integers].

Lesson plans 3 and 4 and PPT slide 15 was used to address the above conceptual gaps. The next concept was that of identifying the position of Irrational numbers with reference to Integers in the set of Real numbers. It is mentally challenging to visualize the positions of given Irrational numbers with respect to Integers (Voskoglou & Kosyvas, 2012). Estimation strategy was used in the present Study (Lesson Plan 12) to deliver this concept. The responses to the test-items in Evaluation-2 proved that fifty percent of the students got a proper hold on the above mentioned concepts.

- **Representation of Irrational numbers on number line**

This was the most complex and abstract topic in the Unit - Real numbers (Yilmaz & Sonay, 2018; Schwarzenberger & Tall, 1978; Ely, 2010; Voskoglou & Kosyvas, 2012). Developing teaching strategies that could cater to its conceptual clarity as well to its algorithmic requirements was a challenge. Moreover, concept clarity with a number of previously learnt concepts like geometrical constructions; applications of Pythagoras theorem; concept of 'unit'- were pre-requisites. Integrating the same, with the concepts of Real numbers learnt so far in the Chapter and then explaining two different techniques- one to represent Irrational numbers of the square-root-of-Integer form- (like $\sqrt{5}$) and one of the square-root-of-decimal form- (like $\sqrt{5.2}$), was challenging. Expecting students to retain so much knowledge and then apply the same for higher order tasks seemed unrealistic. But the investigator made efforts to create all the relevant connections, fill up gaps that could be apprehended in student thinking and use student friendly approaches to design and implement this Concept. Inspite of devoting considerable time and energy behind this concept the Evaluation-2 test item responses revealed lack in concept clarity with the Hypotenuse Geometric method to represent Irrational numbers on Number line, especially for the larger numbers like $\sqrt{32}$, $\sqrt{85}$, etc. where multiple calculations of the value of 'hypotenuse' using the Pythagoras theorem as well as multiple representations on the Number line to reach the final result was required.

Thus, a suggestion for the teachers and the textbook developers would be to include only the 'Perpendicular Geometric method' to represent Irrational numbers on Number line in accordance to the Lesson plans 16 and 17.

- **Mathematical Operations on Real numbers**

In Conventional classroom teaching, this topic is limited only to the goal of developing computational skills of students using Irrational numbers in their square root representations. Only a number of algorithmic procedures that are limited to the scope of the textbook or sometimes in a prescribed Practice book constitute the teaching material as well as the teaching strategy. This results into inefficiency in performing operations on Irrational numbers (Yilmaz & Sonay, 2018). To address this major inadequacy in the transaction of this important topic, the investigator developed a number of Lesson plans and Worksheets to guide students' thinking and lead them through the concepts with maximum clarity.

Inductive method was used in Worksheets 11 and 13 to guide students to work-out, observe patterns and make generalizations regarding the applicability of the Commutative, Associative and the Closure property on Real numbers, specifically Irrational numbers. The concept of 'Mathematical operations on Irrational numbers' was explained with direct instructions in accordance to Lesson plans 21 and 22, where the analogy of Algebraic expressions with Irrational expressions was shown.

The main teaching strategy used for teaching 'the application of mathematical operations on Irrational numbers' was that of providing thoughtfully chosen examples for students to work-out, guiding them through the mathematical reasoning, cautioning them on probable errors, and along the process aiding students to gain computation efficiency on applying operations on Irrational numbers.

Classroom interactions and the Posttest responses revealed that most of the students had gained computational skills required for simplifying expressions with Irrational numbers and few of them could also use their analysis skill to see more connections.

6.9 Discussion

The development of the Instructional Package with its implementation within stipulated time period and an active participation from students, proved that the first two research questions (developing and implementing instructions involving cognitivist strategies, and constructivist strategies like questioning & probing, generalizing, estimating, visualizing, connecting mathematical ideas- using guided discovery methods) has been responded positively through this Study. The findings report the effectiveness of the Instructional Package over the Conventional method of teaching in terms of higher order thinking (Higher level and Basic level competencies) as well as achievement scores on HOTS questions. Thus the third objective of the Study is also responded in favour of the developed Package. The reaction of the students on the implementation of the Package proves its effectivity in students' understanding and interest. Thus, the envisaged objectives of blending several effective teaching strategies and proving its applicability in regular Indian Mathematics classrooms have been fairly achieved through this Study.

The Instructional Package allowed strengthening of the concepts: Relation between fractions and decimals; Representations of proper and improper fractions on Number line; the Relationship between BODMAS & algebraic operations with operations on Irrational numbers; Positioning of Irrational numbers on Number line with respect to Integers. A number of difficulties that students face in understanding these topics have been highlighted in the studies of Voskoglou & Kosyvas (2012); Yilmaz & Ay (2018); Mereluoto & Lehtinen (2002); Dolma (2002); Simms et al. (2016) – which could be to some extent addressed by the present Study.

The relative comparison of the Experimental and the Control group students proved that more number of students benefited from the Instructional Package rather than from the Conventional form of teaching in terms of Higher level competencies like

- understanding of information, grasping of meaning, interpretation of facts, compare and contrast, order and group;
- use of information, use of methods, concepts, theories in new situations to solve problems or make inferences

- identification of components, organisation of the components, recognition of hidden meaning to solve problem
- old ideas to create new ones, generalise from given facts, relate knowledge from several areas, and draw conclusions
- comparison and discrimination between ideas, making choices based on reasoned argument and verification of value.

As well as Basic level competencies like

- identification and application of concepts, theories and rules;
- calculations; and
- algorithmic procedure used in a mathematical problem from the content 'Real Numbers' at Class IX level.

Thus, though the present Study was conducted in limited time, the above mentioned basic level and higher level competencies could be considerably achieved by students, proving its relevance to be emulated by the teaching fraternity.

Most of the studies in Mathematics education conducted so far have highlighted on the Achievement scores for the comprehension, application, analysis, synthesis and evaluation levels (Sunitha, 2017; Paul 2017, Adams, 2011, Apino & Ratnawati, 2017); the present Study focused on the specific mental skills that define the above cognitive levels. Guided discovery methods used as pedagogical tool in most of the Lessons in the present Study proved successful as also indicated by Varghese (2009); Kirchner, Sweller & Clark (2006); Himmouri (2016); McCarthy (2016) and Gururajan (2013).

This Study stands out from other similar research studies in the area of Mathematics education in India that target development of higher order thinking skills because of the combination of teaching strategies that were used in the Instructional Package. Most of the studies used standard models (Samo et al., 2017; Apino & Retnawati, 2017; Sunitha, 2017; Montague et al., 2014) which may not comprehensibly satisfy the needs of different mathematical concepts most effectively. The present Study offers a liberty to choose from a number of teaching strategies – the one most appropriate to the respective concept, which may be blended with other strategies to result into effective learning and student thinking.

The investigator accepts the fact that designing such a package needs enormous time, energy, mathematical fluency, in-depth subject-knowledge, knowledge of several pedagogical tools, and most important - knowledge of student thinking processes. Thus, to transact such instructional processes, teachers need to be given both exhaustive and intensive training. Also, the implementation of the package can be successful if the teacher has the ability to motivate students and provide them with viable environments to practice mathematical thinking.

Some of the hurdles that the researcher had to face during the implementation phase is mentioned in the paragraphs below.

The Achievement test (Pretest) taken before the implementation of the Instructional Package revealed a considerable lack in the Numeric conceptual understanding and higher order thinking skills of students. The sample students who had been exposed to the traditional forms of teaching (totally procedure-based) throughout their school tenure (almost twelve years) were not conditioned for thinking tasks. Thus, it took considerable effort to develop the required previous knowledge and to direct the minds towards mathematical thinking and reasoning. A task, tough to fully achieve with time constraints.

Leading students towards in-depth exploration, beyond textbook contents or beyond mental abilities, sometimes left them uninterested. For example, the explanation of ‘Why for a Rational number p/q, if q is zero then p/q gets an infinite status?’ and the concept behind ‘Constructions to represent Irrational numbers on Number line’ remained successful for very few cases, evident from formative assessment responses. Most could gain efficiency in the procedure but without actual understanding, thus failing to respond when contexts were varied.

Completion of home tasks and individual learning was not always responded with required sincerity by the students, disrupting the plans.

Only the above-average students could go ahead with the thinking tasks and deliver appropriate conjectures like ‘Characteristics of Irrational numbers with respect to - decimals; Roots of non-perfect squares or cubes…; and Exponents’; ‘Correlation between operations of Real numbers and the Closure property’ etc.

Overall most of the students got motivated and interested in Mathematics when exposed to the Instructional Package, as it gave invariable scope of active mental participation. Leaving out an exception of five cases, most of the students showed complete involvement in the process. Though researcher tried to respond to the conceptual gaps and erroneous thinking processes of students, she was not successful in doing so completely and for all students.

But the Student feedback at the end of the experiment revealed that the students surely were now more aware of Mathematics as a subject in which every word, symbol, statement, definition, formula and activity had a logical background that is ingrained in the real world around them. Their minds were now more curious, looking for valid justifications for each statement made, trying to identify patterns, trying to make connections by visualizing and estimating (as revealed from their feedback). They do ask the questions more frequently now…Why? and How?

The developed Instructional Package was only for the content ‘Real Numbers’ taken across forty-eight sessions, so the result may not be generalized for all the topics of Mathematics of the class IX level. In order to generalize the result, all the topics of class IX Mathematics were needed to be designed as per the strategies used in the Package, and the same had to be implemented on students through-out the year.

6.10 Suggestions

The experiences of the present research, helped the investigator to put forward some suggestions for improving the quality of Mathematics teaching in the classroom. The suggestions are lined out for Mathematics teachers and for Policy makers. That will be followed by some suggestions for further research.

6.10.1 Suggestions for mathematics teachers

- The paradigm shift suggested by NCF 2005 has to be brought about by teachers by switching from complete procedural based teaching to a blended conceptual plus procedural based teaching; with a proper blend of student-centred and teacher-centred approaches.
- Teachers should adopt guided discovery methods of teaching Mathematics, skilfully guiding students to self-study, estimate, generalize, visualize, see the mathematical connections, predict, justify with proper reasoning and discover the mathematical concepts.
- Practice work designed for students need to focus on helping students not only to achieve basic computational skills, but also ready them for HOTS level questions.
- A continuous assessment and feedback mechanism needs to be adopted to eliminate conceptual errors acting as barriers for achieving full potential in Mathematics.
- In the process of fulfilling the demands of the subject, Mathematics teachers generally avoid the affective domain in the classroom exchanges, leaving back an environment not so conducive for Mathematics explorations to happen. It is very important for the teachers to give students the mental ease to go wrong and to approach the teacher with the silliest of doubts. Maximum learning in Mathematics happens from the mistakes made, as it opens up avenues for students to explore in-depth, across topics, across processes to find the solution and that is when they are doing higher order thinking.

- As for the topic Real Numbers, a number of suggestions are given in the Chapter 4 (Implementation of the Instructional Package) in terms of re-organization of the sub-topics, representation of Rational and Irrational numbers, estimation of the values of Irrational numbers with respect to Integers, geometrical representations of Irrational numbers on Number line, mathematical operations on Irrational numbers. The experiences shared in Researcher's Observation section and the Lesson plans, Worksheets, PPTs in the Instructional Package can be used for effective results in class IX.

6.10.2 Suggestions for policy makers

In order to excel in Mathematics, students not only have to gain efficiency in the mathematical language but also have to explore to decode its real life connections. They need to get a grip over the basic computations and the algorithms to bring about accuracy and precision; as well as tread their minds to understand the concept so that it can be applied in variable contexts for which they need to develop their analysis, synthesis and evaluation skills. The subject thus offers an excellent tool to exercise the higher order thinking processes of students, but there are certain limitations that are keeping away teachers to use the subject for its maximum benefit. They are:

- Large exhaustive syllabus
- Crowded classrooms
- Time limitations
- Design of Textbooks
- Teacher's inefficiency to design and implement balanced blended teaching strategies
- Most of the training or learning modules for teachers include only hands-on or activity based instructions, which if not implemented well leaves back conceptual gaps
- Assessments include application level questions but on similar lines as in textbook exercises

Based on the learning of the present research, the investigator makes some suggestions to the policy makers, as follows:

- Integrating the pedagogical practices with the content matter and then transacting the same in the classroom is a challenge for the Mathematics teachers who are absolutely conditioned to disseminate knowledge in the traditional mode. There needs to be regular, frequent and quality in-service training programmes for teachers for primary, upper primary, secondary and higher secondary levels including specific contents aligned with the best effective respective pedagogies. The present ones are mostly pedagogy based, where teachers do not get a proper hold of the content and the best fitting pedagogy to be used and thus enter classrooms to switch back to the conventional procedural easy method of teaching that does not support conceptual understanding among students.

- Teachers' instructional manual and students' set of worksheets (with guided discovery approach and practice) and self-study materials (with content related previous knowledge, common errors made, common misconceptions) can be prepared to help teachers provide quality teaching inside classrooms.
- Monitoring and rewarding Mathematics classroom teaching need be to an agenda to motivate teachers to really perform when unseen. Presently, student performance (marks) is the only parameter in which teachers are judged, and with assessments hardly focusing on HOTS; marks-oriented performance is generally due to procedure-based teaching which can be attained even by coaching classes.
- The aim with which Mathematics textbooks are designed are to be more teacher-friendly guiding teachers to teach. It also needs to harbour student-friendly goals, initiating students towards in-depth exploration of concepts and processes and challenge cognitions appropriately. For this the present syllabus has to be curtailed and reduced by atleast twenty percent.
- Development of higher order thinking skills is a goal of Mathematics teaching and learning and targeted efforts need to be made right from primary to the grade twelve level.
- The efforts must begin in the teacher-training institutions, with Mathematics training designed, implemented and monitored on the same lines as discussed in the present research.

6.10.3 Suggestions for further research

The effectiveness of the developed Instructional Package in terms of higher order thinking skills specifically for the concepts of Real Numbers indicate further implications of the teaching strategies used in the Package.

- All the Mathematics topics of the secondary section can be designed using similar teaching strategies so that teaching-learning can produce learning outcomes as envisaged by the NCF and as per the needs of the evolving generation.
- A longitudinal study can be taken up for the development of higher order thinking skills in Mathematics for the students of the upper primary levels (classes VI to VIII).

- Qualitative study, purely focusing on the student thinking processes with respect to the classroom interactions can also be conducted.
- A study based on observations of classroom practices of Mathematics teachers that are targeted to promote higher order thinking skills can be taken up.
- A study can be taken up to compare the higher order thinking skills of Mathematics students of the different boards – CBSE, GSHSEB, ICSE.
- Textbook analysis to check the inclusion of the pedagogies or strategies needed to develop higher order thinking skills can also be taken as a research work.
- Case studies of high achievers can be useful to understand study habits, thinking mechanisms and learning styles.
- A study that focuses on standardizing tools to develop HOTS promoting lesson plans and assessments can also be very useful.

6.11 Conclusion

Mathematics is a scientific subject; logically interconnected, sequentially organized and built on one's previous knowledge. Interconnections and contexts if ill established, large irreversible gaps are created and the child's progress is lost even if they have intelligence. A rich planned and organized set of instructions can change and mould a child's thinking- intense targeted efforts are thus needed in this direction. Instructions in the classroom need to be well planned and structured around the parameters that have proved to be successful in promoting higher-order thinking skills in students and an affective climate need to be maintained.

It can be concluded on the basis of the Implementation phase data in which the students were continuously being observed and assessed, that the Instructional Package created a positive difference in the students' mathematical thinking and in the understanding of the concepts taught. This was again substantiated by the analysed data, which vividly showed the impact of the Package on the higher and the basic cognitive skills of the students and their achievements in terms of marks for all level of questions (Bloom's taxonomy) in comparison to the effect that was caused due to the Conventional method of teaching.

Thus, the strategies used in the Instructional package can be used as a guiding literature for Mathematics teachers who aspire to develop higher order thinking skills in their students. The aim to develop higher order thinking to a broader spectrum would require all Mathematics topics to be designed and executed in lines with the Package through-out the school curriculum and collective effort of school administration and teachers to passionately act on it. The most important change needed is in the mindset - to shift from marks-oriented learning towards knowledge-oriented learning.